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Some Results on Path Related Even Sum Graphs

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ABSTRACT
Graph labeling is the allocation of integers to the vertex or edges or both subject to certain conditions. The concept of Even Sum Graphs was introduced by Chinju Krishna et al.. A graph G is called an even sum graph if there is a labeling η of its vertices with distinct non - negative even integers, so that for any two distinct vertices a and b; ab is an edge of G if and only if $\eta(a) + \eta(b) = \eta(c)$ for some vertex c in G. The minimum number of isolated vertices required to make the graph G, an even sum graph is called the even sum number of G and is denoted by $\gamma(G)$.

Keywords: Olive tree, spider graph, path, pendant vertex

Mathematics Subject Classification: 05C05, 05C70, 05C75, 05C78.

1. Introduction

In 1990, Harary interpolated Sum graphs[14]. A graph G(V,E) is called a sum graph if there is an bijection f from V to a set of positive integers S such that $xy \in E$ if only if $f(x) + f(y) \in S$. Since the vertex with the highest label in a sum graph cannot be adjacent to any other vertex, every sum graph must contain isolated vertices. For a connected graph G,

let $\sigma(G)$, the sum number of G, denote the minimum number of isolated vertices that must be added to G so that the resulting graph is a sum graph. In 1994, Harary [15] generalized sum graphs by permitting S to be any set of integers. He called these graphs as integral sum graphs. Unlike sum graphs, integral sum graphs need not have isolated vertices. The integral sum number, $\zeta(G)$ of G is the minimum number of isolated vertices that must be added to G so that the resulting graph is an integral sum graph. Thus, by definition we get G is an integral sum graph if and only if $\zeta(G) = 0$.The properties of Sum Graphs and Integral Sum graphs was investigated by many authors, including Chen. Z [1], Mary Florida. L [18], Nicholas. T [16, 17], Soma Sundaram. S [16], Vilfred.V [17 - 21], Surya Kala.V [19, 20] and Rubin Mary. K [20, 21]. An olive tree is a rooted tree consisting of k branches where the i^{th} branch is a path of length i. A spider is a tree with one vertex of degree atleast 3 and all others of degree atmost 2. It is denoted by $K_{(1,n,n)}$. In this paper we investigates some path related even sum graphs[2 - 11]. For all basic ideas, we refer [12, 13].

2. Primary Results

Theorem 3.1 An olive tree G is an even sum graph.

Proof: Let $a_0, a_{i,j}, 1 \le i \le n, i \le j \le n$ be the vertices of G in which a_0 is the root of G. Define a function $\eta : V(G) \to 2Z^+ \cup \{0\}$ by $\eta(a_0) = 2$; $\eta(a_{1,1}) = 4$; $\eta(a_{1,j}) = \eta(a_0) + \eta(a_{1,j-1})$, $2 \le j \le n$; $\eta(a_{2,2}) = \eta(a_0) + \eta(a_{1,n})$; $\eta(a_{i,i}) = \eta(a_{i-1,n}) + \eta(a_{i-2,n})$, $2 \le$

$i \le n-1$; $\eta(a_{n-1,n}) = 0$; $\eta(a_i, j) = \eta(a_i - 1, j-1) + \eta(a_i, j-1)$, $i \ne j$, $2 \le i \le n-2$; $3 \le$

$j \le n$; $\eta(a_{n,n}) = \eta(a_{n-1,n-1}) + \eta(a_{n-2,n-1})$. **Then the labels are distinct and for any edge ab in G, the condition $\eta(a) + \eta(b) = \eta(c)$ holds for some vertex c in G. Hence G is an even**

sum graph.

Illustration: 3.2. An even sum graph of G is shown in the following figure 3.1.

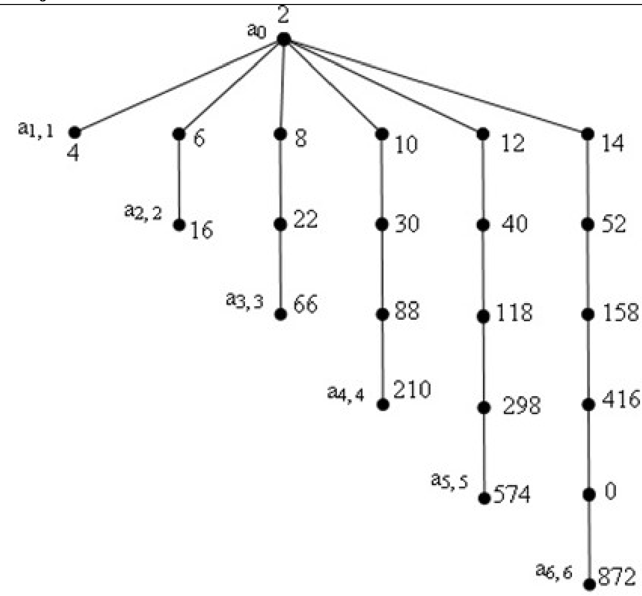

Figure 1: ESG of G

Theorem 3.3 A spider $K_{(1,n,n)}$ is an even sum graph.

Proof: Let G = $K_{(1,n,n)}$ and a_0, $a_{i,j}$, $1 \le i \le n$, $1 \le j \le n$ be the vertices of G in which a_0 is the root. Define a function $\eta : V(G) \to 2Z^+ \cup \{0\}$ by $\eta(a_0) = 2$; $\eta(a_{1,1}) = 4$; $\eta(a_{1,j}) = \eta(a_0) + \eta(a_{1,j-1})$, $2 \le j \le n$; $\eta(a_{2,1}) = \eta(a_0) + \eta(a_{1,n})$; $\eta(a_{i,1}) = \eta(a_{i-1,n}) +$

$\eta(a_{i-2,n})$, $3 \le i \le n-1$; $\eta(a_{n,1}) = \eta(a_{n-1,n-1}) + \eta(a_{n-2,n-1})$; $\eta(a_{n-1,n}) = 0$; $\eta(a_{i,j}) =$

$\eta(a_{i-1,j-1}) + \eta(a_{i,j-1})$, $2 \le i \le n$; $2 \le j \le n$; $\eta(a_{n,n}) = \eta(a_{n-1,n-1)} + \eta(a_{n,n-1})$. Then the labels are distinct and for any edge ab in G, the condition $\eta(a) + \eta(b) = \eta(c)$ holds for some vertex c in G. Hence G is an even sum graph.

Illustration: 3.4. An even sum graph of G is shown in the following figure 3.2.

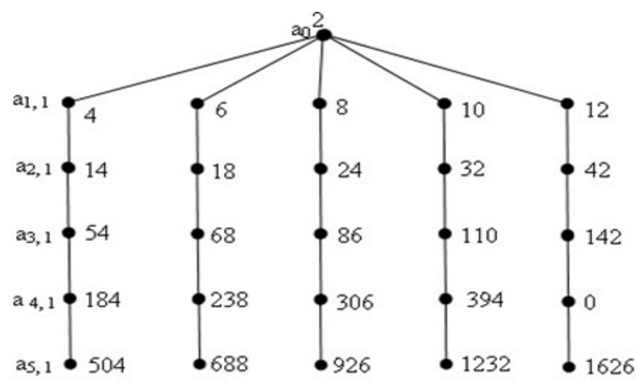

Figure 2: ESG of G

Theorem 3.5 A graph G obtained by joining a pendent vertex with a vertex of degree two is an even sum graph.

Proof: Let $a_1, a_2, \ldots,$ an be the vertices of a path P_n. Let b_i be the vertex adjacent to a_i, $1 \le i \le n$. The resultant graph is $P_n \circledcirc K_1$. Join a new pendent vertex c to the vertex an of $P_n \circledcirc K_1$. The resultant graph G has $2n + 1$ vertices and 2n edges with edge set $E(G) = \{a_ia_i + 1, a_nc/1 \le i \le n-1\} \cup \{a_ib_i/1 \le i \le n\}$. Define a function $\eta : V(G) \to 2Z^+ \cup 0$ by $\eta(a_1) = 2$; ?$\eta(a_2) = 4$; $\eta(a_i) = \eta(a_i - 1) + \eta(a_i - 2)$, $3 \le i \le n$; $\eta(b_1) = \eta(a_n - 1) + \eta(a_n)$, $\eta(b_i) = \eta(a_i - 1) + \eta(b_i - 1)$, $2 \le i \le n-1$; $\eta(b_n) = 0$; $\eta(c) = \eta(a_n - 2) + \eta(b_n - 2)$. Then the labels are distinct and for any edge ab in G, the condition $\eta(a) + \eta(b) = \eta(c)$ is satisfied for some vertex c in G. Hence a graph G obtained by joining

a pendent vertex with a vertex of degree two is an even sum graph.

Illustration: 3.6. An even sum graph of G is shown in the following figure 3.3.

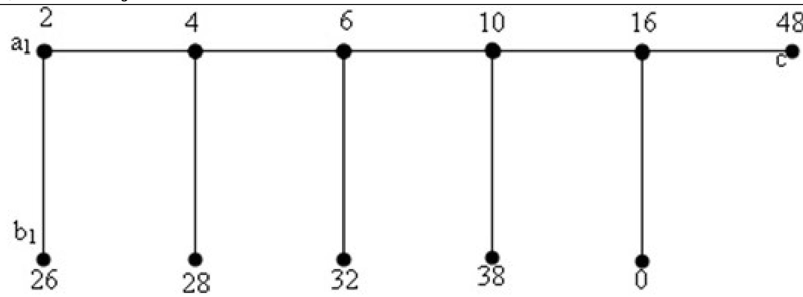

Figure 3: ESG of G

3. Conclusion

In this paper, we have explored the concept of even sum graphs and we investigated different types of path related even sum graphs. This paper motivates to derive similar results on other types of graphs to be an even sum graph and to investigate the characterization of even sum graphs.

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