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Analysis of an Encouraged arrival finite source queue providing unlimited service

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ABSTRACT
An Encouraged arrival finite source queue with unlimited service is proposed in this paper. The proposed model incorparates EA in a self-service scenario where unlimited service is offered for a calling population of finite size. Firms offer discounts to attract customers which result in Encouraged arrivals(EA). A sort of queueing model which provides unlimited service affirms that all arrivals are honored. The cost analysis of the proposed model shows that profit of the firm increases with EA. The mean system size and average waiting time in the system are determined and the graphs are generated using MATLAB. A special case of client impatience and retention of such impatient clients are discussed.To back up the conclusions, numerical results are provided. The proposed model also satisfies Little's formula(LF).

KEYWORDS
Encouraged Arrivals (EA) ; Finite-source queue; Profit ; Incentives; Expenditure.

1. Introduction

In today's business scenario, the main aim of the firms is to operate smoothly and efficiently in order to fulfill the client's demand and provide the best possible service because of the intense competition. Nowadays, even for relatively simple products, clients have a vast array of possibilities. It is quite difficult to manage a business effectively in the present setting of an unpredictable and competitive business environment while yet meeting the client's expectations and attracting new customers. To entice customers to sign up for the system, businesses provide promotions and incentives.

As a result, businesses regularly offer steep discounts and other alluring incentives to draw in new clients. Customers are drawn to the company as a result of these promotions (known as Encouraged Arrivals (EA), a term coined by Som and Seth[11]). The occurrence of arrivals that are encouraged can be thought of as the opposite of those that are discouraged discussed by Kumar et al. [8]. However, if businesses are not prepared to run efficiently during the promotion period to give customers an amazing service experience, their reputation may suffer. Companies must therefore be aware of their performance level in order to manage the service quality. Queueing models assist businesses in understanding their performance in advance, enabling them to plan effectively for offering seamless and effective customer service.

Only one server may be available in some queueing models, whereas multiple servers may be prepared to handle the incoming traffic in other situations. No one wishes to spend time in waiting as it is time consuming, however it is unavoidable in many situations. But with the help of queueing theory, we design a model where every arrival get prompt assistance. In a self-service scenario where the firm provides unlimited service no one has to wait for service. Many organisations make use of this unique instance of queuing model.

This model can be applied in various self-service situations.For example, Buffet system offers a unlimited service running in parallel for a finite population. Many barbeque firms offer discounts to attract customers where they provide a large number of options for clients. (i.e.,) a family of four members can enjoy a variety of nearly 50 dishes in a Barbeque at a minimum cost. The customers need to be satisfied simulatneously the firm should gain profit for the firm to run successfully. The proposed model is inspired from such situations where a firm provides unlimited service with discounts for a finite population with an eye to gain profit. The cost analysis discussed in the paper will be very helpful in planning such offers effectively. The analysis of performance metrics helps the firm to assess the expected number of clients to visit the firm and accordingly the firm can improve the rate at which service is carried out.

This paper is divided into various Sections.In the first section the introduction of the paper is provided. Section 2 comprises of the preliminaries required for the paper. The mathematical notations for the proposed model are listed in Section 3. The mathematical model's derivation and the formula for the system's mean number of consumers are both covered in Section 4. Section 5 presents analysis of performance metrics. The model's economic analysis is covered in Section 6. In section 7, a special instance of client impatience and retention of such impatient clients is described and Section 8 wraps up the paper with remarks and conclusion.

2. Preliminaries

2.1 Encouraged Arrivals (EA)

Before purchasing any product, customers frequently check for profitable offers being offered by numerous business firms. Businesses issue a variety of offers and discounts to attract new consumers and keep their existing ones happy in order to maintain sustained growth. Customers are drawn to the business by the discounts and offerings. In this study, attracted clients are referred to as Encouraged Arrivals **(**EA).

2.2 Finite-source queue with unlimited service

We propose a queueing model which provides unlimited service and has a calling population of finite size.

For example, Buffet system offers a unlimited service running in parallel for a finite population. As people prefer self-service in various sectors which includes shopping dresses, buying groceries and fruits etc. Nowadays people prefer online shopping where they are provided unlimited service.

2.3 System capacity

When the line reaches a specific length, no more customers are permitted to enter until system becomes available as a result of a service being completed. This is because some queueing systems have a physical limit to the quantity of waiting area. These are referred to as finite queueing circumstances, meaning that the maximum system size has a finite upper bound.

The basic queueing literature benefits from the EA. Jain et al. [5] described the idea of customer mobilisation and stated that a system attracts a new consumer by taking a look at its sizable customer base. EA deals with the percentage change in clients as a result of promotions and discounts. One can think of the phenomenon of promoted immigration as being in opposition to discouraged arrivals. Multi-server queuing model is presented with discouragement in [6]. Reynolds et al. discovered the equilibrium distribution of number of clients in the queue and extrapolated it to other performance metrics. By using recursive relations, Roijers et al. [4] examined the busy period formula , all moments and covariance for a IS queue during periods of congestion.

In order to evaluate the effectiveness of the proposed method, Barache et al. [1] dealt with the stationary characteristics of the $GI/M/\infty$ Queueing system using a infinite server(IS) to obtain the stable inequalities of the stationary distribution of the queue size. This algorithm allowed for the computation of a number of theoretical results. Whitt [15] investigated a steady state IS in which a queuing distribution with sinusoidal arrival rate and exponential service are considered. D'Auria [2] took into account the $M/G/\infty$ queue and other cases in a random environment when calculating a stochastic decomposition formula for the total count of clients in the system. Gullu [10] reviewed the batch arrivals of $M/G/\infty$ queue which is characterised by a compound Poisson random variable for the number of jobs, and each batch of arrivals is handled by a single server. In Kumar et al. [7] a limited capacity markovian several server queuing system with retention of defaulting consumers was built. Different effectiveness measures have been established by deriving the model's steady-state solution iteratively. It has been investigated how the likelihood of keeping defaulting clients has an impact on the anticipated system size. Sushil et al. [13] observed the pattern of changing request in the system, with regard to various arrival and service rates in an infinite server queueing system with finite capacity. Som et al. [12] studied a multi- server queuing model with limited capacity for any organization encountering EA and reverse reneging.

3. Mathematical notation

The suggested Queueing model primarily uses the following mathematical notations, with a few exceptions for specific models that use other notations. The following are some examples of common mathematical notations used in queueing theory that we employ in this paper:

i) The arrivals happen sequentially according to a Poisson process with the parameter $\lambda(1+ea\eta)$, where "eaη" denotes the change of percentage in the total count of clients estimated from observed data. For instance, if a firm previously offered discounts and a percentage change in the total count of clients was noticed of +10%, +30% or +50%, then eaη= 0.1, 0.3 or eaη=0.5, respectively.

ii) Service times have an exponential distribution with parameter (μ).

iii) Customers are served in the order of their arrival i.e., FCFS (First Come First Serve) discipline.

iv) The system has a finite calling population, say M.

v) The system provides unlimited service (For example. A self-service buffet)

vi). The state j represents the number of clients in the system. State j's probability is denoted by Pb_j

vii) Mean number of clients in the system is given by L_S

viii) System's waiting time is given by W_S.

In the following section, we propose the mathematical model. The derivation of the balanced equations for the various states of the system based on the transient diagram is obtained by employing the recursive method.

4. Mathematical model

The following are the assumptions of our proposed model:

The proposed model treats a situation where the firm provides unlimited service to a calling population which is finite size M. For instance, consider a Buffet system which offers unlimited service to a finite population. Further we introduce EA into the proposed model to observe how the proposed model benefits by the inclusion of EA.

The following figure 1. depicts the rate transition diagram for the proposed model where the Birth and Death rates are given as follows:

$$\lambda_n(1+ea\eta)=\begin{cases}(M-j)\lambda(1+ea\eta), & (0\leq j<M)\\ 0 & , (j\geq M)\end{cases}$$

Where j is the total count of clients in the system.

$\mu_n = j\mu$, for all j

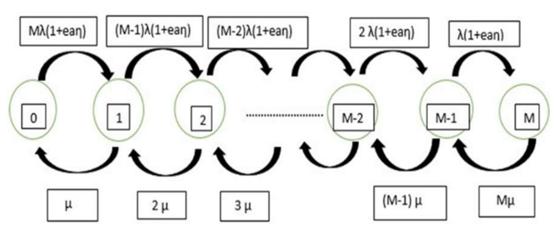

Fig 1. Rate transition diagram of the proposed model

The following are the differential-difference equations of the proposed model:

$$Pb_0{'}(t) = -M\lambda(1+ea\eta)Pb_0{'}(t) + \mu Pb_1(t) \quad (1)$$

$$\begin{aligned}Pb_1{'}(t) &= M\lambda(1+ea\eta)Pb_0(t) - (M-1)\lambda(1+ea\eta)Pb_1(t) - \mu Pb_1(t) + 2\mu Pb_2(t)\\ &= M\lambda(1+ea\eta)Pb_0(t) - [(M-1)\lambda(1+ea\eta)+\mu]Pb_1(t) + 2\mu Pb_2(t) \quad (2)\end{aligned}$$

$$\begin{aligned} Pb_2'(t) &= \left((M-1)\,\lambda(1+ea\eta)\right)Pb_1(t) - \left[(M-2)\,\lambda(1+ea\eta)\right]Pb_2(t) - 2\mu Pb_2(t) \\ &\qquad + 3\mu Pb_3(t) \\ &= \left((M-1)\,\lambda(1+ea\eta)\right)Pb_1(t) - \left[(M-2)\,\lambda(1+ea\eta)+2\mu\right]Pb_2(t) \\ &\qquad + 3\mu Pb_3(t) \qquad (3) \end{aligned}$$

$$\begin{aligned} Pb_j'(t) &= \left((M-(j-1))\right)\lambda(1+ea\eta)Pb_{j-1}(t) - \left[(M-j)\,\lambda(1+ea\eta)\right]Pb_j(t) \\ &\qquad - j\mu Pb_j(t) + (j+1)\mu Pb_{j+1}(t) \\ &= \left((M-(j-1))\right)\lambda(1+ea\eta)Pb_{j-1}(t) - \left[(M-j)\,\lambda(1+ea\eta)+j\mu\right]Pb_j(t) \\ &\qquad + (j+1)\mu Pb_{j+1}(t) \qquad (4) \end{aligned}$$

$$Pb_M'(t) = \lambda(1+ea\eta)\,Pb_{M-1}(t) + M\mu Pb_M(t) \qquad (5)$$

Solving the differential-difference equations from (1) to (5),
Pb_j can be expressed in the product forms as follows

$$Pb_j = \prod_{k=0}^{j-1} \frac{\lambda(1+ea\eta)(\mathrm{M}-\mathrm{k})}{(k+1)\mu} Pb_0$$

Thus,

$$Pb_j = Pb_0 \left(\frac{\lambda(1+ea\eta)}{\mu}\right)^j \binom{M}{j} \qquad (6)$$

Using normalizing condition of probability $\sum_{j=0}^{M} Pb_j = 1$, substitute the value of Pb_j from (6) which implies

$$\sum_{j=0}^{M} Pb_0 \left(\frac{\lambda(1+ea\eta)}{\mu}\right)^j \binom{M}{j} = 1$$

Thus, $Pb_0 = \left[\sum_{j=0}^{M} \left(\frac{\lambda(1+ea\eta)}{\mu}\right)^j \binom{M}{j}\right]^{-1}$

The term enclosed by the bracket has the binomial form, so

$$Pb_0 = \frac{1}{\left(1+\frac{\lambda(1+ea\eta)}{\mu}\right)^M}$$

From the obtained value of Pb_0, Pb_j can be expressed as follows

$$Pb_j = \begin{cases} \dfrac{\left(\frac{\lambda(1+ea\eta)}{\mu}\right)^j \binom{M}{j}}{\left(1+\frac{\lambda(1+ea\eta)}{\mu}\right)^M} & 0 \le j \le M \\ 0 & otherwise \end{cases}$$

5. Analysis of performance metrics with EA:

Let L_S be the mean number of clients in the system, then utilising the formula for the expectation of the probability distribution, it is then determined.

$$L_S = \sum_{j=0}^{M} jPb_j = \sum_{j=0}^{M} j \frac{\left(\frac{\lambda(1+ea\eta)}{\mu}\right)^j \binom{M}{j}}{\left(1+\frac{\lambda(1+ea\eta)}{\mu}\right)^M}$$

The system's average service time, which is given by W_S which serves as the system's mean waiting time as well.

$$W_s = \frac{1}{\mu}$$

The average number of clients in the queue and the average waiting time in the queue are given by L_q and W_q respectively.
Where $L_q = W_q = 0$, since the proposed model provides unlimited service and the calling population is finite.

We give a numerical illustration of the aforementioned model in this section.
The following tables 1 to 4 shows the mean system size grows with respect to EA. The figures 2 to 4 represents the graphs generated
using MATLAB which shows the increase in system size of the proposed model with respect to EA for various service rate. The data for Table 1 without EA is obtained from [13].

Table 1: Calculating mean system size with poisson arrivals(PA) (i.e.,) eaη=0

eaη	λ	μ=12	μ=13	μ=14	μ=15
0	1	1.5378	1.3735	1.3328	1.2382
	2	2.862	2.67	2.502	2.35
	3	3.99	3.753	3.525	3.33
	4	4.962	4.733	4.45	4.215
	5	5.875	5.549	5.262	4.96
	6	6.67	6.311	5.994	5.714
	7	7.365	6.99	6.67	6.37
	8	8.024	7.616	7.27	6.95
	9	8.571	8.179	7.83	7.5
	10	9.08	8.694	8.331	8.002

Table 2: Calculating mean system size with eaη = 0.1

eaη	λ	μ=12	μ=13	μ=14	μ=15
0.1	1	1.6792	1.5601	1.4569	1.3664
	2	3.0986	2.8948	2.7161	2.5582
	3	4.3137	4.0492	3.8150	3.6065
	4	5.3661	5.0574	4.7826	4.5361
	5	6.2858	5.9464	5.6412	5.3661
	6	7.0967	6.7347	6.4078	6.1111
	7	7.8172	7.4396	7.0967	6.7842
	8	8.4617	8.0734	7.7193	7.3952
	9	9.0410	8.6463	8.2845	7.9518
	10	9.5655	9.1666	8.8000	8.4617

Fig 2. Increase in mean system size when eaη = 0.1

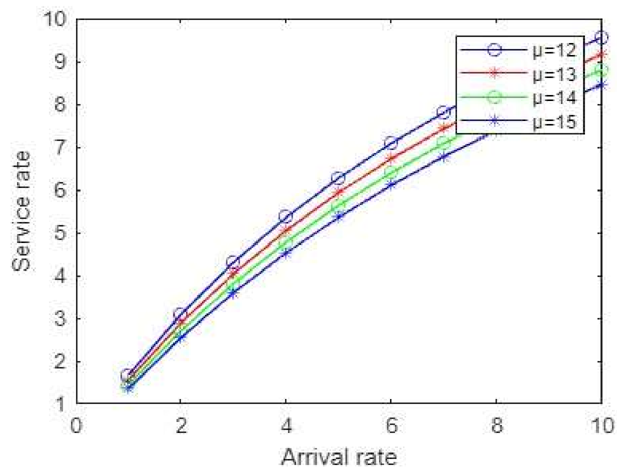

Table 3: Calculating mean system size with eaη = 0.3

eaη	λ	μ=12	μ=13	μ=14	μ=15
0.3	1	1.9548	1.8181	1.6913	1.5951
	2	3.5620	3.3333	3.1365	2.9497
	3	4.9056	4.6153	4.3505	4.1269
	4	6.0432	5.7142	5.4121	5.1521
	5	7.0298	7.0129	6.3387	6.0432
	6	7.8787	7.5000	7.1547	6.8421
	7	8.6363	8.2352	7.8787	7.5776
	8	9.2876	8.8888	8.5189	8.1866
	9	9.8734	9.4736	9.1008	8.7640
	10	10.3846	10	9.6265	9.2876

Fig 3. Increase in mean system size when eaη = 0.3

eaη	λ	μ=12	μ=13	μ=14	μ=15
0.5	1	2.2222	2.0689	1.9354	1.8181
	2	4	3.7530	3.55255	3.3333
	3	5.4545	5.1411	4.8599	4.6153

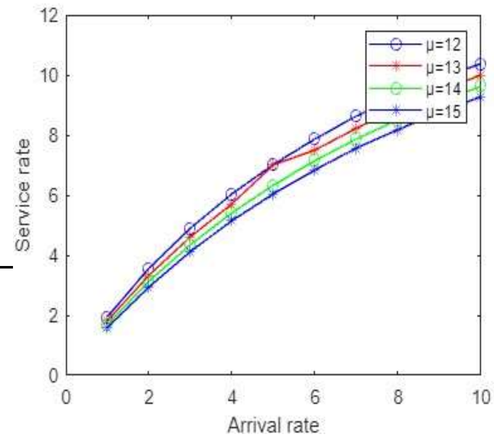

Table 4: Calculating mean system size with eaη = 0.5

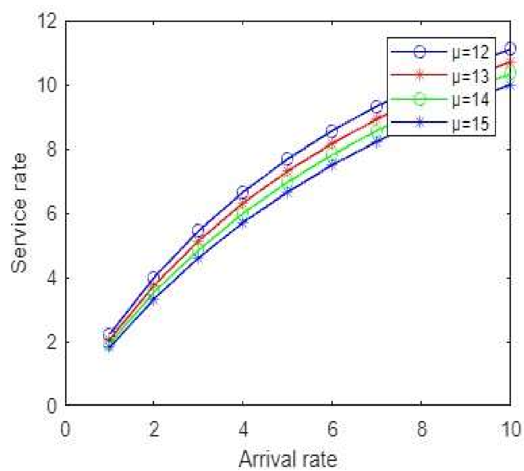

Fig 4. Increase in mean system size when eaη = 0.5

5.1 Reduction in waiting time with respect to service rates and EA

The system's average service time, which is given by $W_S = \frac{1}{\mu}$, serves as the system's mean waiting time as well. As the service rate increases the system's waiting time falls which is depicted in fig 5.

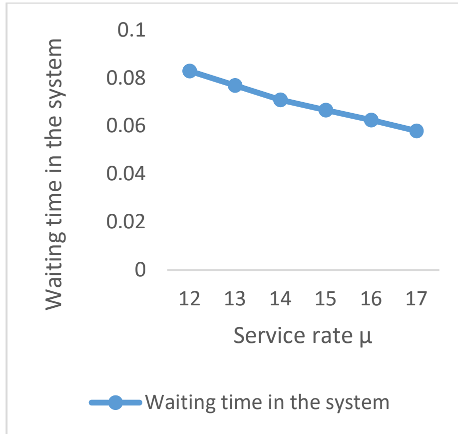

Fig 5. Mean waiting time in the system with respect to µ

Using LF, $L_s = \lambda W_s$
For eaη we have the following
$L_s = \lambda(1+ ea\eta)\, W_s$
$W_s = \frac{\lambda(1+ ea\eta)}{Ls}$
We analyse the system's waiting time based on the EA.
The following tables 5 to 8 shows system's waiting time gradually reduces with respect to EA. The figures 6 to 9 represents the graphs generated using MATLAB which shows the reduction in waiting time of the proposed model with respect to EA for various service rate when compared with PA[13]

Table 5. Calculating W_s with respect to EA when µ=12

λ	eaη=0	eaη = 0.1	eaη = 0.3	eaη = 0.5
1	1.537	1.5265	1.503	1.481
2	1.431	1.408	1.37	1.333
3	1.33	1.307	1.257	1.212
4	1.24	1.219	1.162	1.111
5	1.175	1.142	1.091	1.025
6	1.111	1.072	1.010	0.952
7	1.052	1.015	0.949	0.888
8	1.003	0.961	0.893	0.833
9	0.952	0.913	0.843	0.784
10	0.908	0.869	0.798	0.740

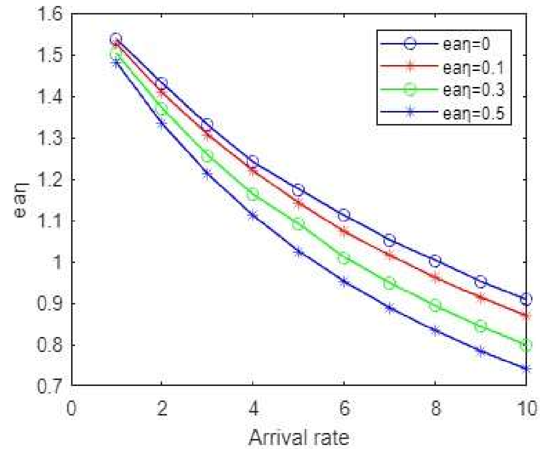

Fig 6 . Reduction in waiting time when μ=12

Table 6. Calculating W_s with respect to EA when μ=13

λ	eaη=0	eaη = 0.1	eaη = 0.3	eaη = 0.5
1	1.373	1.418	1.398	1.371
2	1.335	1.315	1.282	1.251
3	1.251	1.227	1.183	1.142
4	1.183	1.149	1.098	1.053
5	1.109	1.081	1.078	0.975
6	1.051	1.020	0.961	0.908
7	0.998	0.966	0.904	0.851
8	0.952	0.917	0.854	0.799
9	0.908	0.873	0.809	0.755
10	0.869	0.833	0.769	0.714

Fig 7 . Reduction in waiting time when μ=13

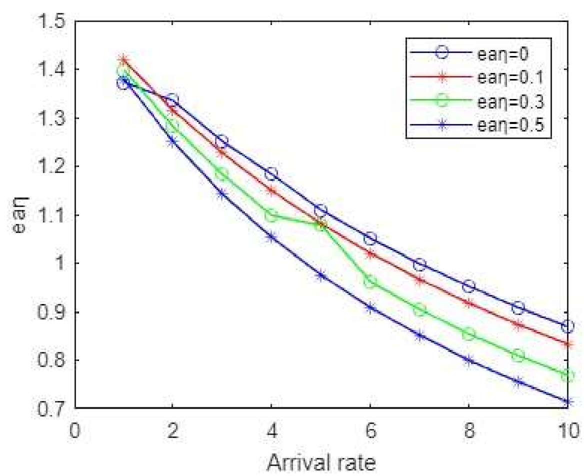

Table 7. Calculating W_s with respect to EA when μ=14

λ	eaη=0	eaη = 0.1	eaη = 0.3	eaη = 0.5
1	1.332	1.324	1.301	1.290
2	1.251	1.234	1.206	1.184
3	1.175	1.156	1.115	1.079
4	1.112	1.086	1.040	1.000
5	1.052	1.025	0.975	0.930
6	0.999	0.970	0.917	0.869
7	0.952	0.921	0.865	0.816
8	0.908	0.877	0.819	0.769
9	0.87	0.836	0.777	0.727
10	0.833	0.80	0.740	0.689

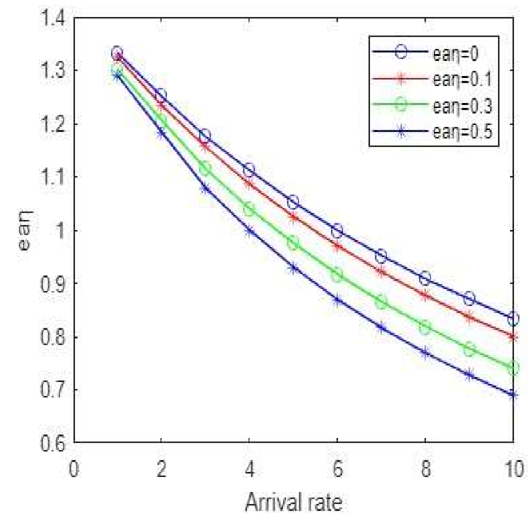

Fig 8 . Reduction in waiting time when μ=14

Table 8. Calculating W_s with respect to EA when μ=15

λ	eaη=0	eaη = 0.1	eaη = 0.3	eaη = 0.5
1	1.238	1.242	1.227	1.212
2	1.175	1.162	1.134	1.111
3	1.11	1.092	1.058	1.025
4	1.053	1.030	0.990	0.952
5	0.992	0.975	0.929	0.888
6	0.952	0.925	0.877	0.833
7	0.91	0.881	0.832	0.784

8	0.8687	0.840	0.787	0.740
9	0.833	0.803	0.749	0.701
10	0.800	0.769	0.714	0.666

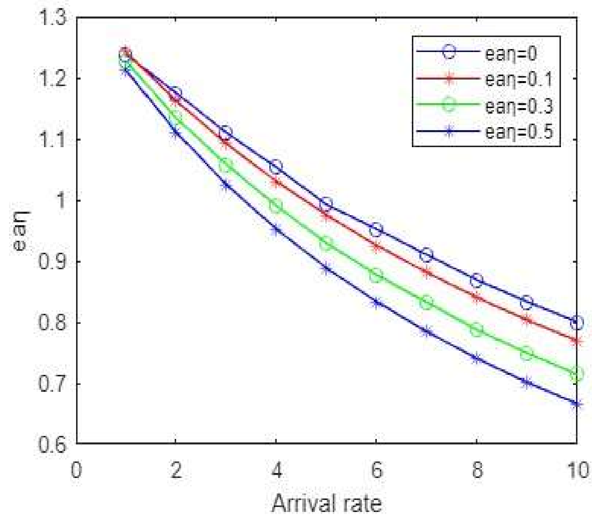

Fig 9 . Reduction in waiting time when μ=15

6 Economic analysis of the proposed model

An economic analysis is a procedure that helps business owners to get a vivid understanding of the current economic environment as it relates to their company's potential for success.

The development of total anticipated cost (TAC), total anticipated revenue (TAR), and total anticipated profit (TAP) functions is used to analyse the model's economic analysis.

The system's TAC is given as follows

$$TAC = S_e\mu + H_eL_S + L_e\lambda\left[\frac{\lambda(1+ea\eta)}{\mu}\right]^M Pb_0$$

Where $Pb_0 = \left[\frac{1}{1+\frac{\lambda(1+ea\eta)}{\mu}}\right]^M$

The system's TAR is given as follows

TAR = Re × μ × (1- Pb_0)

The system's TAP is given as follows

TAP = TAR – TAC

Where

S_e denotes the service expense per unit time

H_e denotes the holding expense per unit per unit time

L_e denotes the expense for each unit lost per unit of time

Table 9. TAC, TAR and TAP variation with regard to λ

Let N=20, μ=12, eaη=0.3, S_e=10, H_e=2, L_e=15, R_e=200

λ	TAC	TAR	TAP
1	123.909	2091.36	1976.45
2	127.124	2352.39	2225.266
3	129.81	2394.24	2264.43
4	132.086	2398.08	2265.994
5	134.6	2399.52	2264.92
6	135.74	2399.8	2264.06
7	137.28	2399.97	2262.69
8	138.574	2399.991	2261.417
9	139.746	2399.997	2260.251
10	140.76	2399.999	2259.239

The above table demonstrates that when λ rises, the total projected profit rises as well. It then hits a maximum value and begins to decline. This is due to the fixed service rate, which causes costs to rise more quickly than revenues after a given level of service load. The following figure shows the graph generated using MATLAB for the cost analysis with respect to EA.

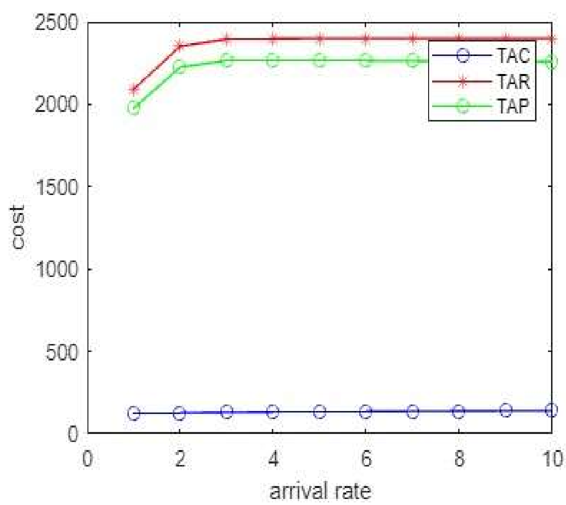

Fig 10. Cost analysis with respect to EA

Table 10. TAC, TAR and TAP variation with regard to μ

Let N=20, λ=2, eaη=0.3, S_e=10, H_e=2, L_e=15, R_e=200

μ	TAC	TAR	TAP
12	127.124	2352.39	2225.266
13	136.6	2352.72	2396.12

14	146.272	2800	2653.73
15	155.9	2876.68	2720.78

It is clear from the table that as the service quality increases, revenues increase and the company's profit keeps growing. The following figure 10 represents the increase in profit with respect to service rate.

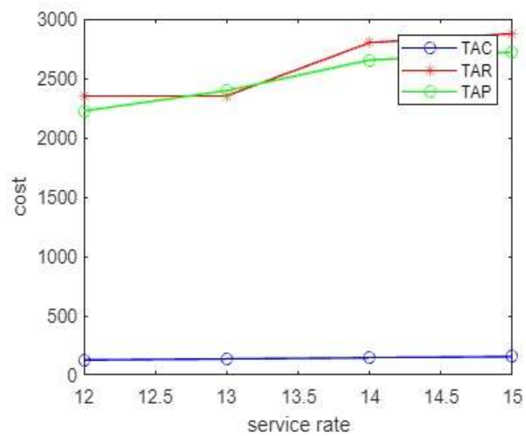

Fig 11. Cost analysis with respect to service rate

7 Special cases

i) Reneging and retention of reneged customers

Taking customer impatience into account in the proposed model , the reneging times are exponentially distributed with parameter ψ. Let Q be the probability of those dissatisfied clients who may rejoin the queue to get their service completed. A client may leave queue satisfied with probability P = 1-Q. The probability of retention of retention of reneged customer is given by Q'. The probability that a client may not be retained is given by P' = 1- Q'.

Then Pb_j can be expressed as follows

$$Pb_j = \begin{cases} \dfrac{\left(\dfrac{\lambda(1+ea\eta)}{\mu P+\varphi P'}\right)^j \binom{M}{j}}{\left(1+\dfrac{\lambda(1+ea\eta)}{\mu P+\varphi P'}\right)^M} & 0 \le j \le M \\ 0 & otherwise \end{cases}$$

The mean number of clients in this case is given by

$$L_S = \sum_{j=0}^{M} jPb_j = \sum_{j=0}^{M} j \frac{\left(\dfrac{\lambda(1+ea\eta)}{\mu P+\varphi P'}\right)^j \binom{M}{j}}{\left(1+\dfrac{\lambda(1+ea\eta)}{\mu P+\varphi P'}\right)^M},$$

With increase in reneging times the mean number of clients in the system decreases .

ii) When there is no EA in the proposed model (i.e.,) eaη = 0

Probability that there are no clients in the system is given by

$$Pb_0 = \left[\sum_{j=0}^{M} \left(\frac{\lambda}{\mu} \right)^j \binom{M}{j} \right]^{-1}$$

$$Pb_0 = \frac{1}{\left(1 + \frac{\lambda}{\mu} \right)^M}$$

Probability that there are M clients in the system is given by

$$Pb_M = \left(\frac{\lambda}{\mu} \right)^j \binom{M}{j} Pb_0$$

The system becomes a simple, finite-source queuing paradigm.

7.1 Verification of LF

For eaη we have the following

$L_s = \lambda(1+ ea\eta)\ W_s$

Table 11. verfication of LF for eaη=0.3

λ	eaη	λ(1+ eaη)	L_s	W_s	λ(1+ eaη) W_s
1	0.3	1.3	1.9548	1.503	1.95
2	0.3	2.6	3.5620	1.37	3.56
3	0.3	3.9	4.9056	1.257	4.90
4	0.3	5.2	6.0432	1.162	6.04
5	0.3	6.5	7.0298	1.091	7.02
6	0.3	7.8	7.8787	1.010	7.87
7	0.3	9.1	8.6363	0.949	8.63
8	0.3	10.4	9.2876	0.893	9.28
9	0.3	11.7	9.8734	0.843	9.87
10	0.3	13	10.3846	0.798	10.38

Table 12. verfication of LF for eaη=0.5

λ	eaη	λ(1+ eaη)	L_s	W_s	λ(1+ eaη) W_s
1	0.5	1.5	2.2222	1.481	2.22
2	0.5	3	4	1.333	3.99
3	0.5	4.5	5.4545	1.212	5.45
4	0.5	6	6.6666	1.111	6.66
5	0.5	7.5	7.6923	1.025	7.69

6	0.5	9	8.5714	0.952	8.57
7	0.5	10.5	9.3333	0.888	9.32
8	0.5	12	10	0.833	9.99
9	0.5	13.5	10.5882	0.784	10.58
10	0.5	15	11.1111	0.740	11.1

Remarks

1. We observe from the proposed model that with increase in EA there is an increase in system size from tables 1 to 4 .
2. With increasing service rates, we notice that there is drop in system's waiting time with EA from tables 5 to 8, which is the main objective of organisations to run successfully.
3. Further the cost analysis of the model highlights the increase in profit of the firm with EA from tables 9 and 10.Thus the proposed queueing model benefits by introducing EA.

8 Conclusion

The proposed queuing model of a firm providing unlimited service to a finite population with EA is examined in this paper.The performance measures of the proposed model are analysed . The mean system size and the system's waiting time are computed and the graphs are generated using MATLAB.

A special case of introducing client impatience and retention of such impatient clients in the proposed model is discussed. This proposed model is practically applicable in various firms as people prefer self-service in various sectors. For any organisation dealing with the concept of incentivized consumers and load on service, the paper's findings are of great value. This concept can be used to plan an effective strategy.

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