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Relativistic Study of the Recoil Energy of Photon due to Nuclear and Atomic Transition in Mossbauer Effect

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Abstract

In this paper, we have discussed the non-relativistic and relativistic treatment of the recoil energy of photon due to the nuclear transition and atomic transition and calculated the total energy of the recoil event for atomic and nuclear transition which concluded that the relativistic recoil energy of photon due to nuclear transition is 10^5 times greater than that of atomic transition.

Keywords: Nuclear resonance absorption, Nuclear transition and Atomic transition

1. Introduction

The Mossbauer Effect is a physical phenomenon discovered by Mossbauer in 1958 and involves the resonant and recoil free emission and absorption of Gamma radiation by atomic nuclei bound in a solid [1].

Mössbauer was the first person who observed this effect and used Gamma rays of energy 0.129 MeV, corresponding to the transition from ground to the first excited state. He was awarded by the Nobel Prize in 1961 for this work. This effect has also been used to verify the prediction of gravitational red shift. Pound and Rebka first carried out such experiments in 1959[2].

Petra Agota Silagyi studied the speciation of the ferric ethylene di-amine tetra acetate in aqueous solutions, its photochemical properties, its reaction with hydrogen peroxide and its solid state behaviour [3].

A.V. Kirichok, V.M. Kuklin and A.G. Zagorodny had shown that the intensity of the absorption and emission lines at the natural frequency ω_0 of the oscillator which significantly exceeds the intensity of other spectral lines, particularly at frequencies $\omega_0 \pm \Omega$ [4].
In this paper, the relativistic study of the recoil energy of photon due to nuclear transition and atomic transition has been studied.

2. Theoretical Discussion

The phenomenon of either emission or absorption of a gamma-photon without loss of recoil energy and without thermal broadening is known as Mossbauer effect [3]. This effect is used for the production of a monochromatic electromagnetic radiation with a very accurately defined energy spectrum so that it may be used to resolved minute energy differences gamma radiation belonging to a part of electromagnetic spectrum. The recoil energy increases the kinetic energy of vibrating nucleus and since the nuclei are bound in a lattice, a phonon may be emitted.

The uncertainties in energy and in time are related to Planck's constant $h = 2\pi\hbar$ by Heisenberg uncertainty principle [3].

$$\Delta E.\Delta t \geq \hbar \tag{1}$$

Since, the energy of a gamma (γ) ray quantum is of a few keV and energy of photon is responsible for atomic excitation of about few electron volt or less. The energy of emitted photon is defined as the amount of energy of emitted radiation (E_γ) equals to the magnitude of the recoil energy. According to the principle of conservation of momentum [3], we have

$$\frac{E_\gamma}{c} = Mv \tag{2}$$

Where M=mass after the recoiling process
v =speed of recoil mass
c= velocity of light.
or

or $$\frac{E_\gamma{}^2}{c^2} = M^2v^2$$

or $$\frac{E_\gamma{}^2}{Mc^2} = Mv^2$$

or $$\frac{E_\gamma{}^2}{2Mc^2} = \frac{1}{2}Mv^2$$

or $$\frac{E_\gamma{}^2}{2Mc^2} = E_R \tag{3}$$

where $E_R = \frac{1}{2}Mv^2 =$ Recoil kinetic energy of nucleus (4)

The eq^n (3) shows that the recoil kinetic energy of nucleus is directly proportional to the square of the gamma ray energy.
The term Mc^2 in the R.H.S. of equation (3) gives the total energy of recoil event and denoted by E_{tot}
Hence

$$E_{tot} = Mc^2 \tag{5}$$

Putting the above value in eq^n (3), we have

$$E_R = \frac{E_\gamma^2}{2E_{tot}} \tag{6}$$

Or $$E_\gamma = \sqrt{2E_{tot}E_R} \tag{7}$$

Or $$E_{total} = \frac{E_\gamma^2}{2E_R} \tag{8}$$

For the atomic transition [3].

E_γ =1 electron-volt.

E_R =10^{-11} electron-volt.

The total energy of the recoil event can be calculated with the help of equation (8) as follows:

$$E_{total} = \frac{1}{2\times10^{-11}} eV$$

Or $$E_{total} = 0.5\times10^{11} eV \tag{9}$$

Hence the total energy required to recoil event due to atomic transition is $0.5\times10^{11} eV$

For nuclear transition [3].

E_γ =10^5 electron-volt.

E_R =10^{-3} electron-volt.

The total energy of the recoil event can be calculated with the help of equation (8) as follows:

$$E_{total} = \frac{\left(10^5\right)^2}{2\times10^{-3}} eV = \frac{10^{10}}{2\times10^{-3}}$$

Or $$E_{total} = 0.5\times10^{13} eV \tag{10}$$

Hence the total energy required to recoil event due to nuclear transition is $0.5\times10^{13} eV$

Thus comparing the equation (9) & (10), we may conclude that the total energy of recoiling event needed for the nuclear transition is 100 times greater than to that of the atomic transition.

3. Relativistic study of recoil energy

From equation(3), we have

$$E_R = \frac{E_\gamma^2}{2Mc^2} = \frac{1}{2}\frac{E_\gamma^2}{Mc^2} \tag{11}$$

We know from the variation of mass with velocity [5,6].

$$M = \frac{M_0}{\sqrt{1-\frac{v^2}{c^2}}} \tag{12}$$

Or $$M = M_0\left(1-\frac{v^2}{c^2}\right)^{-1/2}$$

The above expression can be expanded by using the binomial theorem to obtain the following equation.

$$M = M_0\left(1+\frac{v^2}{2c^2}\right) \tag{13}$$

Putting the above value in the equation (11), we have

$$E_{Rel} = \frac{E_\gamma^2}{2M_0\left(1+\frac{v^2}{2c^2}\right)c^2} \tag{14}$$

$$E_{Rel} = \frac{E_\gamma^2}{2M_0c^2}\left(1+\frac{v^2}{2c^2}\right)^{-1} \tag{15}$$

Now using the binomial theorem for the expansion of above equation, we have

$$E_{Rel} = \frac{E_\gamma^2}{2M_0c^2}\left(1-\frac{v^2}{2c^2}\right) \tag{16}$$

$$\text{or } E_{Rel} = \frac{E_\gamma^2}{2M_0c^2} - \frac{E_\gamma^2 v^2}{4M_0c^4} \tag{17}$$

For atomic transition [3].

$$E_\gamma = 1eV$$

The equation (17) becomes

$$E_{Rel} = \frac{1}{2M_0c^2} - \frac{v^2}{4M_0c^4} \tag{18}$$

The equation (18) indicates that the relativistic recoil energy of gamma photon due to atomic transition slightly less than by amount $\frac{v^2}{4M_0c^4}$ to that of the non-relativistic recoil energy of gamma photon as clear from equation(11).

The equation (18) can be written as

$$\text{or } E_{Rel} = \frac{1}{2M_0c^2} - \frac{1}{4M_0c^2}(v^2/c^2) \tag{19}$$

For the velocity of the recoil photon is nearly equal to the velocity, then $v/c \approx 1$ [7].
from equation (18), we have

$$\text{or } \quad E_{Rel} = \frac{1}{2M_0c^2} - \frac{1}{4M_0c^2} \tag{20}$$

$$\text{or } \quad E_{Rel} = -\frac{1}{4M_0c^2} \tag{21}$$

For nuclear transition [3]

$$E_\gamma = 10^5 eV \tag{22}$$

The equation (18) becomes

$$\text{or } \quad E_{Rel} = -\frac{1}{4M_0c^2}(10^5) \tag{23}$$

In the case of relativistic treatment, when the equation (21) is compared with equation (23), it is observed that the recoil energy of photon due to nuclear transition is 10^5 times greater than that of atomic transition.

4. Result and Discussion

In the present work, we have discussed the basics of the Mossbauer Effect. We also have calculated the total energy required to recoil event due to atomic transition is $0.5\times10^{11} eV$ and nuclear transition is $0.5\times10^{13} eV$. This result shows that the total energy to recoil the photon needed for the nuclear transition is 10^2 times greater than to that of the atomic transition. But when we apply the relativistic treatment on the same event to obtain the total energy for the recoiling photon represented by the equation (21) due to atomic transition and by the equation (23) due to nuclear transition. When we compare both the equations (21) & (23), it is concluded that the total energy to recoil the photon needed for the nuclear transition is 10^5 times greater than to that of the atomic transition.

5. Conclusion

We have concluded the following facts during the research work as follows;

1. The total non-relativistic energy of recoiling event needed for the nuclear transition is 10^2 times greater than to that of the atomic transition.
2. The relativistic recoil energy of photon due to nuclear transition is 10^5 times greater than to that of atomic transition.

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