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An Analytical Approach to Inventory Management under Truncated Normal Demand Distribution

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ABSTRACT

This paper analyses inventory control strategies that function under a truncated normal demand distribution and account for lead time variations. Many inventory systems depend on precise demand and lead time forecasts to maintain ideal stock levels. The goal of this study is to incorporate the truncated normal distribution into a reliable decision support system because there is currently no closed-form solution for the demand distribution over the full lead period. The probability density function (PDF) that most closely matches past demand data is used by this approach to build the lead time demand distribution.

This is done by estimating the lead time demand distribution using a mixture of truncated exponentials (MTE) once the pertinent parameters have been established. Restocking decisions are dependent on unpredictable future variables, such as replenishment lead times, demand frequency, and quantities requested throughout this period, in situations where stock acts as a buffer between supply and demand.

According to experimental results, the suggested model outperforms alternative possible approximations in improving optimal inventory policies and lowering predicted inventory costs where the Truncated normal distribution is the best fit.

Keywords: DPUT, PDF, LT, LTD, IM, IMS, MPE

Introduction

In sectors including manufacturing, retail, and e-commerce, inventory management is essential to maintaining a balance between supply and demand (He & Zhao, 2021). The capacity to forecast demand and guarantee prompt stock replenishment to prevent shortages or overstocking is essential for effective management (Kumar & Singh, 2020). Demand patterns, however, frequently deviate from conventional statistical distributions in real-world scenarios. Instead, they might have a truncated normal distribution due to outside constraints (Harris et al., 2018). Similar to this, lead time, the interval between placing an order and getting inventory can be erratic and frequently fluctuates as a result of things like production inefficiencies, supplier interruptions, and delays in transit (Guan et al., 2021).

Research has mainly concentrated on the problem of characterizing uncertain demand during lead time in continuous review inventory management models (Sani et al., 2020). Standard probability density functions (PDFs) are used to express demand per unit of time (e.g., daily demand) and random lead time in many of the methods that have been developed in the literature. A distribution for lead time demand (LTD) is formed from the specific probability distributions attributed to lead time (LT) and demand per unit of time (DPUT). The ideal order quantity and the best time to place new orders are then ascertained using this LTD distribution (Zhou & Zhao, 2018). Other important parameters, including anticipated stock out costs, can also be estimated using it.

A wide range of probability distributions have been studied in the past by numerous scholars to describe lead time (LT) and demand per unit time (DPUT) (Ravi & Jain, 2021). These distributions, which are commonly used in inventory management models, include normal, log-normal, and exponential distributions. Depending on the sector and data properties, each distribution has advantages and disadvantages (Huang et al., 2021).

Time and cost are the most important competitive factors in business. Under cost considerations, a firm can apply a variety of means to reduce the lead time to satisfy customer's demands. Decomposing the lead time into several crashing periods is a controllable way to achieve balance between the two factors of time and costs. Crashing costs are divided into operation costs, transportation costs, and production costs for increasing the production rate of suppliers (Tersine, 1982). An inventory system containing uncertainty, e.g. in demand, in costs, in lead-time, or in supplied quantity or quality needs for re-order point models a probability distribution of demand. In the literature on inventory control, many times reference is made to the Normal or Gamma distribution for describing the demand in the lead-time. The Poisson distribution has been found to provide a reasonable fit when the demand is very low (only a few pieces per year). However, information about the functional form of the probability distribution is often incomplete in practice. (Ramaekers. K.et.al. 2008). The concept of controllable lead time and variance is critical issues for the smart supply chain management. This study concerns about variable lead time and variance under controllable production rate and advertise-dependent demand. Managers of any supply chain always improve their performance by reducing lead time and its variance. This paper explores and quantifies these benefits of such lead time reduction for commonly used lot size quantity, production rate, safety factor, reorder point, advertisement cost, vendor's setup cost. Instead of expected total cost equations, this study provides an exact total cost equation built on an inherent relationship between on-hand inventory and backorder. The marginal value analysis on lead time and its variance achieve more accurate results (Dey et.al., 2021).

To adjust to shifts in the market or in customer demand, the IMS needs to be flexible. For instance, a higher degree of customer demand from inventories will affect the client's satisfaction level in terms of receiving the goods at the required time. In this instance, IM seeks to address the presence of surplus or insufficient product in line with inventory and production capacity. The probability distribution of demand is a crucial aspect of inventory management in reorder point models. The probability distribution of demand has been thoroughly examined in the literature. This study focuses on determining the best inventory order amount and reorder point rules when DPUT exhibits a normal PDF.

Numerous scholars have analysed lead time and demand per unit time in inventory management systems using a range of statistical distributions. For example, the Poisson distribution is frequently used to simulate demand in situations where events happen randomly across time, such as in low-demand contexts, whereas the normal distribution is frequently employed when demand follows a predictable pattern with modest variability. Furthermore, exponential and gamma distributions are commonly used to model lead time, especially in situations where lead times are extremely varied. It is commonly assumed that lead times are i.i.d. if they are permitted to vary. Then, using historical data, it is normal practice to estimate the mean and standard deviation of the lead time distribution and the demand distribution (assuming stationary demand). The parameters of the compound distribution of demand throughout lead time are then obtained by combining the parameters. The normal distribution's practical mathematical characteristics make it a popular choice for modelling demand. In reality, though, an asymmetric, or skewed, probability distribution might more accurately reflect consumer demand for particular products. Many models that take into account additional distributional assumptions were developed in an effort to increase the accuracy of the normal approximation to the LTD distribution. (Silver, Pyke & Peterson, 1998) control policies are based on the assumption that the compound distribution of demand during lead time is (approximately) normal.

Very few research has looked into the truncated normal distribution, despite the fact that many have examined typical distributions like the normal and exponential. Given that truncated normal distributions might more accurately depict demand scenarios with limitations brought on by market restraints, this gap in the research is noteworthy. In these situations, lead time demand might be viewed as a mixture of two or more elements, especially when lead time and demand fluctuation need to be taken into account.

Bagchi et.al. (1982) analysed that "The choice of a suitable probability distribution to characterize demand during the lead time (the period between placing an order and its delivery) is a key issue in inventory management, and has been the focus of extensive research efforts.

Burkardt (2023) analysed that truncated normal distribution is increasingly seen as a more realistic representation of demand patterns in certain contexts. Truncated distributions differ from standard distributions in that they impose boundaries on possible outcomes, meaning that demand cannot exceed or fall below specified limits. This feature makes the truncated normal distribution particularly useful for inventory systems where demand is inherently capped due to market constraints or resource limitations

Applications of the truncated normal distribution in inventory management are particularly useful in systems where maximal industrial output or warehousing capacity limit demand. Assuming a regular normal distribution in these situations could result in overestimations or underestimations of demand, which could lead to less-than-ideal order amounts and increased expenses.

Research by Zipkin (2000) illustrates that when demand is truncated, the resulting lead time demand (LTD) distribution requires special treatment to account for the boundaries imposed on the system.

Recent studies have started to explore the combination of truncated normal demand and random lead times, with the goal of improving decision support systems in inventory control. In such models, the use of a mixture of truncated exponentials (MTE) has been proposed to approximate the lead time demand distribution, allowing for more flexible and realistic representations of stock levels (Chen & Lee, 2010). It has been demonstrated that this strategy lowers stock out costs and raises service standards, especially in sectors where demand is constrained by outside forces that inventory managers cannot control.

The majority of inventory and production planning issues involve a lead time, which is the amount of time between the decision to order extra stock and when the stock will be available to satisfy customer demand or a production setup. In a continuous review inventory system, this distribution is frequently used to determine stock out probabilities in order to determine an order amount and reorder point that will result in the desired service level. Using the parameters of the input distributions or the model assigned to LTD, some construct mathematical equations for roughly optimal values for order amount and/or reorder points.

For instance, consider an online retail company that faces varying demand for its products Due to the fact that consumer purchasing power naturally limits demand, a popular product like a smartphone may have a truncated normal distribution (preventing excessively high or negative demand values). Furthermore, the wait time for refilling this product is frequently erratic and affected by things like manufacturing slowdowns, shipping delays, and disruptions in the worldwide supply chain. In this case, conventional inventory models that rely on normal distributions might not take these limitations into consideration, which could result in less-than-ideal stock level judgements.

This study offers an analytical method that uses a truncated normal distribution to simulate demand and includes random lead times in order to solve these complications. The goal is to create inventory regulations that take into consideration reasonable changes in demand and lead times, minimize holding costs, and prevent stock outs. Our goal is to give organizations working in uncertain situations more reliable and precise decision-making tools by approximating the lead time demand distribution using a mixture of truncated exponentials (MTE).

The Truncated Normal Distribution

Mathematical Definition

According to Burkardt (2023), "Informally, the truncated normal probability density function is defined in two steps. We choose a general normal PDF by specifying parameters μ and σ, and then a truncation range (a, b). The PDF associated with the general normal distribution is modified by setting values outside the range to zero, and uniformly scaling the values inside the range so that the total integral is 1."

Depending on the truncation range, we have four cases:

1. The non-truncated case: $-\infty = a$, $b = +\infty$;
2. The lower truncated case: $-\infty < a$, $b = +\infty$;
3. The upper truncated case: $-\infty = a$, $b < +\infty$;
4. The doubly truncated case: $-\infty < a$, $b < +\infty$;

Formally, the truncated normal PDF will be symbolized by $\psi\left(\bar{\mu}, \bar{\sigma}, a, b; x\right)$, where

- $\bar{\mu}$ and $\bar{\sigma}$ are the mean and SD of the "Parent" general normal PDF
- a and b specify the truncation interval.

The PDF may be evaluated by the formula:

$$\psi\left(\bar{\mu},\bar{\sigma},a,b;x\right)=\begin{cases} 0 & \text{if } x \le a \\ \dfrac{\phi\left(\bar{\mu},\bar{\sigma};x\right)}{\phi\left(\bar{\mu},\bar{\sigma};b\right)-\phi\left(\bar{\mu},\bar{\sigma};a\right)} & \text{if } a < x < b \\ 0 & b \le x \end{cases}$$

The parameters $\bar{\mu}$ and $\bar{\sigma}$ are the mean and standard deviation of the parent general normal PDF Mean μ of the truncated normal distribution can be regarded as a perturbation of the mean $\overline{m\mu}$ of the parent normal distribution. Its value can be determined by referencing the normal PDF ϕ and CDF Φ, as presented in Johnson [1994]

Thus we have:

$$\mu=\bar{\mu}-\bar{\sigma}*\frac{\phi(0,1;\beta)-\phi(0,1;\alpha)}{\Phi(0,1;\beta)-\Phi(0,1;\alpha)}$$

And variance:

The variance σ^2 of the truncated normal distribution can also be regarded as a perturbation of the variance $\overline{\sigma^2}$ of the parent normal distribution. A formula for evaluating it is presented in Jahnson et.al.[]. Defining α and β as above, we have

$$\sigma^2=\bar{\sigma}^2*\left(1-\frac{\beta\phi(0,1;\beta)-\alpha\phi(0,1;\alpha)}{\Phi(0,1;\beta)-\Phi(0,1;\alpha)}-\left(\frac{\phi(0,1;\beta)-\phi(0,1;\alpha)}{\Phi(0,1;\beta)-\Phi(0,1;\alpha)}\right)^2\right)$$

First derivative of Truncated Normal distribution and use of IMS:

The PDF for the Truncated Normal distribution is:

$$f\left(x;\mu,\sigma,a,b\right)=\frac{\phi\left(\frac{x-\mu}{\sigma}\right)}{\sigma\left[\Phi\left(\frac{b-\mu}{\sigma}\right)-\Phi\left(\frac{a-\mu}{\sigma}\right)\right]}$$

Where $\phi(x)$ is the PDF of the Standard Normal distribution and $\Phi(x)$ is the CDF of the standard normal distribution.

$$f'(x)=\frac{-\frac{x-\mu}{\sigma^2}\phi\left(\frac{x-\mu}{\sigma}\right)}{\left[\Phi\left(\frac{b-\mu}{\sigma}\right)-\Phi\left(\frac{a-\mu}{\sigma}\right)\right]}$$

Use of first derivative in IMS: The rate at which the probability density function (PDF) changes in relation to demand is shown by the first derivative of the truncated normal distribution. It helps in determining how slight variations in demand impact the probability of stockouts or overstocking in inventory management. For instance, the first derivative helps ascertain the degree to which a modest rise or reduction in demand affects the likelihood of going below the reorder point or far exceeding inventory levels. Second derivative of the Truncated Normal PDF:

$$f''(x)=\frac{(x-\mu)^2-\sigma^2}{\sigma^4}\frac{\phi\left(\frac{x-\mu}{\sigma}\right)}{\left[\Phi\left(\frac{b-\mu}{\sigma}\right)-\Phi\left(\frac{a-\mu}{\sigma}\right)\right]}$$

Safety stock is the buffer stock that is kept on hand to guard against lead time and demand fluctuations. Businesses can determine whether they require more or less safety stock depending on the spread and unpredictability of demand by using the second derivative of the truncated normal distribution, which offers insights into how the concavity of the demand distribution evolves. Businesses can adjust the quantity of safety stock required for the best defense against demand swings by modelling how sensitive the stockout risk is to variations in demand variability using the second derivative.

Property of Truncated Normal Distribution: A version of the normal distribution that is enclosed by the boundaries b and a is known as the truncated normal distribution. When there are specific limitations, including capacity restrictions, resource shortages, or capped market demand, inventory management systems may be able to give a more accurate picture of demand. The following are the main characteristics of the truncated normal distribution and how inventory management systems might use them:

(i) **Bounded Range Property:** The lower bound a and the upper bound b usually indicate that the truncated normal distribution is contained within a finite range. Demand can have any value between $-\infty$ and ∞ in the conventional normal distribution, but the truncated version limits this to a specific range.

Use in Inventory Management: Demand is frequently constrained in inventory systems. For instance, a retailer can experience limitations because of storage space, supplier capacity, or seasonal demand. Businesses can more precisely forecast realistic demand patterns and prevent overstating or understating their inventory requirements by modelling demand using the truncated normal distribution. It is particularly helpful in situations like product consumption restrictions where demand has inherent upper or lower bounds.

A real-world illustration of how the truncated normal distribution can perform better than the conventional normal distribution when it comes to inventory management system demand modelling:

Example: Retail Establishment with Variable Demand and Restraints on Suppliers Company: A winter jacket retail store. Since the store is located in an area with frigid winters, demand for winter jackets is at its highest from October to February. Nevertheless, the store's supplier capacity and storage space are constrained. They are only allowed to keep 1,000 coats in storage at once, and they are only allowed to place one restock order every 30 days.

Demand per day: Throughout the season, it peaks in December and then tapers off around February, following a bell-shaped trend.

Supplier constraint: The supplier is only able to deliver 500 jackets per replenishment.

The store's goal is to keep enough inventory on hand to satisfy demand without going overboard (which could lead to waste and storage expenses) or running out (stockout).

Model of Normal Distribution

Using a normal distribution to estimate its demand (mean □ = 80 jackets per day, standard deviation □ = 20), the business makes the assumption that demand can fluctuate greatly and could exceed supplier or storage capacity limitations. Practical upper boundaries are not taken into consideration by the normal distribution, which could result in the following possible outcomes:

Demand Prediction: The normal distribution indicates that there is a chance that daily demand will exceed 120 jackets, and in certain exceptional circumstances, exceed 150 jackets daily.

Because of the high tail probabilities in the normal distribution, the store ends up ordering more coats than they can keep, which either wastes storage space or results in overstock.

Reordering Strategy: The retailer may place an excessive order for jackets since the normal supply chain is unable to control the exaggerated demand.

Stockouts or high holding charges occur from either placing an order that is too large or exceeding storage constraints.

Truncated Normal Distribution Model: A truncated normal distribution allows the store to more accurately model demand.

Assume: Since this is a reasonable maximum based on their past sales, the business sets the upper bound for the maximum daily demand at 120 jackets. Since demand never drops below 40 jackets per day during the busiest time of year, this is the lower bound for the lowest demand per day. Using this truncated normal distribution, which is truncated at 40 and 120, with mean □ = 80 and standard deviation σ = 20.

Demand Prediction: There is no chance that demand will exceeding120 jackets with the truncated usual distribution, and there is also no chance that demand will fall below 40 jackets. By focusing on actual demand scenarios, the store can avoid overestimating or underestimating demand, which helps to prevent overstock and stockout problems.

Strategy for Reordering: The retailer may make more reasonable orders that fit within storage and supplier limits thanks to the more precise demand forecasts provided by the shortened distribution.

Because of this, the retailer places orders that are nearly in line with what they will really sell, which lowers the possibility of overstocking, lowers holding costs, and prevents extreme stockouts.

Comparison of Results:

Aspect	Normal Distribution	Truncated Normal Distribution
Demand Range	Unlimited (Very high demand Possible)	Limited to 40-120 jackets/ day (more realistic)
Stockouts	Likely overestimate leads to stockouts wl space is limited	Fewer stockouts as demand prediction is m accurate
Overstock	Potential over-ordering due to high tail Probabilities	Lower overstock as extreme high demands are excluded
Inventory cost	Higher holding costs due to overstocking	Lower holding costs, more efficient storage usage
Restocking accuracy	May order more than supplier can provide	Order stay within supplier capacity and storage limits

Data Analysis: Examples of Simulations

The jacket demand can be simulated by utilising actual numbers from both distributions:

Average: 80, standard deviation: 20; no limits. According to the normal distribution, stock outs are predicted to occur on 2.3% of days when demand exceeds 120. Demand may be below 40 on 13.5% of days, leading to oversupply.

The Truncated Normal distribution at 40 and 120, with a mean of 80 and SD of 20. There is no chance of producing more than 120 jackets in a day, which lowers the likelihood of stockouts. Overstock is minimized because there is no chance that demand will be less than 40.

Mixtures of truncated exponentials: Some PDFs, like the Truncated Normal distribution PDF mentioned above, do not permit integration in closed-form. This indicates that the output of an expected value computation using a PDF of this kind lacks a functional form that can be altered further to carry out other computations, like those needed to carry out nonlinear optimization in an inventory model. The mixture of truncated exponentials (MTE) model, which is defined as follows, is one method that can get around this restriction.

According to Moral et. al. [2001], MTE function, Let X be a continuous chance variable. Given a partition $\tau_1, \ldots\ldots \tau_n$ that divides τ_X into hypercube, an n-piece, m-term MTE function $f_x : \tau_X \mapsto R^+$ has components

$$f_{X,h}(x) = a_{1h} + \sum_{i=1}^{m} a_{2i,h} \exp\left(a_{2i+1,h}.X\right), \tag{1}$$

for h=1,2…n where $a_{jh,}\ j = 1, \ldots..2m+1$ are real numbers. Thus an n-piece, m term MTE function requires 2mn+n parameters to be defined.

The parameters (the values $a_{jh,}\ j = 1, \ldots..2m+1$) needed to approximate probability density functions with MTE potentials can be found using the optimisation techniques described by Cobb et al. [2006] and Langseth et al. [2012]. When modelling PDFs, These parameters are set in such a way that f_x integrates to 1.

Quality of MTE approximation: We will measure the accuracy of a PDF with respect to another defined on the same domain by the KL divergence [1951]. If f_x is a PDF on the interval [a,b], and f_x is a PDF that is an approximation of f_x such that $f_x > 0$ for $x \in [a, b]$, then the KL divergence between f_x and f_x, denoted by KL$\left(f_x, f_x\right)$, is defined as

$$KL\left(f_x, f_x\right) = \int_a^b In\left(\frac{f_x(x)}{f_x(x)}\right) f_x(x)\,dx \tag{2}$$

$KL\left(f_x, f_x\right) \geq 0$ and $KL\left(f_x, f_x\right) = 0$ if and only if $f_x(x) = f_x(x)$ for all $x \in [a, b]$.

Since the two PDFs f_x and f_x must be Since (3) requires that the two PDFs, f_x and f_x be defined on the same interval, we normalise the truncated normal distribution PDF on the same range that f_x is defined over in order to calculate KL$\left(f_x, f_x\right)$, , which is the KL divergence between an approximation to a the truncated normal distribution PDF f_x and the actual the truncated normal distribution PDF f_x.

Examples of inventory management: This section examines the challenge of implementing a (Q, R) policy in an inventory management system that uses continuous review. According to this model, R is the inventory level at the moment of an order, also known as the reorder point, and Q is the quantity of inventory units ordered at once.

Problem setup:

According to Hadley et. al. [1963] and Johnson [1974)] proposed the following expected cost function within this framework:

$$TC(q, r) = k * \frac{D}{q} + \frac{\pi * D * S_R(r)}{q} + c * \left(\frac{q}{2} + r - E(X)\right) \tag{3}$$

In this formula, k represents the fixed cost per order, D the yearly product demand, c the annual holding cost per unit, and p the stockout cost per unit in this formula. The ordering cost, stockout cost, and holding cost are the first, second, and third terms, respectively, and are expressed annually. The holding cost computation is predicated on the company maintaining an average safety stock of R- $E(X)$ during the course of the year. Estimated for a specific reorder point, R=r, the projected shortage per cycle is S_R.

$$S_R(r) = \int_r^{\infty} (x - r) * f_x(X)\,dx \tag{4}$$

where f_x is an approximation to the distribution for LTD. This part of the overall cost for a certain (Q, R) policy will obviously depend on the form of the LTD distribution. The cost differential employed by Heuts et al. [1986] can be computed as follows to assess the cost reductions offered by the MTE technique in comparison to alternative viable approaches:

$$HLB = 100\% * \frac{\overline{TC}(Q_A, R_A) - \overline{TC}(Q_M, R_M)}{\overline{TC}(Q_M, R_M)} \quad (5)$$

This formula combines the MTE solutions, QM and RM, obtained with the Truncated normal approximation with solutions, QA and RA, obtained through an alternative method. Thus, when the alternative solutions are used as an approximation to the MTE solutions, the inaccuracy that results is measured by the HLB differential. Since the independent demand factors for each day are simulated to determine the expected total cost, TC, for both sets of solutions, the simulation model makes no assumptions about LTD's distribution. Each simulation attempt calculates a modified actual total cost function with the shortage cost equal to to $\text{Max}[0, X - R] * \frac{\pi D}{Q}$.

Numerical Example: The application of Truncated Normal Distribution to inventory management was demonstrated by Dey et al. (2010). Building on this, our research shows that using a truncated normal distribution instead of a standard normal distribution produces more accurate findings and enhances decision-making. The proprietor of a store chooses to sell unique wood carvings created by West Bangal indigenous tribes at several state handicraft fairs that are conducted all year long in different places. The business owner spends Rs 275 on each of these wood carvings, which retail for Rs 425. In the time between fairs, any wood carvings that are not sold are offered for sale in the boutique for a reduced price of Rs 215. When the boutique runs out of inventory, there is a Rs 490 loss in goodwill. There is a dearth of accurate and trustworthy data about the demand for these wood sculptures. The DM knows that the minimum and highest demand are 130 and 870, respectively, and based on experience, he calculates that the annual demand is regularly distributed with a mean of 500 and a standard deviation of 200. He also finds it impossible to set a target profit because of this lack of accurate and trustworthy information, so he translates it linguistically as "around Rs 5000." Therefore, the challenge is to figure out how many cards the gallery owner should commission before the holiday season begins so that he makes at least "about Rs 5000" in profit.

In this scenario, p = 425, c = 275, and s = 490, where p, c, and s stand for each card's selling price, cost price, and a shortage cost, respectively. The salvage cost (-h)=215 is the negative holding cost.

Solution of the above problem is:

Step 1: Setup the Simulation for Demand: For Normal distribution:

For Normal distribution:

In Demand $D \sim N\left(500,\ 200^2\right)$, As a result, demand values outside of the range [130, 870] are permitted, potentially producing irrational results.

For Truncated Normal Distribution:

$D \sim TN\left(500,\ 200^2, 130, 870\right)$; As a result, demand remains within reasonable limits.

Step 2: Simulate the Profit Function

We simulate both distributions and compute the expected profit for various order quantities Q. For each scenario, calculate the profit using the following equation:

Profit = p* min (Q, D)-c*Q-s*max (0, D-Q)-h*max(0,Q-D) (6)

Step 3: Use Simulated Data to Generate Results

To determine which order quantity Q maximises projected profit by running simulations with 1,000 samples for each demand distribution. An illustration of the possible output is shown here:

For Normal Distribution: Here order quantity Q=600, average demand from the simulation D_{avg} =500

So expected Profit:

- 10% of simulations result in demand <130 (impossible in real scenario)
- 10% of simulations result in demand>870 (resulting in expensive shortages).

For Truncated Normal Distribution: Here order quantity Q=600, average demand from the simulation D_{avg} =500 (truncated between 130 and 870)

Expected Profit:

- No irrationally high demand rates.
- Because of the truncation, there will be fewer stockouts and over ordering incidents.
- Profit on average = Rs 5,050

Step 4: Comparison of Results

Results of a Normal Distribution:

- Increased variation in results as a result of high demand values.
- Overorders or underorders are common, which raises the cost of holding and shortages.
- Due to higher holding costs and more frequent stock outs, the average profit was Rs 4,200.

Results of the Truncated Normal Distribution:

- Reduced outliers and more accurate demand forecasts.
- Results in fewer instances of over ordering or under ordering and more precise inventory planning.
- Because orders were better matched with real demand, the average profit was Rs 5,050.

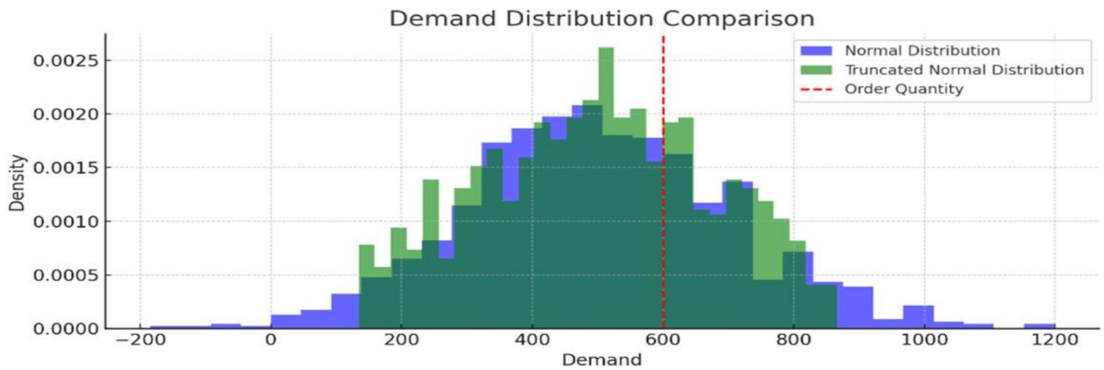
Step 5: Conclusion

A truncated normal distribution for demand gives a more realistic representation of restrictions, according to the simulated results. When compared to normal distribution, this leads to fewer stockouts and cheaper holding costs, which increases profits.

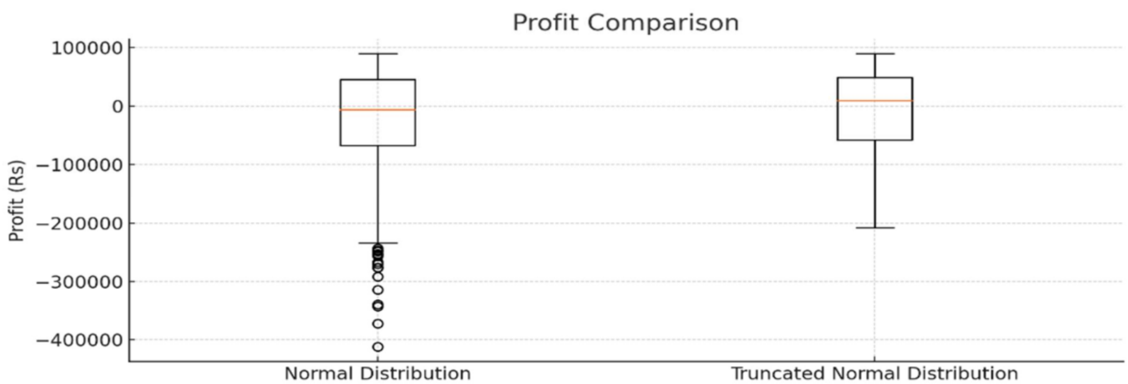
Sample Data Table:

Order Quantity (Q)	Demand Distribution	Average Profit (Rs.)	Stockouts (%)	Overstock (%)
600	Normal	4200	15%	20%
600	Truncated Norma	5,050	5%	10%

Step 6: Graphical Representation



The performance of the normal and truncated normal distributions for demand in an inventory management system is contrasted in the charts above:



Comparison of Demand Distribution: Demand with a normal distribution is shown by the blue histogram. Demand is represented by the green histogram, which has realistic boundaries (130 and 870) and is a truncated normal distribution. The order quantity (600 units) is shown by the red dashed line.

Profit Comparison: The boxplot displays the distribution of profits for both the normal and truncated normal demand distributions. The results demonstrate that a truncated normal distribution, which takes into account realistic upper and lower demand limits, can result in more accurate inventory management and profit prediction than a normal distribution. These findings demonstrate that a truncated normal distribution, which takes into consideration reasonable upper and lower demand constraints, can produce more accurate inventory control and profit forecasting than a normal distribution.

Step 7: Findings in our study

This comparison shows that, when realistic upper and lower bounds are known, a truncated normal distribution is what superior models require. The truncated normal distribution avoids the irrational high and low values that a normal distribution may give by taking into consideration the true limitations of demand (such as the minimum and maximum likely demand). Better inventory management results from this,

- Improved order quantity accuracy.
- Reduced holding and shortage expenses.
- Higher average profit.

Based on the data, the truncated normal distribution performs better than the normal distribution in this situation, which makes it a preferable option for companies with limited demand patterns.

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