Library Progress International *Print version ISSN 0970 1052*
Vol.44 No.3, July-December 2024: P. 22320-22324 *Online version ISSN 2320 317X*

Original Article *Available online at www.bpasjournals.com*

ANUJ TRANSFORM FOR THE CLOSED FORM SOLUTION OF GENERALIZED ABEL'S INTEGRAL EQUATION OF SECOND KIND

Sudhanshu Aggarwal[1*], Anuj Kumar[2] and Shikha Bansal[3]

[1]Assistant Professor, Department of Mathematics, National Post Graduate College, Barhalganj, Gorakhpur-273402, Uttar Pradesh, India
[2]Research Scholar, SRM Institute of Science and Technology, Delhi-NCR, Campus, Modinagar, Ghaziabad-201204, Uttar Pradesh, India
[3]Assistant Professor, SRM Institute of Science and Technology, Delhi-NCR, Campus, Modinagar, Ghaziabad-201204, Uttar Pradesh, India
sudhanshu30187@gmail.com, anuj.doc84@gmail.com, shikhab@srmist.edu.in

How to cite this article: Sudhanshu Aggarwal, Anuj Kumar and Shikha Bansal (2024) Anuj Transform for the Closed Form Solution of Generalized Abel's Integral Equation of Second Kind. *Library Progress International,* 44(3), 22320-22324.

ABSTRACT
Abel's integral equations have numerous applications in mathematical physics, chemical kinetics, potential theory, crystal growth, electrostatics, astrophysics, and stereology. In this paper, the Anuj transform for the closed form solution of the generalized Abel's integral equation of the second kind (GAIESK) is discussed by the authors. Several examples of GAIESK are considered for demonstrating the reliability of the Anuj transform. The outcomes of these examples support the reliability and efficiency of the present scheme. It is also visualized from the outcomes that the Anuj transform provides the closed form solution of our study's problem without the need for complicated calculation work.

Keywords: Abel integral equation, Anuj transform, integral transform, integral equation, fundamental functions.

MSC2010: 35A22, 45A05, 45D05

1. INTRODUCTION

Integral transforms have wide range of application in mathematics to solve differential equations, integral equations and their systems [1-2]. These transforms convert the original problem into the simpler problem that is easy to handle compare to the original problem. Integral transforms [3-10] are easy to apply and less time consuming to solve the problems of engineering and applied sciences compare to other methods like Adomian decomposition method [11], Homotopy perturbation method [12], Rayleigh-Ritz method [13], Galerkin method [14], Variational iteration method [15], Quadrature method [16], and Sawi decomposition method [17]. Nowadays, researchers [18-20] developed various integral transforms and used these transforms for solving the problems of different kinds of integral equations. The duality relations of various integral transforms are well documented [21-28] and useful for developing theories of these transforms. Various situations of nature and sciences such as population problem of species, hanging chain problem, spring-mass problem, mortality of equipment and rate of replacement [29-33] properly handled by expressing them into integral equations because integral equations are easily solvable compare to differential equations. In this paper, we intend to find the closed form solution of GAIESK by the use of Anuj transform.

2. BASIC DEFINITIONS AND PROPERTIES OF ANUJ TRANSFORMS:

2.1 DEFINITION OF ANUJ TRANSFORM

A function $\omega(x) \in \mathcal{C}, x \geq 0$, where $\mathcal{C}$ is the collection of the piecewise continuous exponential order functions, has the Anuj transform and it is given by [20]

$$\mathcal{A}\{\omega(x)\} = q^2 \int_0^\infty \omega(x) e^{-\left(\frac{x}{q}\right)} dx = \oint(q),\ q > 0$$

The Anuj's transformations of fundamental mathematical functions are given in Table 1 (see Table 1).

Table-1: The Anuj's transformations of fundamental mathematical functions [20]

S.N.	$\omega(x) \in C, x \geq 0$	$\mathcal{A}\{\omega(x)\} = f(q)$
1	1	q^3
2	e^{ax}	$\left(\frac{q^3}{1-qa}\right)$
3	$x^a, a \in N$	$a!\, q^{a+3}$
4	$x^a, a > -1, a \in R$	$q^{a+3}\Gamma(a+1)$
5	$\sin(ax)$	$\left(\frac{a\, q^4}{1+q^2a^2}\right)$
6	$\cos(ax)$	$\left(\frac{q^3}{1+q^2a^2}\right)$
7	$\sinh(ax)$	$\left(\frac{aq^4}{1-q^2a^2}\right)$
8	$\cosh(ax)$	$\left(\frac{q^3}{1-q^2a^2}\right)$

2.2 LINEARITY PROPERTY OF ANUJ TRANSFORMS [34]

If $\omega_i(x) \in C, x \geq 0$ and $\mathcal{A}\{\omega_i(x)\} = f_i(q)$ then

$\mathcal{A}\{\sum_{i=1}^{n} a_i\omega_i(x)\} = \sum_{i=1}^{n} a_i\, \mathcal{A}\{\omega_i(x)\} = \sum_{i=1}^{n} a_i\, f_i(q)$, where a_i are arbitrary constants.

2.3 TRANSLATION PROPERTY OF ANUJ TRANSFORMS [10]

If $\omega(x) \in C, x \geq 0$ and $\mathcal{A}\{\omega(x)\} = f(q)$ then $\mathcal{A}\{e^{ax}\omega(x)\} = (1-qa)^2 f\left(\frac{q}{1-qa}\right)$, where a is arbitrary constant.

2.4 CHANGE OF SCALE PROPERTY OF ANUJ TRANSFORMS [6]

If $\omega(x) \in C, x \geq 0$ and $\mathcal{A}\{\omega(x)\} = f(q)$ then $\mathcal{A}\{\omega(ax)\} = \frac{1}{a^3} f(aq)$, where a is arbitrary constant.

2.5 FALTUNG (CONVOLUTION) PROPERTY OF ANUJ TRANSFORMS [20]

If $\omega_i(x) \in C, x \geq 0, i = 1,2$ and $\mathcal{A}\{\omega_i(x)\} = f_i(q), i = 1,2$ then

$\mathcal{A}\{\omega_1(x) * \omega_2(x)\} = \frac{1}{q^2}\prod_{i=1}^{2} \mathcal{A}\{\omega_i(x)\} = \frac{1}{q^2}\prod_{i=1}^{2} f_i(q)$.

3. INVERSE ANUJ TRANSFORM [20]

The inverse Anuj transform of $f(q)$, assigned by $\mathcal{A}^{-1}\{f(q)\}$, is another function $\omega(x)$ having the characteristic that $\mathcal{A}\{\omega(x)\} = f(q)$.

Table-2: The inverse Anuj's transformations of fundamental mathematical functions [20, 34]

S.N.	$f(q)$	$\mathcal{A}^{-1}\{f(q)\} = \omega(x)$
1	q^3	1
2	$\left(\frac{q^3}{1-qa}\right)$	e^{ax}
3	$a!\, q^{a+3}, a \in N$	x^a
4	$q^{a+3}\Gamma(a+1), a > -1, a \in R$	x^a
5	$\left(\frac{q^4}{1+q^2a^2}\right)$	$\frac{\sin(ax)}{a}$
6	$\left(\frac{q^3}{1+q^2a^2}\right)$	$\cos(ax)$
7	$\left(\frac{aq^4}{1-q^2a^2}\right)$	$\frac{\sinh(ax)}{a}$
8	$\left(\frac{q^3}{1-q^2a^2}\right)$	$\cosh(ax)$

4. ANUJ TRANSFORM FOR THE CLOSED FORM SOLUTION OF GENERALIZED ABEL'S INTEGRAL EQUATION OF SECOND KIND:

The generalized Abel's integral equation of second kind is given by [12]

$$\omega(x) = F(x) + \int_0^x \frac{\omega(t)}{(x-t)^\alpha} dt, 0 < \alpha < 1 \quad (1)$$

where $F(x)$, $\omega(x)$, and α are known function, unknown function, and constant respectively.

Operating Anuj transform on Eq. (1) gives

$\mathcal{A}\{\omega(x)\} = \mathcal{A}\{F(x)\} + \mathcal{A}\left\{\int_0^x \frac{\omega(t)}{(x-t)^\alpha}dt\right\}, 0 < \alpha < 1$

$\Rightarrow \mathcal{A}\{\omega(x)\} = \mathcal{A}\{F(x)\} + \mathcal{A}\{\omega(x) * x^{-\alpha}\}, 0 < \alpha < 1$ (2)

Use of convolution property of Anuj transform in Eq. (2) suggests

$\mathcal{A}\{\omega(x)\} = \mathcal{A}\{F(x)\} + \frac{1}{q^2}\mathcal{A}\{\omega(x)\}\mathcal{A}\{x^{-\alpha}\}, 0 < \alpha < 1$

$\Rightarrow \mathcal{A}\{\omega(x)\} = \mathcal{A}\{F(x)\} + \frac{1}{q^2}\mathcal{A}\{\omega(x)\}\Gamma(1-\alpha)q^{3-\alpha}, 0 < \alpha < 1$

$\Rightarrow \mathcal{A}\{\omega(x)\} = \left[\frac{1}{1-\Gamma(1-\alpha)q^{1-\alpha}}\right]\mathcal{A}\{F(x)\}, 0 < \alpha < 1$ (3)

Operating inverse Anuj transform Eq. (3) gives

$\omega(x) = \mathcal{A}^{-1}\left\{\left[\frac{1}{1-\Gamma(1-\alpha)q^{1-\alpha}}\right]\mathcal{A}\{F(x)\}\right\}, 0 < \alpha < 1$, which gives the required closed form solution of Eq. (1).

5. NUMERICAL EXAMPLES: In this section several examples of GAIESK have considered to visualization the efficiency of Anuj transform.

Example: 5.1 Consider the GAIESK

$\omega(x) = x - \frac{9}{4}\left(x^{4/3}\right) + \int_0^x \frac{\omega(t)}{(x-t)^{2/3}}dt$

(4)

Operating Anuj transform on Eq. (4) gives

$\mathcal{A}\{\omega(x)\} = \mathcal{A}\{x\} - \frac{9}{4}\mathcal{A}\left\{x^{4/3}\right\} + \mathcal{A}\left\{\int_0^x \frac{\omega(t)}{(x-t)^{2/3}}dt\right\}$

$\Rightarrow \mathcal{A}\{\omega(x)\} = \mathcal{A}\{x\} - \frac{9}{4}\mathcal{A}\left\{x^{4/3}\right\} + \mathcal{A}\left\{\omega(x) * x^{-(2/3)}\right\}$ (5)

Use of convolution property of Anuj transform in Eq. (5) suggests

$\mathcal{A}\{\omega(x)\} = q^4 - \frac{9}{4}\Gamma\left(\frac{7}{3}\right)q^{\frac{13}{3}} + \frac{1}{q^2}\mathcal{A}\{\omega(x)\}\mathcal{A}\left\{x^{-(2/3)}\right\}$

$\Rightarrow \mathcal{A}\{\omega(x)\} = q^4 - \frac{9}{4}\Gamma\left(\frac{7}{3}\right)q^{\frac{13}{3}} + \frac{1}{q^2}\mathcal{A}\{\omega(x)\}\Gamma\left(\frac{1}{3}\right)q^{\frac{7}{3}}$

$\Rightarrow \mathcal{A}\{\omega(x)\} = q^4$ (6)

Operating inverse Anuj transform Eq. (6) gives

$\omega(x) = \mathcal{A}^{-1}\{q^4\} = x$, which provides the required closed form solution of Eq. (4).

Example: 5.2 Consider the GAIESK

$\omega(x) = x^2 - \frac{128}{45}\left(x^{9/4}\right) + \int_0^x \frac{\omega(t)}{(x-t)^{3/4}}dt$ (7)

Operating Anuj transform on Eq. (7) gives

$\mathcal{A}\{\omega(x)\} = \mathcal{A}\{x^2\} - \frac{128}{45}\mathcal{A}\left\{x^{9/4}\right\} + \mathcal{A}\left\{\int_0^x \frac{\omega(t)}{(x-t)^{3/4}}dt\right\}$

$\Rightarrow \mathcal{A}\{\omega(x)\} = \mathcal{A}\{x^2\} - \frac{128}{45}\mathcal{A}\left\{x^{9/4}\right\} + \mathcal{A}\left\{\omega(x) * x^{-(3/4)}\right\}$ (8)

Use of convolution property of Anuj transform in Eq. (8) suggests

$\mathcal{A}\{\omega(x)\} = 2q^5 - \frac{128}{45}\Gamma\left(\frac{13}{4}\right)q^{\frac{21}{4}} + \frac{1}{q^2}\mathcal{A}\{\omega(x)\}\mathcal{A}\left\{x^{-(3/4)}\right\}$

$\Rightarrow \mathcal{A}\{\omega(x)\} = 2q^5 - \frac{128}{45}\Gamma\left(\frac{13}{4}\right)q^{\frac{21}{4}} + \frac{1}{q^2}\mathcal{A}\{\omega(x)\}\Gamma\left(\frac{1}{4}\right)q^{\frac{9}{4}}$

$\Rightarrow \mathcal{A}\{\omega(x)\} = 2q^5$ (9)

Operating inverse Anuj transform Eq. (9) gives

$\omega(x) = \mathcal{A}^{-1}\{2q^5\} = 2\mathcal{A}^{-1}\{q^5\} = x^2$, which provides the required closed form solution of Eq. (7).

Example: 5.3 Consider the GAIESK

$\omega(x) = x - \frac{25}{14}\left(x^{7/5}\right) + \int_0^x \frac{\omega(t)}{(x-t)^{3/5}}dt$ (10)

Operating Anuj transform on Eq. (10) gives

$\mathcal{A}\{\omega(x)\} = \mathcal{A}\{x\} - \frac{25}{14}\mathcal{A}\left\{x^{7/5}\right\} + \mathcal{A}\left\{\int_0^x \frac{\omega(t)}{(x-t)^{3/5}}dt\right\}$

$\Rightarrow \mathcal{A}\{\omega(x)\} = \mathcal{A}\{x\} - \frac{25}{14}\mathcal{A}\left\{x^{7/5}\right\} + \mathcal{A}\left\{\omega(x) * x^{-(3/5)}\right\}$ (11)

Use of convolution property of Anuj transform in Eq. (11) suggests

$\mathcal{A}\{\omega(x)\} = q^4 - \frac{25}{14}\Gamma\left(\frac{12}{5}\right)q^{\frac{22}{5}} + \frac{1}{q^2}\mathcal{A}\{\omega(x)\}\mathcal{A}\left\{x^{-(3/5)}\right\}$

$\Rightarrow \mathcal{A}\{\omega(x)\} = q^4 - \frac{25}{14}\Gamma\left(\frac{12}{5}\right) q^{\frac{22}{5}} + \frac{1}{q^2}\mathcal{A}\{\omega(x)\}\Gamma\left(\frac{2}{5}\right) q^{\frac{12}{5}}$

$\Rightarrow \mathcal{A}\{\omega(x)\} = q^4$ (12)

Operating inverse Anuj transform Eq. (12) gives

$\omega(x) = \mathcal{A}^{-1}\{q^4\} = x$, which provides the required closed form solution of Eq. (10).

Example: 5.4 Consider the GAIESK

$\omega(x) = x - \frac{36}{55}\left(x^{11/6}\right) + \int_0^x \frac{\omega(t)}{(x-t)^{1/6}} dt$ (13)

Operating Anuj transform on Eq. (13) gives

$$\mathcal{A}\{\omega(x)\} = \mathcal{A}\{x\} - \frac{36}{55}\mathcal{A}\left\{x^{11/6}\right\} + \mathcal{A}\left\{\int_0^x \frac{\omega(t)}{(x-t)^{1/6}} dt\right\}$$

$\Rightarrow \mathcal{A}\{\omega(x)\} = \mathcal{A}\{x\} - \frac{36}{55}\mathcal{A}\left\{x^{11/6}\right\} + \mathcal{A}\left\{\omega(x) * x^{-(1/6)}\right\}$ (14)

Use of convolution property of Anuj transform in Eq. (14) suggests

$$\mathcal{A}\{\omega(x)\} = q^4 - \frac{36}{55}\Gamma\left(\frac{17}{6}\right) q^{\frac{29}{6}} + \frac{1}{q^2}\mathcal{A}\{\omega(x)\}\mathcal{A}\left\{x^{-(1/6)}\right\}$$

$\Rightarrow \mathcal{A}\{\omega(x)\} = q^4 - \frac{36}{55}\Gamma\left(\frac{17}{6}\right) q^{\frac{29}{6}} + \frac{1}{q^2}\mathcal{A}\{\omega(x)\}\Gamma\left(\frac{5}{6}\right) q^{\frac{17}{6}}$

$\Rightarrow \mathcal{A}\{\omega(x)\} = q^4$ (15)

Operating inverse Anuj transform Eq. (15) gives

$\omega(x) = \mathcal{A}^{-1}\{q^4\} = x$, which provides the required closed form solution of Eq. (13).

Example: 5.5 Consider the GAIESK

$\omega(x) = x - \frac{9}{10}\left(x^{5/3}\right) + \int_0^x \frac{\omega(t)}{(x-t)^{1/3}} dt$ (16)

Operating Anuj transform on Eq. (16) gives

$$\mathcal{A}\{\omega(x)\} = \mathcal{A}\{x\} - \frac{9}{10}\mathcal{A}\left\{x^{5/3}\right\} + \mathcal{A}\left\{\int_0^x \frac{\omega(t)}{(x-t)^{1/3}} dt\right\}$$

$\Rightarrow \mathcal{A}\{\omega(x)\} = \mathcal{A}\{x\} - \frac{9}{10}\mathcal{A}\left\{x^{5/3}\right\} + \mathcal{A}\left\{\omega(x) * x^{-(1/3)}\right\}$ (17)

Use of convolution property of Anuj transform in Eq. (17) suggests

$$\mathcal{A}\{\omega(x)\} = q^4 - \frac{9}{10}\Gamma\left(\frac{8}{3}\right) q^{\frac{14}{3}} + \frac{1}{q^2}\mathcal{A}\{\omega(x)\}\mathcal{A}\left\{x^{-(1/3)}\right\}$$

$\Rightarrow \mathcal{A}\{\omega(x)\} = q^4 - \frac{9}{10}\Gamma\left(\frac{8}{3}\right) q^{\frac{14}{3}} + \frac{1}{q^2}\mathcal{A}\{\omega(x)\}\Gamma\left(\frac{8}{3}\right) q^{\frac{14}{3}}$

$\Rightarrow \mathcal{A}\{\omega(x)\} = q^4$ (18)

Operating inverse Anuj transform Eq. (18) gives

$\omega(x) = \mathcal{A}^{-1}\{q^4\} = x$, which provides the required closed form solution of Eq. (16).

6. CONCLUSION: In the present research, authors implemented Anuj transform to determine the closed form solution of the generalized Abel's integral equation of second kind. The efficiency of Anuj transform was visualized by solving several examples of GAIESK. The outcomes of these examples suggested that the method of Anuj transform is efficient and accurate in finding the closed form solutions for GAIESK. In future, Anuj transform can be use to solve the system of GAIESK.

REFERENCES:

1. Debnath, L. and Bhatta, D., Integral Transforms and their Applications, Chapman & Hall/ CRC, Boca Raton, 2007.
2. Davies, B., Integral Transforms and their Applications, Springer-Verlag, New York, 2002.
3. Aggarwal, S., Gupta, A.R., Singh, D.P., Asthana, N. and Kumar, N., Application of Laplace transform for solving population growth and decay problems, International Journal of Latest Technology in engineering, Management & Applied Science, 7(9), 141-145, 2018.
4. Jafari, H. and Aggarwal, S., Upadhyaya integral transform: A tool for solving non-linear Volterra integral equations, Mathematics and Computational Sciences, 5(2), 63-71, 2024. https://doi.org/10.30511/MCS.2024.2029301.1174
5. Aggarwal, S., Kumar, R. and Chandel, J., Solution of linear Volterra integral equation of second kind via Rishi transform, Journal of Scientific Research, 15(1), 111-119, 2023. https://dx.doi.org/10.3329/jsr.v15i1.60337
6. Kumar, A., Bansal, S. and Aggarwal, S., Determination of the concentrations of the reactants of first order consecutive chemical reaction using Anuj transform, European Chemical Bulletin, 12(7), 2528-2534, 2023.
7. Aggarwal, S., Kumar, R. and Chandel, J., Exact solution of non-linear Volterra integral equation of first kind using Rishi transform, Bulletin of Pure and Applied Sciences- Math & Stat. 41E(2), Jul-Dec 2022, 159-166, 2022. https://doi.org/10.5958/2320-3226.2022.00022.4
8. Jafari, H., A new general integral transform for solving integral equations, Journal of Advanced Research, 32, 133-138, 2021. https://doi.org/10.1016/j.jare.2020.08.016

9. Kumar, A., Bansal, S. and Aggarwal, S., Determination of the blood glucose concentration of a patient during continuous intravenous injection using Anuj transform, Neuroquantology, 19(12), 303-306, 2021.
10. Bansal S., Kumar, A. and Aggarwal, S., Application of Anuj transform for the solution of bacteria growth model, Sis Science Journal, 9(6), 1465-1472, 2022.
11. Adomian, G., Solving Frontier Problems of Physics: The Decomposition Method, Kluwer, 1994.
12. Wazwaz, A.M., Linear and Nonlinear Integral Equations: Methods and Applications, Higher Education Press, Beijing, 2011.
13. Kaya, R. and Taseli, H., A Rayleigh-Ritz method for numerical solutions of linear Fredholm integral equations of the second kind, Journal of Mathematical Chemistry, 60, 1107-1129, 2022. https://doi.org/10.1007/s10910-022-01344-9
14. Thomas, K.S., Galerkin methods for singular integral equations, Mathematics of Computation, 36(153), 193-205, 1981.
15. Xu, L., Variational iteration method for solving integral equations, Computers and Mathematics with Applications, 54, 1071-1078, 2007.
16. Nadjafi, J.S. and Heidari, M., Anew modified quadrature method for solving linear weakly singular integral equations, World Journal of Modelling and Simulation, 10(1), 69-78, 2014.
17. Higazy, M., Aggarwal, S. and Taher, A. N., Sawi decomposition method for Volterra integral equation with application, Journal of Mathematics, 2020, 1-13, 2020. https://doi.org/10.1155/2020/6687134
18. Upadhyaya, L. M., Introducing the Upadhyaya integral transform, Bulletin of Pure and Applied Sciences, 38E (Math & Stat.) (1), 471-510, 2019. https://doi.org/10.5958/2320-3226.2019.00051.1
19. Kumar, R., Chandel J. and Aggarwal, S., A new integral transform "Rishi Transform" with application, Journal of Scientific Research, 14(2), 521-532, 2022. https://dx.doi.org/10.3329/jsr.v14i2.56545
20. Kumar, A., Bansal S. and Aggarwal, S., A new novel integral transform "Anuj transform" with application, Design Engineering, 2021(9), 12741-12751, 2021.
21. Aggarwal, S. and Gupta, A.R., Dualities between Mohand transform and some useful integral transforms, International Journal of Recent Technology and Engineering, 8(3), 843-847, September 2019.
22. Aggarwal, S. and Gupta, A.R., Dualities between some useful integral transforms and Sawi transform, International Journal of Recent Technology and Engineering, 8(3), 5978-5982, September 2019.
23. Aggarwal, S., Bhatnagar, K. and Dua, A., Dualities between Elzaki transform and some useful integral transforms, International Journal of Innovative Technology and Exploring Engineering, 8(12), 4312-4318, October 2019.
24. Chauhan, R., Kumar, N. and Aggarwal, S., Dualities between Laplace-Carson transform and some useful integral transforms, International Journal of Innovative Technology and Exploring Engineering, 8(12), 1654-1659, October 2019.
25. Aggarwal, S. and Bhatnagar, K., Dualities between Laplace transform and some useful integral transforms, International Journal of Engineering and Advanced Technology, 9(1), 936-941, October 2019.
26. Chaudhary, R., Sharma, S.D., Kumar, N. and Aggarwal, S., Connections between Aboodh transform and some useful integral transforms, International Journal of Innovative Technology and Exploring Engineering, 9(1), 1465-1470, November 2019.
27. Aggarwal, S., Sharma, N. and Chauhan, R., Duality relations of Kamal transform with Laplace, Laplace-Carson, Aboodh, Sumudu, Elzaki, Mohand and Sawi transforms, SN Applied Sciences, 2, 135, 2020.
28. Mishra, R., Aggarwal, S., Chaudhary, L. and Kumar, A., Relationship between Sumudu and some efficient integral transforms, International Journal of Innovative Technology and Exploring Engineering, 9(3), 153-159, January 2020.
29. Jerri, A., Introduction to Integral Equations with Applications, Wiley, New York, 1999.
30. Estrada, R. and Kanwal, R., Singular Integral Equations, Birkhauser, Berlin, 2000.
31. Linz, P., Analytical and Numerical Methods for Volterra Equations, SIAM, Philadelphia, 1985.
32. Moiseiwitsch, B.L., Integral Equations, Longman, London and New York, 1977.
33. TeBeest, K.G., Numerical and analytical solutions of Volterra's population model, SIAM Review, 39(3), 484-493, 1997.
34. Aggarwal, S., Kumar, A. and Bansal, S., Bessel functions of first kind and their Anuj transforms, World Journal of Advanced Research and Reviews, 19(01), 672-681, 2023. https://doi.org/10.30574/wjarr.2023.19.1.1381