

Strongly Contra Neutrosophic $G\alpha^*$ -Continuous Function In Neutrosophic Topological Spaces

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Abstract: In this paper, the concept of Strongly Contra Neutrosophic $g\alpha^*$ -continuous function and Contra Neutrosophic $g\alpha^*$ -irresolute mapping are introduced in Neutrosophic Topological Spaces. We also discussed about the interrelations of these functions with the already existing neutrosophic functions. We also discussed about the properties and characterizations of these functions with examples.

Keywords: Neutrosophic $g\alpha^*$ -closed sets, Strongly Contra Neutrosophic $g\alpha^*$ -continuous, Contra Neutrosophic $g\alpha^*$ -irresolute mapping.

1 Introduction

The N_{eu} set theory was introduced by F.Smarandache[11]. It consists of three components namely truth, indeterminacy and false membership function. The idea of N_{eu} set comes from two concepts. One is intuitionistic fuzzy sets and it is introduced by K.Atanassov's[6]. The other idea is fuzzy sets and it is introduced by L.A.Zadeh's[15]. R.Dhavaseelan, S.Jafari, C.Ozel and M.A.Al-Shumrani[10] has introduced the concept of Strongly Contra $N_{eu}g$ -continuous and he also discussed about the idea of Contra $N_{eu}g$ -irresolute mapping. The real life application of N_{eu} topology is applied in Information Systems, Applied Mathematics etc.

In this paper, we introduce some new concepts in $N_{eu}TS$ such as Strongly Contra $N_{eu}g\alpha^*$ -continuous and Contra $N_{eu}g\alpha^*$ -irresolute mapping.

2 Preliminaries

Definition 2.1:[14] Let $\mathbb{P}$ be a non-empty fixed set. A N_{eu} set H on the universe $\mathbb{P}$ is defined as $H = \{ \langle p, (t_H(p), i_H(p), f_H(p)) \rangle : p \in \mathbb{P} \}$ where $t_H(p)$, $i_H(p)$, $f_H(p)$ represent the degree of membership function $t_H(p)$, the degree of indeterminacy $i_H(p)$ and the degree of non-membership function $f_H(p)$ respectively for each element $p \in \mathbb{P}$ to the set H . Also, $t_H, i_H, f_H : \mathbb{P} \rightarrow]0, 1^+[$ and $0 \leq t_H(p) + i_H(p) + f_H(p) \leq 3^+$. Set of all Neutrosophic set over $\mathbb{P}$ is denoted by $N_{eu}(\mathbb{P})$.

Definition 2.2:[14] Let $\mathbb{P}$ be a non-empty set. $A = \{ \langle p, (t_A(p), i_A(p), f_A(p)) \rangle : p \in \mathbb{P} \}$ and $B = \{ \langle p, (t_B(p), i_B(p), f_B(p)) \rangle : p \in \mathbb{P} \}$ are N_{eu} sets, then

- (i) $A \subseteq B$ if $t_A(p) \leq t_B(p)$, $i_A(p) \leq i_B(p)$, $f_A(p) \geq f_B(p)$ for all $p \in \mathbb{P}$.
- (ii) $A \cap B = \{ \langle p, (\min(t_A(p), t_B(p)), \min(i_A(p), i_B(p)), \max(f_A(p), f_B(p))) \rangle : p \in \mathbb{P} \}$.
- (iii) $A \cup B = \{ \langle p, (\max(t_A(p), t_B(p)), \max(i_A(p), i_B(p)), \min(f_A(p), f_B(p))) \rangle : p \in \mathbb{P} \}$.
- (iv) $A^c = \{ \langle p, (f_A(p), 1 - i_A(p), t_A(p)) \rangle : p \in \mathbb{P} \}$.
- (v) $0_{N_{eu}} = \{ \langle p, (0, 0, 1) \rangle : p \in \mathbb{P} \}$ and $1_{N_{eu}} = \{ \langle p, (1, 1, 0) \rangle : p \in \mathbb{P} \}$.

Definition 2.3:[14] A neutrosophic topology ($N_{eu}T$) on a non-empty set $\mathbb{P}$ is a family $\tau_{N_{eu}}$ of neutrosophic sets in $\mathbb{P}$

satisfying the following axioms ,

- (i) $0_{N_{eu}}, 1_{N_{eu}} \in \tau_{N_{eu}}$.
- (ii) $\mathbb{A}_1 \cap \mathbb{A}_2 \in \tau_{N_{eu}}$ for any $\mathbb{A}_1, \mathbb{A}_2 \in \tau_{N_{eu}}$.
- (iii) $\cup \mathbb{A}_i \in \tau_{N_{eu}}$ for every family $\{\mathbb{A}_i / i \in \Omega\} \subseteq \tau_{N_{eu}}$.

In this case , the ordered pair $(\mathbb{P}, \tau_{N_{eu}})$ or simply $\mathbb{P}$ is called a neutrosophic topological space ($N_{eu}TS$) . The elements of $\tau_{N_{eu}}$ is neutrosophic open set ($N_{eu} - OS$) and $\tau_{N_{eu}}^c$ is neutrosophic closed set ($N_{eu} - CS$) .

Definition 2.4:[1] A neutrosophic set $\mathbb{A}$ in a $N_{eu}TS$ $(\mathbb{P}, \tau_{N_{eu}})$ is neutrosophic generalized semi alpha star closed set ($N_{eu}gs\alpha^* - CS$) if $N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathbb{A})) \subseteq N_{eu} - int(G)$, whenever $\mathbb{A} \subseteq G$ and G is $N_{eu}\alpha^* - OS$.

Definition 2.5:[2] A $N_{eu}TS$ $(\mathbb{P}, \tau_{N_{eu}})$ is $N_{eu}gs\alpha^* - T_{1/2}$ space if every $N_{eu}gs\alpha^* - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$ is a $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$.

Definition 2.6: A N_{eu} function $f_N: (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ is

- (1) $N_{eu} -$ continuous[7] if f_N^{-1} of $N_{eu} - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$ is a $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$.
- (2) $N_{eu}gs\alpha^* -$ continuous[2] if f_N^{-1} of $N_{eu} - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$ is a $N_{eu}gs\alpha^* - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$.
- (3) $N_{eu}gs\alpha^* -$ irresolute map[2] if f_N^{-1} of $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$ is a $N_{eu}gs\alpha^* - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$.
- (4) strongly $N_{eu}gs\alpha^* -$ continuous[3] if f_N^{-1} of $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$ is a $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$.
- (5) perfectly $N_{eu}gs\alpha^* -$ continuous[3] if f_N^{-1} of $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$ is both $N_{eu} - OS$ and $N_{eu} - CS$ (ie , $N_{eu} -$ clopen set) in $(\mathbb{P}, \tau_{N_{eu}})$.
- (6) almost $N_{eu}gs\alpha^* -$ continuous[4] if f_N^{-1} of $N_{eu}R - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$ is a $N_{eu}gs\alpha^* - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$.
- (7) almost contra $N_{eu}gs\alpha^* -$ continuous[5] if f_N^{-1} of $N_{eu}R - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$ is a $N_{eu}gs\alpha^* - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$.
- (8) contra $N_{eu} -$ continuous[10] if f_N^{-1} of $N_{eu} - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$ is a $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$.
- (9) contra $N_{eu}gs\alpha^* -$ continuous[5] if f_N^{-1} of $N_{eu} - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$ is a $N_{eu}gs\alpha^* - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$.

Definition 2.7: A $N_{eu} -$ function $f_N: (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ is

- (1) strongly contra $N_{eu}g -$ continuous[10] if f_N^{-1} of $N_{eu}g - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$ is a $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$.
- (2) strongly contra $N_{eu}gR -$ continuous[7] if f_N^{-1} of $N_{eu}gR - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$ is a $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$.
- (3) strongly contra $N_{eu}g\alpha -$ continuous[9] if f_N^{-1} of $N_{eu}g\alpha - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$ is a $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$.
- (4) contra $N_{eu}gR -$ irresolute[7] if f_N^{-1} of $N_{eu}gR - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$ is a $N_{eu}gR - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$.
- (5) contra $N_{eu}g\alpha -$ irresolute[9] if f_N^{-1} of $N_{eu}g\alpha - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$ is a $N_{eu}g\alpha - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$.
- (6) contra $N_{eu}g -$ irresolute[10] if f_N^{-1} of $N_{eu}g - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$ is a $N_{eu}g - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$.

Definition 2.8:[8] A space $(\mathbb{P}, \tau_{N_{eu}})$ is $N_{eu} -$ locally indiscrete space if every $N_{eu} - OS$ is a $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$.

Definition 2.9:[5] A space $(\mathbb{P}, \tau_{N_{eu}})$ is $N_{eu}gs\alpha^* -$ locally indiscrete space if every $N_{eu}gs\alpha^* - OS$ is a $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$.

Definition 2.10:[13] Let $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}\}$ is $N_{eu}TS$ over $\mathbb{P}$. Then $(\mathbb{P}, \tau_{N_{eu}})$ is $N_{eu} -$ discrete topological space .

3 Strongly Contra Neutrosophic $gs\alpha^* -$ Continuous Function

Definition 3.1: A $N_{eu} -$ function $f_N: (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ is strongly contra $N_{eu}gs\alpha^* -$ continuous if the inverse image of every $N_{eu}gs\alpha^* - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$ is a $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. (ie) $f_N^{-1}(\mathbb{A})$ is a $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$ for every $N_{eu}gs\alpha^* - OS$ $\mathbb{A}$ in $(\mathbb{Q}, \sigma_{N_{eu}})$.

Theorem 3.2: Every strongly contra $N_{eu}gs\alpha^* -$ continuous is contra $N_{eu}gs\alpha^* -$ continuous , but not conversely .

Proof: Let $f_N: (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be any $N_{eu} -$ function . Let $\mathbb{A}$ be any $N_{eu} - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then $\mathbb{A}$ is a $N_{eu}gs\alpha^* - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Given f_N is strongly contra $N_{eu}gs\alpha^* -$ continuous , then $f_N^{-1}(\mathbb{A})$ is a $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f_N^{-1}(\mathbb{A})$ is a $N_{eu}gs\alpha^* - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f_N$ is contra $N_{eu}gs\alpha^* -$ continuous.

Example 3.3: Let $\mathbb{P} = \{p\}$ and $\mathbb{Q} = \{q\}$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ and $\sigma_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{B}\}$ are $N_{eu}TS$ on $(\mathbb{P}, \tau_{N_{eu}})$ and $(\mathbb{Q}, \sigma_{N_{eu}})$. Also $\mathbb{A} = \{p, (0.4, 0.6, 0.8)\}$ and $\mathbb{B} = \{q, (0.2, 0.4, 0.6)\}$ are $N_{eu}(\mathbb{P})$ and $N_{eu}(\mathbb{Q})$. Define a map $f_N: (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ by $f_N(p) = q$. Let $\mathbb{B} = \{q, (0.2, 0.4, 0.6)\}$ be a $N_{eu} - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then $f_N^{-1}(\mathbb{B}) = \{p, (0.2, 0.4, 0.6)\} \Rightarrow f_N^{-1}(\mathbb{B})$ is a $N_{eu}gs\alpha^* - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f_N$ is contra $N_{eu}gs\alpha^* -$ continuous . But f_N is not strongly contra $N_{eu}gs\alpha^* -$ continuous .

Theorem 3.4: Every strongly contra $N_{eu}gs\alpha^*$ – continuous is contra N_{eu} – continuous , but not conversely .

Proof: Let $f_N : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be any N_{eu} – function . Let $\mathcal{A}$ be any N_{eu} – OS in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then $\mathcal{A}$ is a $N_{eu}gs\alpha^*$ – OS in $(\mathbb{Q}, \sigma_{N_{eu}})$. Given f_N is strongly contra $N_{eu}gs\alpha^*$ – continuous , then $f_N^{-1}(\mathcal{A})$ is a N_{eu} – CS in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f_N$ is contra N_{eu} – continuous .

Example 3.5: Let $\mathbb{P} = \{p\}$ and $\mathbb{Q} = \{q\}$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathcal{A}\}$ and $\sigma_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathcal{B}\}$ are N_{eu} TS on $(\mathbb{P}, \tau_{N_{eu}})$ and $(\mathbb{Q}, \sigma_{N_{eu}})$ respectively . Also $\mathcal{A} = \{\{p, (0.6, 0.6, 0.2)\}\}$ and $\mathcal{B} = \{\{q, (0.2, 0.4, 0.6)\}\}$ are $N_{eu}(\mathbb{P})$ and $N_{eu}(\mathbb{Q})$. Define a map $f_N : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ by $f_N(p) = q$. Let $\mathcal{B} = \{\{q, (0.2, 0.4, 0.6)\}\}$ be a N_{eu} – OS in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then $f_N^{-1}(\mathcal{B}) = \{\{p, (0.2, 0.4, 0.6)\}\} \Rightarrow f_N^{-1}(\mathcal{B})$ is a N_{eu} – CS in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f_N$ is contra N_{eu} – continuous . But f_N is not strongly contra $N_{eu}gs\alpha^*$ – continuous.

Theorem 3.6: Every strongly contra $N_{eu}gs\alpha^*$ – continuous is almost contra $N_{eu}gs\alpha^*$ – continuous , but not conversely .

Proof: Let $f_N : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be any N_{eu} – function . Let $\mathcal{A}$ be any $N_{eu}R$ – OS in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then $\mathcal{A}$ is a N_{eu} – OS in $(\mathbb{Q}, \sigma_{N_{eu}})$ [12] $\Rightarrow \mathcal{A}$ is a $N_{eu}gs\alpha^*$ – OS in $(\mathbb{Q}, \sigma_{N_{eu}})$. Given f_N is strongly contra $N_{eu}gs\alpha^*$ – continuous function , then $f_N^{-1}(\mathcal{A})$ is a N_{eu} – CS in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f_N^{-1}(\mathcal{A})$ is a $N_{eu}gs\alpha^*$ – CS in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f_N$ is almost contra $N_{eu}gs\alpha^*$ – continuous .

Example 3.7: Let $\mathbb{P} = \{p\}$ and $\mathbb{Q} = \{q\}$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathcal{A}\}$ and $\sigma_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathcal{B}\}$ are N_{eu} TS on $(\mathbb{P}, \tau_{N_{eu}})$ and $(\mathbb{Q}, \sigma_{N_{eu}})$. Also $\mathcal{A} = \{\{p, (0.4, 0.5, 0.7)\}\}$ and $\mathcal{B} = \{\{q, (0.3, 0.4, 0.7)\}\}$ are $N_{eu}(\mathbb{P})$ and $N_{eu}(\mathbb{Q})$. Define a map $f_N : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ by $f_N(p) = q$. Let $\mathcal{B} = \{\{q, (0.3, 0.4, 0.7)\}\}$ be a $N_{eu}R$ – OS in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then $f_N^{-1}(\mathcal{B}) = \{\{p, (0.3, 0.4, 0.7)\}\} \Rightarrow f_N^{-1}(\mathcal{B})$ is a $N_{eu}gs\alpha^*$ – CS in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f_N$ is almost contra $N_{eu}gs\alpha^*$ – continuous . But f_N is not strongly contra $N_{eu}gs\alpha^*$ – continuous .

Theorem 3.8: Every perfectly $N_{eu}gs\alpha^*$ – continuous is strongly contra $N_{eu}gs\alpha^*$ – continuous , but not conversely .

Proof: Let $f_N : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be any N_{eu} – function . Let $\mathcal{A}$ be any $N_{eu}gs\alpha^*$ – OS in $(\mathbb{Q}, \sigma_{N_{eu}})$. Given f_N is perfectly $N_{eu}gs\alpha^*$ – continuous, then $f_N^{-1}(\mathcal{A})$ is both N_{eu} – OS and N_{eu} – CS in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f_N^{-1}(\mathcal{A})$ is a N_{eu} – CS in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f_N$ is strongly contra $N_{eu}gs\alpha^*$ – continuous .

Example 3.9: Let $\mathbb{P} = \{p\}$ and $\mathbb{Q} = \{q\}$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathcal{A}, \mathcal{C}\}$ and $\sigma_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathcal{B}\}$ are N_{eu} TS on $(\mathbb{P}, \tau_{N_{eu}})$ and $(\mathbb{Q}, \sigma_{N_{eu}})$. Also $\mathcal{A} = \{\{p, (0.4, 0.6, 0.2)\}\}$, $\mathcal{B} = \{\{q, (0.4, 0.6, 0.2)\}\}$ and $\mathcal{C} = \{\{p, ([0, 0.2], [0, 0.4], [0.4, 1])\}\}$ are $N_{eu}(\mathbb{P})$ and $N_{eu}(\mathbb{Q})$. Define a map $f_N : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ by $f_N(p) = q$. Let $\mathcal{C} = \{\{q, ([0, 0.2], [0, 0.4], [0.4, 1])\}\}$ be a $N_{eu}gs\alpha^*$ – CS in $(\mathbb{Q}, \sigma_{N_{eu}}) \Rightarrow f_N^{-1}(\mathcal{C})$ is a N_{eu} – OS in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f_N$ is strongly contra $N_{eu}gs\alpha^*$ – continuous . But f_N is not perfectly $N_{eu}gs\alpha^*$ – continuous .

Theorem 3.10: Let $f_N : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be strongly contra $N_{eu}gs\alpha^*$ – continuous and $(\mathbb{P}, \tau_{N_{eu}})$ is N_{eu} – locally indiscrete space , then f_N is strongly $N_{eu}gs\alpha^*$ – continuous .

Proof: Let $\mathcal{A}$ be any $N_{eu}gs\alpha^*$ – CS in $(\mathbb{Q}, \sigma_{N_{eu}})$. Given f_N is strongly contra $N_{eu}gs\alpha^*$ – continuous, then $f_N^{-1}(\mathcal{A})$ is a N_{eu} – OS in $(\mathbb{P}, \tau_{N_{eu}})$. Given $(\mathbb{P}, \tau_{N_{eu}})$ is N_{eu} – locally indiscrete space , then $f_N^{-1}(\mathcal{A})$ is a N_{eu} – CS in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f_N$ is strongly $N_{eu}gs\alpha^*$ – continuous .

Theorem 3.11: Let $f_N : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be strongly contra $N_{eu}gs\alpha^*$ – continuous and $(\mathbb{P}, \tau_{N_{eu}})$ is N_{eu} – locally indiscrete space , then f_N is $N_{eu}gs\alpha^*$ – irresolute .

Proof: Let $\mathcal{A}$ be any $N_{eu}gs\alpha^*$ – CS in $(\mathbb{Q}, \sigma_{N_{eu}})$. Given f_N is strongly contra $N_{eu}gs\alpha^*$ – continuous , then $f_N^{-1}(\mathcal{A})$ is a N_{eu} – OS in $(\mathbb{P}, \tau_{N_{eu}})$. Given $(\mathbb{P}, \tau_{N_{eu}})$ is N_{eu} – locally indiscrete space , then $f_N^{-1}(\mathcal{A})$ is a N_{eu} – CS in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f_N^{-1}(\mathcal{A})$ is a $N_{eu}gs\alpha^*$ – CS in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f_N$ is $N_{eu}gs\alpha^*$ – irresolute .

Theorem 3.12: Let $f_N : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be strongly contra $N_{eu}gs\alpha^*$ – continuous and $(\mathbb{P}, \tau_{N_{eu}})$ is N_{eu} – locally indiscrete space , then f_N is $N_{eu}gs\alpha^*$ – continuous .

Proof: Let $\mathcal{A}$ be any N_{eu} – CS in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then $\mathcal{A}$ is a $N_{eu}gs\alpha^*$ – CS in $(\mathbb{Q}, \sigma_{N_{eu}})$. Given f_N is strongly contra $N_{eu}gs\alpha^*$ – continuous , then $f_N^{-1}(\mathcal{A})$ is a N_{eu} – OS in $(\mathbb{P}, \tau_{N_{eu}})$. Given $(\mathbb{P}, \tau_{N_{eu}})$ is neutrosophic locally indiscrete space , then $f_N^{-1}(\mathcal{A})$ is a N_{eu} – CS in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f_N^{-1}(\mathcal{A})$ is a $N_{eu}gs\alpha^*$ – CS in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f_N$ is $N_{eu}gs\alpha^*$ – continuous .

Theorem 3.13: Let $f_N : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be contra $N_{eu}gs\alpha^*$ – continuous . Then f_N is strongly contra

$N_{eu}gs\alpha^*$ – continuous , if $(\mathbb{P}, \tau_{N_{eu}})$ and $(\mathbb{Q}, \sigma_{N_{eu}})$ are $N_{eu}gs\alpha^* - T_{1/2}$ space .

Proof: Let A be any $N_{eu}gs\alpha^* - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Since $(\mathbb{Q}, \sigma_{N_{eu}})$ is a $N_{eu}gs\alpha^* - T_{1/2}$ space , then A is a $N_{eu} - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Given f_N is contra $N_{eu}gs\alpha^* -$ continuous , then $f_N^{-1}(A)$ is a $N_{eu}gs\alpha^* - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Given $(\mathbb{P}, \tau_{N_{eu}})$ is a $N_{eu}gs\alpha^* - T_{1/2}$ space , then $f_N^{-1}(A)$ is a $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f_N$ is strongly contra $N_{eu}gs\alpha^* -$ continuous .

Theorem 3.14: Let $f_N : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be contra $N_{eu} -$ continuous and $(\mathbb{Q}, \sigma_{N_{eu}})$ is a $N_{eu}gs\alpha^* - T_{1/2}$ space , then f is strongly contra $N_{eu}gs\alpha^* -$ continuous .

Proof: Let A be any $N_{eu}gs\alpha^* - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Given $(\mathbb{Q}, \sigma_{N_{eu}})$ is a $N_{eu}gs\alpha^* - T_{1/2}$ space , then A is a $N_{eu} - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Given f_N is contra $N_{eu} -$ continuous , then $f_N^{-1}(A)$ is a $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f_N$ is strongly contra $N_{eu}gs\alpha^* -$ continuous .

Theorem 3.15: Let $f_N : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be strongly contra $N_{eu}gs\alpha^* -$ continuous and $(\mathbb{P}, \tau_{N_{eu}})$ is $N_{eu} -$ locally indiscrete space , then f_N is almost $N_{eu}gs\alpha^* -$ continuous.

Proof: Let A be any $N_{eu}R - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then A is a $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Given f_N is strongly contra $N_{eu}gs\alpha^* -$ continuous , then $f_N^{-1}(A)$ is a $N_{eu} - OS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Given $(\mathbb{P}, \tau_{N_{eu}})$ is a $N_{eu} -$ locally indiscrete space , then $f_N^{-1}(A)$ is a $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f_N^{-1}(A)$ is a $N_{eu}gs\alpha^* - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f_N$ is almost $N_{eu}gs\alpha^* -$ continuous .

Theorem 3.16: Let $f_N : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be strongly contra $N_{eu}gs\alpha^* -$ continuous and $(\mathbb{P}, \tau_{N_{eu}})$ is $N_{eu} -$ discrete topological space , then f is perfectly $N_{eu}gs\alpha^* -$ continuous .

Proof: Let A be any $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Given f_N is strongly contra $N_{eu}gs\alpha^* -$ continuous, then $f_N^{-1}(A)$ is a $N_{eu} - OS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Given $(\mathbb{P}, \tau_{N_{eu}})$ is a neutrosophic discrete topological space, then $f_N^{-1}(A)$ is a $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f_N^{-1}(A)$ is both $N_{eu} - OS$ and $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f_N$ is perfectly $N_{eu}gs\alpha^* -$ continuous .

Theorem 3.17: Let $f_N : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be any $N_{eu} -$ function . Then the following are equivalent .

(1) f_N is strongly contra $N_{eu}gs\alpha^* -$ continuous.

(2) The inverse image of every $N_{eu}gs\alpha^* - CS$ A in $(\mathbb{Q}, \sigma_{N_{eu}})$ is a $N_{eu} - OS$ in $(\mathbb{P}, \tau_{N_{eu}})$.

Proof: (1) $\Rightarrow$ (2) , Let A be any $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then A^c is a $N_{eu}gs\alpha^* - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Given f_N is strongly contra $N_{eu}gs\alpha^* -$ continuous, then $f_N^{-1}(A^c) = (f_N^{-1}(A))^c$ is a $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f_N^{-1}(A)$ is a $N_{eu} - OS$ in $(\mathbb{P}, \tau_{N_{eu}})$.

(2) $\Rightarrow$ (1) , Let A be any $N_{eu}gs\alpha^* - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then A^c is a $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. By hypothesis , $f_N^{-1}(A^c) = (f_N^{-1}(A))^c$ is a $N_{eu} - OS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f_N^{-1}(A)$ is a $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f_N$ is strongly contra $N_{eu}gs\alpha^* -$ continuous .

Theorem 3.18: Let $f_N : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be strongly contra $N_{eu}gs\alpha^* -$ continuous and $g_N : (\mathbb{Q}, \sigma_{N_{eu}}) \rightarrow (\mathbb{R}, \gamma_{N_{eu}})$ be contra $N_{eu}gs\alpha^* -$ continuous , then $g_N \circ f_N : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{R}, \gamma_{N_{eu}})$ is $N_{eu} -$ continuous .

Proof: Let A be any $N_{eu} - CS$ in $(\mathbb{R}, \gamma_{N_{eu}})$. Given g_N is contra $N_{eu}gs\alpha^* -$ continuous , then $g_N^{-1}(A)$ is a $N_{eu}gs\alpha^* - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Given f_N is strongly contra $N_{eu}gs\alpha^* -$ continuous , then $f_N^{-1}(g_N^{-1}(A)) = (g_N \circ f_N)^{-1}(A)$ is a $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow g_N \circ f_N$ is $N_{eu} -$ continuous .

Theorem 3.19: Let $f_N : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be strongly contra $N_{eu}gs\alpha^* -$ continuous and $g_N : (\mathbb{Q}, \sigma_{N_{eu}}) \rightarrow (\mathbb{R}, \gamma_{N_{eu}})$ be $N_{eu}gs\alpha^* -$ continuous , then $g_N \circ f_N : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{R}, \gamma_{N_{eu}})$ is a contra $N_{eu} -$ continuous .

Proof: Let A be any $N_{eu} - OS$ in $(\mathbb{R}, \gamma_{N_{eu}})$. Given g_N is $N_{eu}gs\alpha^* -$ continuous , then $g_N^{-1}(A)$ is a $N_{eu}gs\alpha^* - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Given f_N is strongly contra $N_{eu}gs\alpha^* -$ continuous , then $f_N^{-1}(g_N^{-1}(A)) = (g_N \circ f_N)^{-1}(A)$ is a $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow g_N \circ f_N$ is contra $N_{eu} -$ continuous .

Theorem 3.20: Let $f_N : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ and $g_N : (\mathbb{Q}, \sigma_{N_{eu}}) \rightarrow (\mathbb{R}, \gamma_{N_{eu}})$ be strongly contra $N_{eu}gs\alpha^* -$ continuous . Then $g_N \circ f_N : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{R}, \gamma_{N_{eu}})$ is strongly contra $N_{eu}gs\alpha^* -$ continuous , if $(\mathbb{P}, \tau_{N_{eu}})$ is $N_{eu} -$ locally indiscrete space .

Proof: Let A be any $N_{eu}gs\alpha^* - OS$ in $(\mathbb{R}, \gamma_{N_{eu}})$. Given g is strongly contra $N_{eu}gs\alpha^* -$ continuous, then $g_N^{-1}(A)$ is

a $N_{eu} - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}}) \Rightarrow g_N^{-1}(\mathbb{A})$ is a $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Given f_N is strongly contra $N_{eu}gs\alpha^* -$ continuous, then $f_N^{-1}(g_N^{-1}(\mathbb{A})) = (g_N \circ f_N)^{-1}(\mathbb{A})$ is a $N_{eu} - OS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Given $(\mathbb{P}, \tau_{N_{eu}})$ is $N_{eu} -$ locally indiscrete space, then $(g_N \circ f_N)^{-1}(\mathbb{A})$ is a $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow g_N \circ f_N$ is strongly contra $N_{eu}gs\alpha^* -$ continuous.

4 Contra Neutrosophic $gs\alpha^* -$ Irresolute Mapping

Definition 4.1: A $N_{eu} -$ function $f_N: (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ is contra $N_{eu}gs\alpha^* -$ irresolute if the inverse image of every $N_{eu}gs\alpha^* - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$ is a $N_{eu}gs\alpha^* - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. (ie) $f_N^{-1}(\mathbb{A})$ is a $N_{eu}gs\alpha^* - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$ for every $N_{eu}gs\alpha^* - OS$ $\mathbb{A}$ in $(\mathbb{Q}, \sigma_{N_{eu}})$.

Theorem 4.2: Every contra $N_{eu}gs\alpha^* -$ irresolute is contra $N_{eu}gs\alpha^* -$ continuous, but not conversely.

Proof: Let $f_N: (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be any $N_{eu} -$ function. Let $\mathbb{A}$ be any $N_{eu} - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then $\mathbb{A}$ is $N_{eu}gs\alpha^* - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Given f_N is contra $N_{eu}gs\alpha^* -$ irresolute, then $f_N^{-1}(\mathbb{A})$ is a $N_{eu}gs\alpha^* - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f_N$ is contra $N_{eu}gs\alpha^* -$ continuous.

Example 4.3: Let $\mathbb{P} = \{p\}$ and $\mathbb{Q} = \{q\}$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ and $\sigma_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{B}\}$ are $N_{eu}TS$ on $(\mathbb{P}, \tau_{N_{eu}})$ and $(\mathbb{Q}, \sigma_{N_{eu}})$. Also $\mathbb{A} = \{\langle p, (0.7, 0.6, 0.5) \rangle\}$ and $\mathbb{B} = \{\langle q, (0.3, 0.2, 0.8) \rangle\}$ are $N_{eu}(\mathbb{P})$ and $N_{eu}(\mathbb{Q})$. Define a map $f_N: (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ by $f_N(p) = q$. Let $\mathbb{B} = \{\langle q, (0.3, 0.2, 0.8) \rangle\}$ be a $N_{eu} - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then $f_N^{-1}(\mathbb{B}) = \{\langle p, (0.3, 0.2, 0.8) \rangle\} \Rightarrow f_N^{-1}(\mathbb{B})$ is a $N_{eu}gs\alpha^* - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f_N$ is contra $N_{eu}gs\alpha^* -$ continuous. But f_N is not contra $N_{eu}gs\alpha^* -$ irresolute.

Theorem 4.4: Every strongly contra $N_{eu}gs\alpha^* -$ continuous is contra $N_{eu}gs\alpha^* -$ irresolute, but not conversely.

Proof: Let $f_N: (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be any $N_{eu} -$ function. Let $\mathbb{A}$ be any $N_{eu}gs\alpha^* - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Given f_N is strongly contra $N_{eu}gs\alpha^* -$ continuous, then $f_N^{-1}(\mathbb{A})$ is a $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f_N^{-1}(\mathbb{A})$ is a $N_{eu}gs\alpha^* - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f_N$ is contra $N_{eu}gs\alpha^* -$ irresolute.

Example 4.5: Let $\mathbb{P} = \{p\}$ and $\mathbb{Q} = \{q\}$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ and $\sigma_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{B}\}$ are $N_{eu}TS$ on $(\mathbb{P}, \tau_{N_{eu}})$ and $(\mathbb{Q}, \sigma_{N_{eu}})$. Also $\mathbb{A} = \{\langle p, (0.4, 0.5, 0.7) \rangle\}$ and $\mathbb{B} = \{\langle q, (0.2, 0.3, 0.7) \rangle\}$ are $N_{eu}(\mathbb{P})$ and $N_{eu}(\mathbb{Q})$. Define a map $f_N: (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ by $f_N(p) = q$. Let $\mathbb{C} = \{\langle q, (0.3, 0.4, 0.6) \rangle\}$ be a $N_{eu}gs\alpha^* - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then $f_N^{-1}(\mathbb{C}) = \{\langle p, (0.3, 0.4, 0.6) \rangle\} \Rightarrow f_N^{-1}(\mathbb{C})$ is a $N_{eu}gs\alpha^* - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f_N$ is contra $N_{eu}gs\alpha^* -$ irresolute. But f_N is not strongly contra $N_{eu}gs\alpha^* -$ continuous.

Theorem 4.6: Every perfectly $N_{eu}gs\alpha^* -$ continuous is contra $N_{eu}gs\alpha^* -$ irresolute, but not conversely.

Proof: Let $f_N: (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be any $N_{eu} -$ function. Let $\mathbb{A}$ be any $N_{eu}gs\alpha^* - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Given f_N is perfectly $N_{eu}gs\alpha^* -$ continuous, then $f_N^{-1}(\mathbb{A})$ is $N_{eu} - OS$ and $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f_N^{-1}(\mathbb{A})$ is a $N_{eu}gs\alpha^* - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f_N$ is contra $N_{eu}gs\alpha^* -$ irresolute.

Example 4.7: Let $\mathbb{P} = \{p\}$ and $\mathbb{Q} = \{q\}$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ and $\sigma_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{B}\}$ are $N_{eu}TS$ on $(\mathbb{P}, \tau_{N_{eu}})$ and $(\mathbb{Q}, \sigma_{N_{eu}})$. Also $\mathbb{A} = \{\langle p, (0.4, 0.5, 0.8) \rangle\}$ and $\mathbb{B} = \{\langle q, (0.2, 0.4, 0.6) \rangle\}$ are $N_{eu}(\mathbb{P})$ and $N_{eu}(\mathbb{Q})$. Define a map $f_N: (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ by $f_N(p) = q$. Let $\mathbb{C} = \{\langle q, (0.5, 0.2, 0.8) \rangle\}$ be a $N_{eu}gs\alpha^* - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then $f_N^{-1}(\mathbb{C}) = \{\langle p, (0.5, 0.2, 0.8) \rangle\} \Rightarrow f_N^{-1}(\mathbb{C})$ is a $N_{eu}gs\alpha^* - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f_N$ is contra $N_{eu}gs\alpha^* -$ irresolute. But f_N is not perfectly $N_{eu}gs\alpha^* -$ continuous.

Theorem 4.8: Every contra $N_{eu}gs\alpha^* -$ irresolute is almost contra $N_{eu}gs\alpha^* -$ continuous, but not conversely.

Proof: Let $f_N: (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be any $N_{eu} -$ function. Let $\mathbb{A}$ be any $N_{eu} - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then $\mathbb{A}$ is a $N_{eu} - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}}) \Rightarrow \mathbb{A}$ is a $N_{eu}gs\alpha^* - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Given f_N is contra $N_{eu}gs\alpha^* -$ irresolute, then $f_N^{-1}(\mathbb{A})$ is $N_{eu}gs\alpha^* - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f_N$ is almost contra $N_{eu}gs\alpha^* -$ continuous.

Example 4.9: Let $\mathbb{P} = \{p\}$ and $\mathbb{Q} = \{q\}$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{A}\}$ and $\sigma_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbb{B}\}$ are $N_{eu}TS$ on $(\mathbb{P}, \tau_{N_{eu}})$ and $(\mathbb{Q}, \sigma_{N_{eu}})$. Also $\mathbb{A} = \{\langle p, (0.6, 0.8, 0.4) \rangle\}$ and $\mathbb{B} = \{\langle q, (0.7, 0.6, 0.5) \rangle\}$ are $N_{eu}(\mathbb{P})$ and $N_{eu}(\mathbb{Q})$. Define a map $f_N: (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ by $f_N(p) = q$. Let $\mathbb{C} = \{\langle q, (0, 0, 1) \rangle\}$ be a $N_{eu}R - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then $f_N^{-1}(\mathbb{C}) = \{\langle p, (0, 0, 1) \rangle\} \Rightarrow f_N^{-1}(\mathbb{C})$ is a $N_{eu}gs\alpha^* - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f_N$ is almost contra $N_{eu}gs\alpha^* -$ continuous. But f_N is not contra $N_{eu}gs\alpha^* -$ irresolute.

Theorem 4.10: Let $f_N: (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be contra $N_{eu}gs\alpha^* -$ continuous and $(\mathbb{Q}, \sigma_{N_{eu}})$ is $N_{eu}gs\alpha^* - T_{1/2}$

space, then f_N is contra $N_{eu}gs\alpha^*$ – irresolute.

Proof: Let A be any $N_{eu}gs\alpha^* - OS$ in $(Q, \sigma_{N_{eu}})$. Given $(Q, \sigma_{N_{eu}})$ is a $N_{eu}gs\alpha^* - T_{1/2}$ space, then A is a $N_{eu} - OS$ in $(Q, \sigma_{N_{eu}})$. Given f_N is contra $N_{eu}gs\alpha^* -$ continuous, then $f_N^{-1}(A)$ is a $N_{eu}gs\alpha^* - CS$ in $(P, \tau_{N_{eu}}) \Rightarrow f_N$ is contra $N_{eu}gs\alpha^* -$ irresolute.

Theorem 4.11: Let $f_N : (P, \tau_{N_{eu}}) \rightarrow (Q, \sigma_{N_{eu}})$ be contra $N_{eu}gs\alpha^* -$ irresolute and $(P, \tau_{N_{eu}})$ is $N_{eu}gs\alpha^* - T_{1/2}$ space, then f_N is strongly contra $N_{eu}gs\alpha^* -$ continuous.

Proof: Let A be any $N_{eu}gs\alpha^* - OS$ in $(Q, \sigma_{N_{eu}})$. Given f_N is contra $N_{eu}gs\alpha^* -$ irresolute, then $f_N^{-1}(A)$ is a $N_{eu}gs\alpha^* - CS$ in $(P, \tau_{N_{eu}})$. Given $(P, \tau_{N_{eu}})$ is $N_{eu}gs\alpha^* - T_{1/2}$ space, then $f_N^{-1}(A)$ is a $N_{eu} - CS$ in $(P, \tau_{N_{eu}}) \Rightarrow f_N$ is strongly contra $N_{eu}gs\alpha^* -$ continuous.

Theorem 4.12: Let $f_N : (P, \tau_{N_{eu}}) \rightarrow (Q, \sigma_{N_{eu}})$ be strongly $N_{eu}gs\alpha^* -$ continuous and $(P, \tau_{N_{eu}})$ is neutrosophic locally indiscrete space, then f_N is contra $N_{eu}gs\alpha^* -$ irresolute.

Proof: Let A be any $N_{eu}gs\alpha^* - OS$ in $(Q, \sigma_{N_{eu}})$. Given f_N is strongly $N_{eu}gs\alpha^* -$ continuous, then $f_N^{-1}(A)$ is a $N_{eu} - OS$ in $(P, \tau_{N_{eu}})$. Given $(P, \tau_{N_{eu}})$ is $N_{eu} -$ locally indiscrete space, then $f_N^{-1}(A)$ is a $N_{eu} - CS$ in $(P, \tau_{N_{eu}}) \Rightarrow f_N^{-1}(A)$ is a $N_{eu}gs\alpha^* - CS$ in $(P, \tau_{N_{eu}}) \Rightarrow f_N$ is contra $N_{eu}gs\alpha^* -$ irresolute.

Theorem 4.13: Let $f_N : (P, \tau_{N_{eu}}) \rightarrow (Q, \sigma_{N_{eu}})$ be contra $N_{eu}gs\alpha^* -$ irresolute. Then f_N is perfectly $N_{eu}gs\alpha^* -$ continuous, if $(P, \tau_{N_{eu}})$ is both $N_{eu}gs\alpha^* - T_{1/2}$ space and $N_{eu} -$ discrete topological space.

Proof: Let A be any $N_{eu}gs\alpha^* - OS$ in $(Q, \sigma_{N_{eu}})$. Given f_N is contra $N_{eu}gs\alpha^* -$ irresolute, then $f_N^{-1}(A)$ is a $N_{eu}gs\alpha^* - CS$ in $(P, \tau_{N_{eu}})$. Given $(P, \tau_{N_{eu}})$ is $N_{eu}gs\alpha^* - T_{1/2}$ space, then $f_N^{-1}(A)$ is a $N_{eu} - CS$ in $(P, \tau_{N_{eu}})$. Since $(P, \tau_{N_{eu}})$ is $N_{eu} -$ discrete topological space, then $f_N^{-1}(A)$ is a $N_{eu} - OS$ in $(P, \tau_{N_{eu}}) \Rightarrow f_N^{-1}(A)$ is both $N_{eu} - OS$ and $N_{eu} - CS$ in $(P, \tau_{N_{eu}}) \Rightarrow f_N$ is perfectly $N_{eu}gs\alpha^* -$ continuous.

Theorem 4.14: Let $f_N : (P, \tau_{N_{eu}}) \rightarrow (Q, \sigma_{N_{eu}})$ be any $N_{eu} -$ function. Then the following are equivalent.

(1) f_N is contra $N_{eu}gs\alpha^* -$ irresolute.

(2) The inverse image of every $N_{eu}gs\alpha^* - CS$ A in $(Q, \sigma_{N_{eu}})$ is a $N_{eu}gs\alpha^* - OS$ in $(P, \tau_{N_{eu}})$.

Proof: (1) $\Rightarrow$ (2), Let A be any $N_{eu}gs\alpha^* - CS$ in $(Q, \sigma_{N_{eu}})$. Then A^c is a $N_{eu}gs\alpha^* - OS$ in $(Q, \sigma_{N_{eu}})$. Given f_N is contra $N_{eu}gs\alpha^* -$ irresolute, then $f_N^{-1}(A^c) = (f_N^{-1}(A))^c$ is a $N_{eu}gs\alpha^* - CS$ in $(P, \tau_{N_{eu}}) \Rightarrow f_N^{-1}(A)$ is a $N_{eu}gs\alpha^* - OS$ in $(P, \tau_{N_{eu}})$.

(2) $\Rightarrow$ (1), Let A be any $N_{eu}gs\alpha^* - OS$ in $(Q, \sigma_{N_{eu}})$. Then A^c is a $N_{eu}gs\alpha^* - CS$ in $(Q, \sigma_{N_{eu}})$. By hypothesis, $f_N^{-1}(A^c) = (f_N^{-1}(A))^c$ is a $N_{eu}gs\alpha^* - OS$ in $(P, \tau_{N_{eu}}) \Rightarrow f_N^{-1}(A)$ is a $N_{eu}gs\alpha^* - CS$ in $(P, \tau_{N_{eu}}) \Rightarrow f_N$ is contra $N_{eu}gs\alpha^* -$ irresolute.

Theorem 4.15: Let $f_N : (P, \tau_{N_{eu}}) \rightarrow (Q, \sigma_{N_{eu}})$ be contra $N_{eu}gs\alpha^* -$ irresolute and $g_N : (Q, \sigma_{N_{eu}}) \rightarrow (R, \gamma_{N_{eu}})$ be contra $N_{eu}gs\alpha^* -$ continuous, then $g_N \circ f_N : (P, \tau_{N_{eu}}) \rightarrow (R, \gamma_{N_{eu}})$ is $N_{eu}gs\alpha^* -$ continuous.

Proof: Let A be any $N_{eu} - CS$ in $(R, \gamma_{N_{eu}})$. Given g_N is contra $N_{eu}gs\alpha^* -$ continuous, then $g_N^{-1}(A)$ is a $N_{eu}gs\alpha^* - OS$ in $(Q, \sigma_{N_{eu}})$. Given f_N is contra $N_{eu}gs\alpha^* -$ irresolute, then $f_N^{-1}(g_N^{-1}(A)) = (g_N \circ f_N)^{-1}(A)$ is a $N_{eu}gs\alpha^* - CS$ in $(P, \tau_{N_{eu}}) \Rightarrow g_N \circ f_N$ is $N_{eu}gs\alpha^* -$ continuous.

Theorem 4.16: Let $f_N : (P, \tau_{N_{eu}}) \rightarrow (Q, \sigma_{N_{eu}})$ be contra $N_{eu}gs\alpha^* -$ irresolute and $g_N : (Q, \sigma_{N_{eu}}) \rightarrow (R, \gamma_{N_{eu}})$ be $N_{eu}gs\alpha^* -$ continuous, then $g_N \circ f_N : (P, \tau_{N_{eu}}) \rightarrow (R, \gamma_{N_{eu}})$ is contra $N_{eu}gs\alpha^* -$ continuous.

Proof: Let A be any $N_{eu} - OS$ in $(R, \gamma_{N_{eu}})$. Given g_N is $N_{eu}gs\alpha^* -$ continuous, then $g_N^{-1}(A)$ is a $N_{eu}gs\alpha^* - OS$ in $(Q, \sigma_{N_{eu}})$. Given f_N is contra $N_{eu}gs\alpha^* -$ irresolute, then $f_N^{-1}(g_N^{-1}(A)) = (g_N \circ f_N)^{-1}(A)$ is a $N_{eu}gs\alpha^* - CS$ in $(P, \tau_{N_{eu}}) \Rightarrow g_N \circ f_N$ is contra $N_{eu}gs\alpha^* -$ continuous.

Theorem 4.17: Let $f_N : (P, \tau_{N_{eu}}) \rightarrow (Q, \sigma_{N_{eu}})$ be contra $N_{eu} -$ continuous and $g_N : (Q, \sigma_{N_{eu}}) \rightarrow (R, \gamma_{N_{eu}})$ be contra $N_{eu}gs\alpha^* -$ irresolute. Then $g_N \circ f_N : (P, \tau_{N_{eu}}) \rightarrow (R, \gamma_{N_{eu}})$ is strongly $N_{eu}gs\alpha^* -$ continuous, if $(Q, \sigma_{N_{eu}})$ is $N_{eu}gs\alpha^* - T_{1/2}$ space.

Proof: Let A be any $N_{eu}gs\alpha^* - CS$ in $(R, \gamma_{N_{eu}})$. Given g_N is contra $N_{eu}gs\alpha^* -$ irresolute, then $g_N^{-1}(A)$ is a

$N_{eu}gs\alpha^* - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Given $(\mathbb{Q}, \sigma_{N_{eu}})$ is $N_{eu}gs\alpha^* - T_{1/2}$ space, then $g_N^{-1}(A)$ is a $N_{eu} - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Given f_N is contra $N_{eu} -$ continuous, then $f_N^{-1}(g_N^{-1}(A)) = (g_N \circ f_N)^{-1}(A)$ is a $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow g_N \circ f_N$ is strongly $N_{eu}gs\alpha^* -$ continuous.

Theorem 4.18: Let $f_N: (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be $N_{eu}gs\alpha^* -$ irresolute and $g_N: (\mathbb{Q}, \sigma_{N_{eu}}) \rightarrow (\mathbb{R}, \gamma_{N_{eu}})$ be contra $N_{eu}gs\alpha^* -$ irresolute, then $g_N \circ f_N: (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{R}, \gamma_{N_{eu}})$ is contra $N_{eu}gs\alpha^* -$ irresolute.

Proof: Let A be any $N_{eu}gs\alpha^* - OS$ in $(\mathbb{R}, \gamma_{N_{eu}})$. Given g_N is contra $N_{eu}gs\alpha^* -$ irresolute, then $g_N^{-1}(A)$ is a $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Given f_N is $N_{eu}gs\alpha^* -$ irresolute, then $f_N^{-1}(g_N^{-1}(A)) = (g_N \circ f_N)^{-1}(A)$ is a $N_{eu}gs\alpha^* - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow g_N \circ f_N$ is contra $N_{eu}gs\alpha^* -$ irresolute.

Theorem 4.20: Let $f_N: (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be strongly $N_{eu}gs\alpha^* -$ continuous and $g_N: (\mathbb{Q}, \sigma_{N_{eu}}) \rightarrow (\mathbb{R}, \gamma_{N_{eu}})$ be contra $N_{eu}gs\alpha^* -$ irresolute, then $g_N \circ f_N: (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{R}, \gamma_{N_{eu}})$ is contra $N_{eu}gs\alpha^* -$ irresolute.

Proof: Let A be any $N_{eu}gs\alpha^* - OS$ in $(\mathbb{R}, \gamma_{N_{eu}})$. Given g_N is contra $N_{eu}gs\alpha^* -$ irresolute, then $g_N^{-1}(A)$ is a $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Given f_N is strongly $N_{eu}gs\alpha^* -$ continuous, then $f_N^{-1}(g_N^{-1}(A)) = (g_N \circ f_N)^{-1}(A)$ is a $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow (g_N \circ f_N)^{-1}(A)$ is a $N_{eu}gs\alpha^* - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow g_N \circ f_N$ is contra $N_{eu}gs\alpha^* -$ irresolute.

Theorem 4.21: Let $f_N: (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be perfectly $N_{eu}gs\alpha^* -$ continuous and $g_N: (\mathbb{Q}, \sigma_{N_{eu}}) \rightarrow (\mathbb{R}, \gamma_{N_{eu}})$ be contra $N_{eu}gs\alpha^* -$ irresolute, then $g_N \circ f_N: (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{R}, \gamma_{N_{eu}})$ is contra $N_{eu}gs\alpha^* -$ irresolute.

Proof: Let A be any $N_{eu}gs\alpha^* - OS$ in $(\mathbb{R}, \gamma_{N_{eu}})$. Given g_N is contra $N_{eu}gs\alpha^* -$ irresolute, then $g_N^{-1}(A)$ is a $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Given f_N is perfectly $N_{eu}gs\alpha^* -$ continuous, then $f_N^{-1}(g_N^{-1}(A)) = (g_N \circ f_N)^{-1}(A)$ is both $N_{eu} - OS$ and $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow (g_N \circ f_N)^{-1}(A)$ is a $N_{eu}gs\alpha^* - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow g_N \circ f_N$ is contra $N_{eu}gs\alpha^* -$ irresolute.

Theorem 4.22: Let $f_N: (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ and $g_N: (\mathbb{Q}, \sigma_{N_{eu}}) \rightarrow (\mathbb{R}, \gamma_{N_{eu}})$ be contra $N_{eu}gs\alpha^* -$ irresolute. Then $g_N \circ f_N: (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{R}, \gamma_{N_{eu}})$ is contra $N_{eu}gs\alpha^* -$ irresolute, if $(\mathbb{P}, \tau_{N_{eu}})$ is $N_{eu}gs\alpha^* -$ locally indiscrete space.

Proof: Let A be any $N_{eu}gs\alpha^* - OS$ in $(\mathbb{R}, \gamma_{N_{eu}})$. Given g_N is contra $N_{eu}gs\alpha^* -$ irresolute, then $g_N^{-1}(A)$ is a $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Given f_N is contra $N_{eu}gs\alpha^* -$ irresolute, then $f_N^{-1}(g_N^{-1}(A)) = (g_N \circ f_N)^{-1}(A)$ is a $N_{eu}gs\alpha^* - OS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Since $(\mathbb{P}, \tau_{N_{eu}})$ is $N_{eu}gs\alpha^* -$ locally indiscrete space, then $(g_N \circ f_N)^{-1}(A)$ is a $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow (g_N \circ f_N)^{-1}(A)$ is a $N_{eu}gs\alpha^* - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow g_N \circ f_N$ is contra $N_{eu}gs\alpha^* -$ irresolute.

5 Conclusions

In this paper we have discussed about Strongly contra $N_{eu}gs\alpha^* -$ continuous and contra $N_{eu}gs\alpha^* -$ irresolute map. We had an idea to extend this paper to the next level about almost contra $N_{eu}gs\alpha^* -$ continuous and also the application of this paper. In future work, we will discuss and find out the results of this paper application.

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Conflicts of Interest

The authors declare that there is no conflicts of interest in the research.

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