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On Hinge Domination of Tent and Umbrella Graphs

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Abstract
A set D_h of vertices in a graph G with vertex set V and edge set E is considered a hinge-dominating set if, for every vertex u in $V - D_h$ there exists a vertex v in D_h and a vertex $w$in $V - D_h$ such that the edge(v, w) is not present in E.The hinge domination number $\gamma_h(G)$represents the minimum size of the hinge dominating set of graph G. In this paper, we determine the exact values of $\gamma_h(G)$ for a selection of basic graphs, an umbrella graph, and a Mongolian tent graph

Keywords: Dominating set, grid graph, pendant, path graph, Fan Graph

1. Introduction

The concept of domination in graphs has experienced rapid growth recently due to its applications in various fields, including security networks, communication networks, coding theory, chemical graph theory, biological networks, etc. Numerous research articles have been dedicated to domination theory owing to its extensive applications. Various variations of domination theory have also been employed, depending on the context. Some companies have multiple departments, each connected to customers through customer care. As a result, individuals working in customer care must maintain communication with company departments and customers. This necessity for two-way communication gives rise to the need for a variation known as hinge domination.

All graphs considered here are finite, simple, and undirected graphs. A set D_h of vertices in a graph G with vertex set V and edge set E is termed a hinged dominating set if, for every vertex u in $V - D_h$ there exists a vertex v in D_h and a vertex $w$in $V - D_h$ such that (v, w) is not an edge in E. The hinged domination number $\gamma_h(G)$ is the minimum cardinality of the hinged dominating set of graph G. This concept was first introduced by Kavitha B N[4].Later, the concept was more extensively used in [3],[5],[6] and [7]. According to the definition, a V-shapededgeis formed between the three vertices v,u,w. Therefore, for the hinge dominating set to exist, all vertices of degree one, including pendant vertices, must be included in the hinge dominating set.

A Mongolian tent graph, denoted as $M_{m,n}$, is derived from the grid graph $P_m \times P_n$, where n is odd. This is achieved by adding a vertex above the graph and connecting every alternate vertex of the top row to this extra vertex. In this paper, we determine the hinge domination number of the Mongolian tent graph, which we will henceforth refer to as a tent graph.

We determine the hinge domination number of corona product of graphs with isolated vertices. The corona product of a graph G with a graph H, denoted by GoH, is obtained by taking one copy of the graph G, V(G) copies of graph H, and joining each vertex of thei^{th} copy of H to the i^{th} vertex in G by an edge. For example, $P_n o K_1$, the corona product of the path graph with an isolated vertex forms a comb graph. Similarly,$C_n o K_1$, the corona product of a cycle with an isolated vertex,creates a graph in which each cycle vertex is adjacent to a pendant vertex.

We utilize the following result to establish the minimality of the hinge-dominating sets under consideration.

Theorem: *A hinge-dominating set D_h is minimal if and only if, for every vertex $v \in D_h$ one of the following conditions holds:*

i) $deg\ (v) = 0$ *in* D_h

ii) *There exists a vertex u in* $V - D_h$ *such that* $N(u) \cap D_h = \{v\}$

iii) $< (V - D_h) \cup \{v\} >$ *is connected.*

In this paper, vertices in the $P_m \times P_n$ grid of the tent graph $M_{m,n}$ are denoted as(i, j), representing the vertex in the vertex in thei^{th} row and j^{th} column of the grid. Additionally, the extra vertex of the tent graph as denoted as 'a'.

2. Results and Discussion

Proposition 2.1: The hinge domination number of $P_n o K_1$*and* $C_n o K_1$ *is n.*

Proof: As every pendant vertex must be part of the hinge-dominating set, all n vertices from the n copies of K_1 are included in the hinge-dominating set S. Each i^{th} vertex in P_n is adjacent to i^{th} copy of K_1in S, as well as the $(i-1)^{th}$ or $(i+1)^{th}$(or both) vertices in $P_n$that is not in S. Importantly, these $(i-1)^{th}$ or $(i+1)^{th}$ vertex are not adjacent to i^{th} copy of K_1. Therefore, S is the hinge-dominating set with the minimumnumber of vertices. Consequently,$\gamma_h(P_n o K_1) = n$.

Figure 1 shows the hinge domination of the comb graph $P_5 o K_1$.

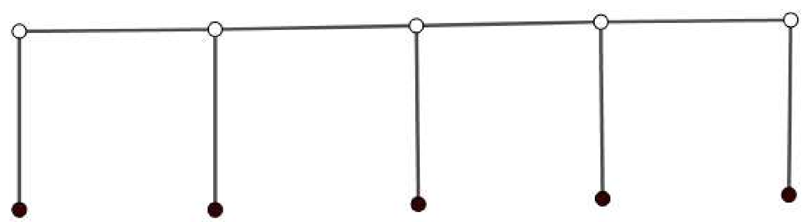

Figure 1

In $C_n o K_1$, we include all n vertice of n copies of K_1in the set S. A similar argument, as mentionedabove,clearly demonstrated that S is a dominating set withthe minimum number of vertices. Therefore, $\gamma_h(C_n o K_1) = n$.

Figure 2 shows the hinge domination of $C_4 o K_1$

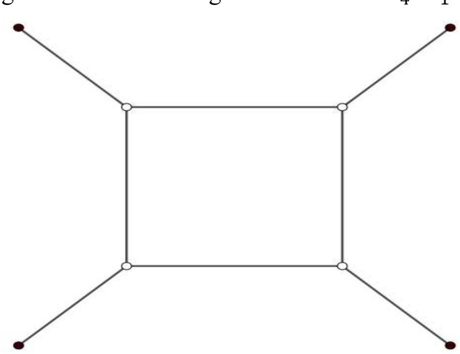

Figure 2

Proposition 2.2:The hinge domination number of a double-star graph $S_{m,n}$*is m+n.*

Proof: The double-star graph$S_{m,n}$consists of two vertices,'a' and 'b'connected by an edge.Additionally, 'm' pendant vertices are adjacent to 'a', and 'n' pendant vertices are adjacent to 'b.'

Since pendant vertices must be included in the hinge-dominating set, we incorporate all *m+n*of these pendant vertices. Vertex 'a' is adjacent to all these 'm' vertices in the dominating set and is adjacent to 'b,'while none of these 'm' vertices are adjacent to 'b.'Similarly, vertex 'b' is adjacent to all 'n' pendant vertices.Thus,it is adjacent to 'a', but 'a' is not adjacent to any of these 'n' pendant vertices. Thus, it satisfies the conditionsfor hinge domination. Therefore,$\gamma(S_{m,n}) = m + n$.

Hinge-dominating set for $S_{2,3}$ is shown below in Figure 3:

Figure 3

Proposition 2.3:

The hinge domination number of the Umbrella graph $U_{m,n}$*is* $\gamma_h\left(U_{m,n}\right) = \{m+2, \quad if\ n = 2\ m+k+2,\ if\ n = 3k+2\ \lceil\frac{n}{3}-1\rceil + m + 1,\ if\ n \neq 3k+2$

Proof: For any integers $m > 2 and n > 1, the$Umbrella graph $U_{m,n}$ is obtained by connecting the end vertex of the path graph P_n with the central vertex of a fan graph F_m.

Firstly,when n=2, it is necessary to include all the vertices of the umbrella graph in the hinge domination set to satisfy the hinge property.Therefore, for $n = 2$, all m+2 vertices of the umbrella graph are included.

Now, for $n > 2$, we include all vertices of the fan graph F_m in the hinge domination set, except for the central vertex, which is also an end vertex of the path graph P_n. We excluded this central and adjacent vertex from the hinge domination set. Subsequently, we include all the vertices fromthe hinge domination set of the remaining part of the path graph P_{n-2}. While constructingthe hinge dominating set for the path graph, we observe that there are either 2 non-dominating vertices between every two dominating vertices or no non-dominating vertices.

In (1), the hinge domination number of the path graph is given by $\gamma_h(P_n) = \{2,\ \ if\ n = 2\ \ k+2\ \ \ if\ n = 3k\ \lceil\frac{n-1}{3}\rceil + 1\ \ if\ n \neq 3k$

So, taking *n-2* in place of *n* in the above formula, we have a hinge domination number of P_{n-2} for $n > 2$ as $\gamma_h(P_{n-2}) = \{k+2\ ,\ if\ n = 3k+2\ \lceil\frac{n}{3}-1\rceil + 1,\ if\ n \neq 3k+2$

Now, we add 'm' to the previously determined hinge domination number to obtain the hinge domination number of the umbrella graph. The addition accounts for the inclusion of 'm' extra vertices in the hinge-dominating set of P_{n-2}. Hence, we get that.

$$\gamma_h\left(U_{m,n}\right) = \{m+2, \quad if\ n = 2\ m+k+2, \qquad if\ n = 3k+2\ \lceil\frac{n}{3}-1\rceil + m + 1, \qquad if\ n \neq 3k+2$$

Hinge domination sets of $U_{4,4}, U_{5,7}$ are shown below in Figure 4 and Figure 5, respectively.

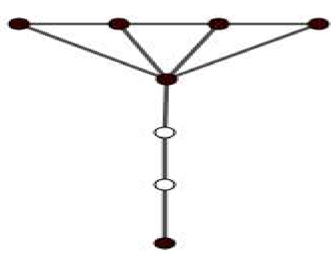

Figure 4

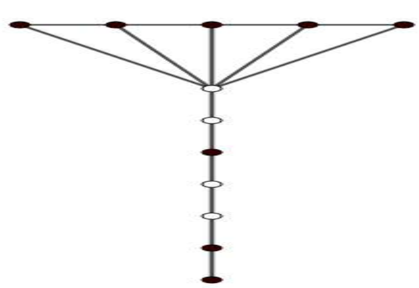

Figure 5

Proposition2.4:The Hinge domination number of the tent graph $M_{2,2n+1}$ is $n+1$

Proof: The degree of the vertex 'a' of the tent graph $M_{2,2n+1}$ is n.

When we include 'a' in the hinge dominating set, all 'n' vertices adjacent to 'a' are dominated. Additionally, we incorporate the vertices $(2,2),(2,4),(2,6)\ldots,(2,2n)$ in the dominating set. Thus, the set $S=\{(2,2),(2,4),(2,6)\ldots,(2,2n),a\}$forms a dominating set. Moreover, itserves as a hinge-dominating set,as explained in the following argument:

Every vertex $(1,k)$ for odd'k' is adjacent to the vertex'a', which is in set S,while the vertex (2,k) is not in S because k is odd. Additionally, the vertices 'a' and (2,k) are not adjacent.

(Note that 'a' is adjacent to only the vertices (1,1),(1,3),…,(1,2n+1))

Now, every vertex (1,p) for even 'p' is adjacent to (2,p), which is in set S, and (1,p+1), which is not in S. Additionally, (2,p) and (1,p+1) are not adjacent.

Now, every vertex (2,k) for odd 'k' is adjacent to $(2,k+1) or\ (2,k-1) or\ both$, all of which are in set S. Additionally, they are adjacent to (1,k), which is not in S. Furthermore, there is no edge between (1,k) and (2,k+1), and no edge between (1,k) and (2,k-1).

Hence, it contributes a dominating set. Furthermore, S is minimal, as per the theorem mentioned earlier. Therefore, S is a hinge-dominating set with the minimum number of vertices, totaling$n+1$. Thus,$\gamma_h(M_{2,2n+1})=n+1$.

The hinge-dominating set of the graph $M_{2,7}$ is Figure 6.

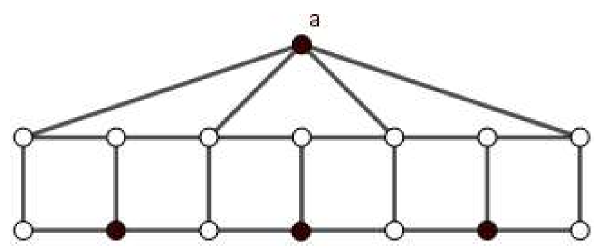

Figure 6

Proposition2.5:

The Hinge domination number of the tent graph $M_{m,2n+1}$ is $1+n(m-1)$

Proof: The proof is similar to the one we provided for m =2,with the difference that we include all vertices of the form (p,q)where q is odd and $p\geq 2$.

We take all vertices in the even columns from the second row onwards as the set S.

$$S=\{(2,2),(2,4),\ldots,(2,2n),(3,2),(3,4),\ldots,(3,2n),\ldots,(m,2),(m,4),\ldots,(m,2n),a\}$$

This set comprises 1+n(m-1) vertices. Additionally, we observe that it is a hinge-dominating set,satisfying the necessary conditions for minimality. Hence, it is a dominating set with the minimum number of vertices. Therefore,$\gamma_{h(M_{m,2n+1})}=1+n(m-1)$.

The hinge-dominating set for $M_{4,5}$ is shown in Figure 7.

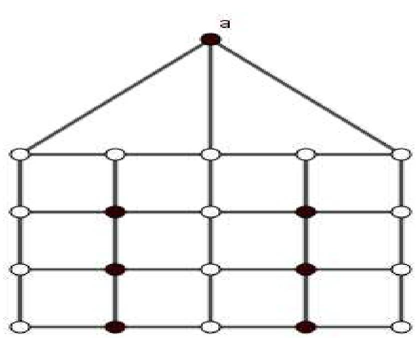

Figure 7

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