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A Comprehensive Study On The Applications Of Fractional Chromatic Numbers

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ABSTRACT

1. In "the investigation of fractional chromatic numbers within the domain of graph theory, seminal references provide extensive insights. Zhang's comprehensive survey published in 2009 presents an elaborate synthesis of definitions, methodologies, and applications pertinent to this area of study. The foundational textbook authored by Chartrand and Lesniak (2005) functions as a pivotal resource for grasping fundamental concepts inherent to graph theory, while West's introductory volume (2001) imparts essential knowledge regarding graph structures and their properties. The monograph by Chen and Zhu (2009) thoroughly examines fractional graph theory, proffering a logical framework for understanding graph theory principles, inclusive of fractional chromatic numbers. Diestel's extensive textbook (2005) encompasses a multitude of facets pertaining to graph theory, notably including graph coloring and fractional chromatic numbers. The research conducted by Guan and Zhu (2011) investigates fractional chromatic numbers within the framework of Mycielski's theorem and its subsequent generalizations. Furthermore, the contributions of Haas, Nowakowski, and Rousseau (2002) yield insights into the fractional chromatic number associated with various graphs. Kostochka and Yancey (2003) scrutinize the fractional chromatic number of triangle-free subcubic graphs, while the study by Chen, Kierstead, and Zhu (2006) delves into this numerical characteristic concerning triangle-free graphs characterized by substantial minimum degree. Caro, Lévêque, and Reed (2007) engage with a generalization of Reed's conjecture, whereas Havet and Sereni (2008) explore the interplay between fractional chromatic numbers, odd holes, and perfect graphs. Finally, Nešetřil and Raspaud (1999) enhance the comprehension of the fractional chromatic number associated with cubic graphs."

1. PRELIMINARIES

2. Definition 2.1: Vertex Colouring:

The vertex coloring of a graph involves assigning colors to its vertices in such a way that no two adjacent vertices share the same color. This can be represented using a coloring function $c: V \rightarrow N$, where V is the set of vertices, and N is the set of natural numbers.

The"condition for a vertex coloring is that adjacent vertices must have different color assignments. This condition can be expressed using the inequality:

$c(u) \neq c(v)$ **for** $u, v \in V$ **such that** $u, v \in E$

Here, *E* represents the edge set of the graph. The goal is to find a coloring function *c* that satisfies this condition for all pairs of adjacent vertices.

3. **Definition 2.2: Edge Coloring:**

4. Edge coloring is the process of assigning colors to the vertices of a graph in such a way that neighboring vertices are coloured differently. Edge coloring of a graph involves assigning colors to its edges in such a way that no two incident edges of the same vertex share the same color. The process is governed by an edge coloring function c: $E \rightarrow$N, where E is the set of edges, and N is the set of natural numbers."

5. The "condition for a valid edge coloring is that edges incident to the same vertex must have different color assignments. This can be expressed using the inequality: $c(uv) \neq c(vw)$ for $u \in V$ such that $v, w \in E$

Example: Consider a simple graph with three vertices V={1,2,3}E={1,2,3} and edges A valid edge coloring could be achieved by assigning different colors to the edges, for instance:

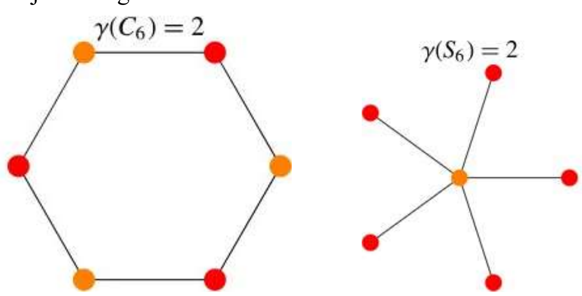
$c(12) = 1, c(23) = 2, c(31) = 3\,c$

This satisfies the condition that incident edges at each vertex have different colors.

6. **Definition 2.3: Face Coloring:**

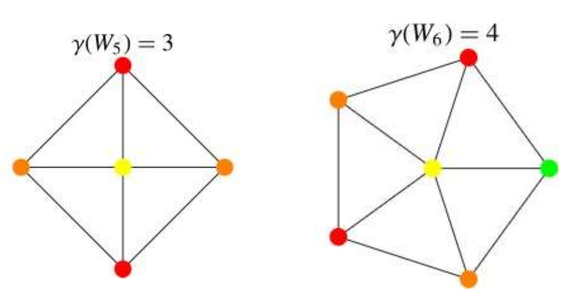
7. Face coloring is the process of assigning colors to the faces of a planar graph in such a way that neighboring faces are coloured with two separate colors and no two faces share the same color boundary.

8. Face coloring in graph theory involves assigning colors to the faces of a planar graph in such a way that no two faces sharing an edge have the same color. This concept is closely related to the coloring of maps or regions in a plane such that adjacent regions have distinct colors."

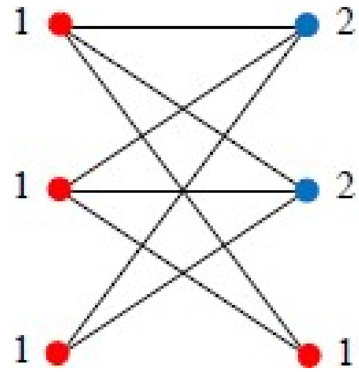


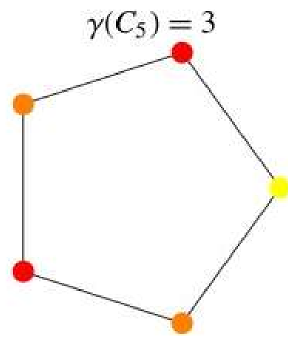
9. **Definition 2.4: Coloring Parameters of Graph with Illustration Chromatic number:**

10. The chromatic number of a graph is the minimum number of colors needed to color the vertices in such a way that no two adjacent vertices have the same color. It is denoted by χ (G), where G is the graph.



11. Consider a simple graph G with vertices V and edges E. The chromatic number χ (G) represents the minimum number of colors needed to color the vertices of G. Let's illustrate this with an example:





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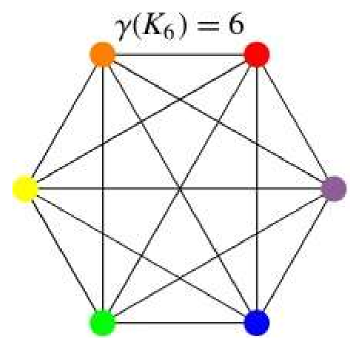

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14. Figure 1: Chromatic number

15. Definition"2.6: Computation of Chromatic Numbers for New Class of Graphs

In"this paper, we compute the chromatic for new class of graphs like central graphs of cycle graph Cn and jelly fish graph $J(m, n)$ [16],[17].

16. Central graph:

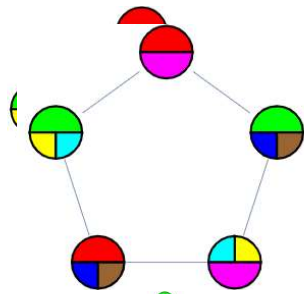
17. Let G be a undirected graph with no loops and parallel edges. The graph is formed by subdividing the each edge exactly once and joining all the non-adjacent vertices of the graph is called the central graph. It is denoted by the symbol (G).

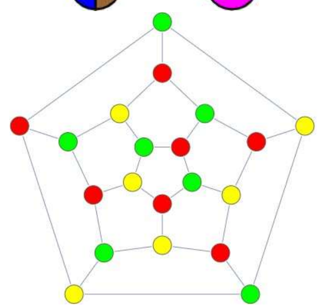
18. Definition 2.7: Fractional coloring

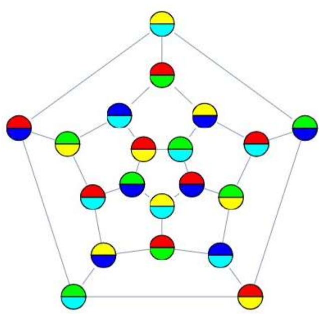
19. Let *I (G)* denote the set of all independent sets of vertices of a graph G, and let I (G, u) denote the independent sets of G that contain the vertex u. A fractional coloring of G is then a nonnegative real function f on *I (G)* such that for any vertex u of G,

20. $\sum_{s \in I(G,u)} f(s) \geq 1$

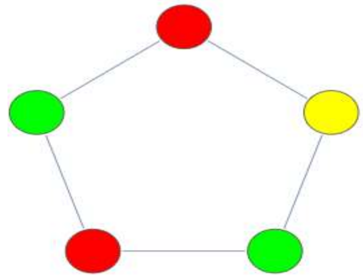
21. The sum of values of *f* is called its weight, and the minimum possible weight of a fractional coloring" is called the fractional chromatic number $\chi^{*}(G)$."



22. 

23. 

24. The above definition of fractional coloring is equivalent to allowing multiple colors at each vertex, each with a specified weight fraction, such that adjacent vertices contain no two colors alike. For example, while the dodecahedral graph is 3-colorable since the chromatic number is 3 (left figure above; red, yellow, green), it is 5/2-multicolorable since the fractional chromatic number is 5/2 (5 colors-red, yellow, green, blue, cyan-each with weight 1/2, giving 5.(1/2)=5/2."



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Note ”that in fractional coloring, each color comes with a fraction which indicates how much of itis used in the coloring. So if red comes with a fraction 1/4, it counts as 1/4 in the weight. There can therefore be more actual colors used in a fractional coloring than in a non-fractionalcoloring. For example, as illustrated above, the 5-cycle graph C_5 is 3-vertex chromatic (left figure) but is 5/2-fractional chromatic (middle figure). However, somewhat paradoxically, the fractional coloring of C_5 (right figure) using *seven* colors still only count as only "5/2 colors. Since the colors come with weights 1/2 (red, green, violet) and 1/4 (the other four), giving a fractional chromatic number of

$$\chi^*(c_5) = 3(\frac{1}{2}) + 4(\frac{1}{4}) = \frac{5}{2}$$

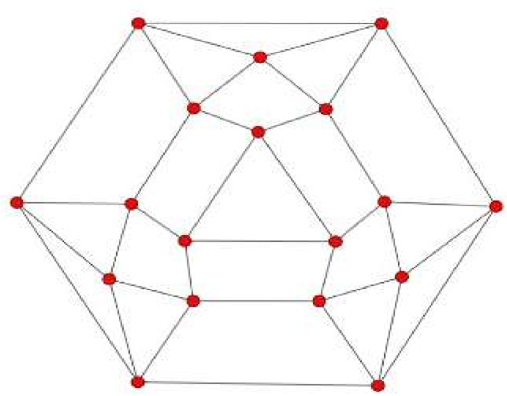
As a result, the question of how to minimize the "actual" number of colors used is not (usually) considered in fractional coloring.

A fractional coloring is said to be regular if for each vertex *u* of a graph *G*,

$$\sum_{s \in I(G,u)} f(s) \geq 1$$

Every graph *G* has a regular fractional coloring with rational or integer values.

Chiu Graph



Definition 2.8: Fractional Chromatic Number

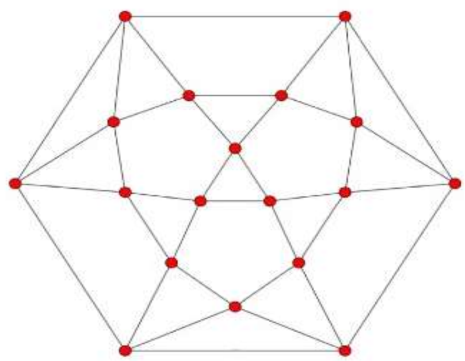
Let be a fractional coloring of a graph *G*. Then the sum of values of f is called its weight, andthe minimum possible weight of a fractional coloring is called the fractional chromatic number3 $^*(G)$ .Every simple graph has a fractional chromatic number which is a rational number or integer.

The fractional chromatic number satisfies

$$\omega(G) \leq \omega^*(G) = \chi^*(G) \leq \chi(G)$$

The fractional chromatic number of a graph may be an integer that is less than the chromatic number. For example, for the Chvátal graph $\chi^* = 3$, but $\chi^* = 4$. Integer differences greater than one are also possible, for example, at least four of the non-Cayley vertex-transitive graphs on 28 vertices have $\chi - \chi^* = 2$, and many Kneser graphs have larger integer differences.

Johnson solid skeleton 92



Gimbel *et al.* (2019) conjectured that every 4-chromatic planar graph has fractional chromatic number strictly greater than 3. Counterexamples are provided by the 18-node Johnson skeleton graph j_{92} as well as the 18-node example given by Chiu *et al.* (2021) illustrated above. Chiu *etal.* (2021) further demonstrated that there are exactly 17 4-regular 18-vertex planar graphs with chromatic number 4 and fractional chromatic number 3, and that there are no smaller graphs having these values."

For "any graph G,

$$\chi^*(G) \le \frac{|G|}{\alpha(G)}$$

where $|G|$ is the vertex count and α (G) is the independence number of G. Equality always holds for a vertex transitive G, in which case $\chi^*(G) \le \frac{|G|}{\alpha(G)}$

2. PROPOSED THEOREMS

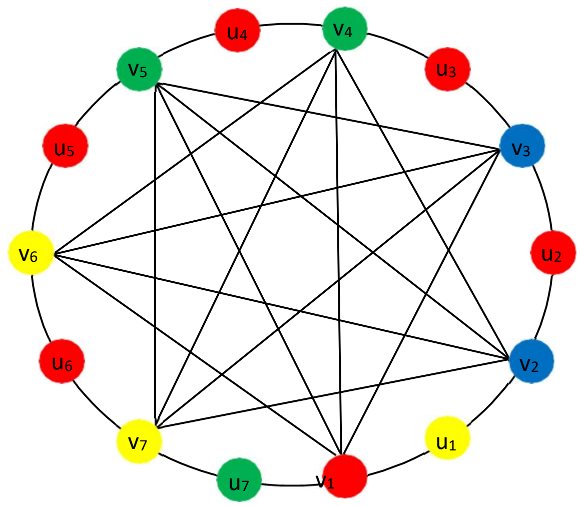
Theorem 3.1:

Prove that the chromatic number for central graph of cycle of length n is 4. (i.e) ((Cn)) = 4.

Proof:

Construction of Central graph of cycleC_n, (C_n): The cycle graph is denoted by the symbol C_n.The vertex set $V(C_n) = \{v_1, v_2, v_3, ...v_n\}$ **and edge set** $E(C_n) = \{q_1, q_2, q_3, ..., q_n\}$ **where q_i = v_i, v_{i+1} for1 ≤ i ≤ n and q_n = v_n, v_1.**

The central graph of cycle of length n is denoted by (C_n). The central (C_n) is formed by subdividing each edge v_i, v_{i+1} for 1 ≤ i ≤ $n-1$ of C_n exactly once by adding a new vertex u_i and subdividing v_n, v_{1+1} and joining v_i with v_i, 1 ≤ i, j ≤ n, i ≠ j and v_i, v_j (C_n). The new vertex set formed is $C(C_n) = (V_1 \cup U_1)$ **where V_1= {v_1'v_2'v_3'…,} and U_1={$u_1$$u_2$$u_3$' … … , u_n}. The new edge set formed is** $C(C_n) = (E_1 \cup E_1)$ **such that** $E_1 = \{e_1^{'}, e_2^{'}, e_3^{'}, ...e_n^{'}\}$ **where** $e_k^{'} = v_i, v_j\ 1 \le i, j \le n = 1, 2, .., n$; $i \ne j; (v_i, v_j) \in E(C_n)$ **and** $E_1 = \{e_1^{"}, e_2^{"}, e_3^{"}, ...e_n^{"}\}$ **where** $e_i^{"} = u_i, v_{j+1} \le i \ne j = v_i, v_j$; **for** $i = j$.



26. **Figure 3: Central graph of cycle graph $C(C_n)$**

Define "the mapping ψ:$V(C(C_n) \rightarrow \{1, 2, 3, 4\}$ such that $\psi(v_1)$= 1, $\psi(v_2) = \psi(v_3) = 2$, $(v_4) = (v_5) = 3$, $(v_6) = \psi(v_7) = and\ \psi(u_2) = \psi(u_3) = \psi(u_4) = (u_5) = \psi(u_6) = 1$; $\psi(u_1) = 2$, $(u_7) = 3$. Clearly it is easy to check that it is a proper coloring on vertices and hence we have $(C_n) \geq 4$. Suppose assume that $(C_n) = 5$ by some optimal coloring β.

Then the coloring β assigns distinct colors to higher degree non-adjacent vertices. Therefore colors on v_1, v_2, v_4, v_7 must be distinct. Now the fifth color must appear on any of the remaining vertices. If this happens, then there exists any one pair of color with non-adjacent vertices does not have edge between them which is a contradiction. So we have $(C(C\text{n})) \leq 4$. Therefore from the inequalities we arrive the result. Hence the chromatic number for central graph of cycle of length n is 4.

(i.e.) χ $(C(C_n)) \geq 4$.

Theorem 3.2:

Prove that the chromatic number for central graph of jelly fish graph 4. (i.e) $(J(m, n)) = 4$."

"Proof:

Construction of Central Graph of Jelly Fish Graph, $((\boldsymbol{m},))$:

The Jelly fish graph is denoted by(m, n). The vertex set V($j_{m,n}$) = $\{x_1, x_2, x_3, x_4 \cup\ u_1 u_2, u_3, \ldots, u_m \cup\ v_1, v_2, v_3, v_4 \ldots \ldots \ldots v_n\}$ and edge set E (C_n) = $\{(x_1, x_2), (x_2, x_3), (x_3, x_4), (x_4, x_1), (x_1, x_3) \cup (x_4, u_i) \cup (x_2, v_j)\}$. where $\{i = 1, 2, 3 \ldots n; j = 1, 2, 3 \ldots m\}$. The central graph of jelly fish graph is denoted by the symbol C(J(m, n)). The central graph of C(J(m, n)) is formed by subdividing each edge x_1, x_{i+1}, $1 \leq i \leq n$ exactly once by adding a new vertex c_i and joining x_i with x_{i+2} = 1, 2, 3, 4 and inclusion of vertex c_5 between the edge (x_1, x_3). Also subdividing the pendant vertices connected by x_4 to p_i and x_2 to q_j and joining x_4 with p_i and x_2 with q_j where i = 1, 2, 3 …m; j = 1, 2, 3 … n."

The new vertex set formed is

V ($j_{m,}$) = {x_1, x_2, x_3, $x_4 U$ $c_{1'} c_2$, $c_{3'} c_5 U p_i$ U $u_i U q_j U v_j$}.

(x_1, c_1), ((x_1, c_2), (c_2, x_2)(x_2, x_3), (c_2, x_3), (c_4, x_3), (c_4, x_1), (c_4, x_4), (x_1, c_5), U(p_i, u_i)U(x_2, q_j)U(q_j, v_j) *where* i = 1, 2, 3, … n; j = 1, 2, 3, …

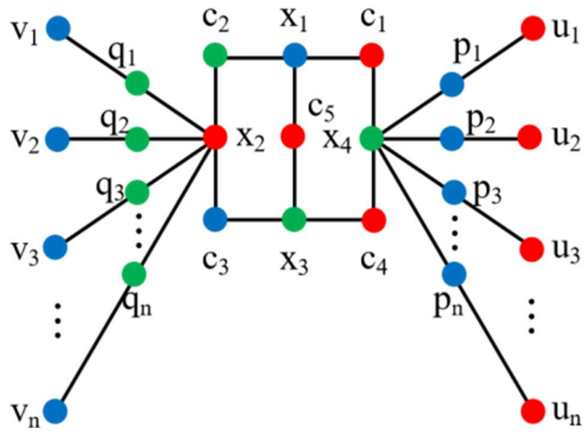


Figure 4: Central graph of jelly fish graph (($\boldsymbol{m}$, n))

Define "the mapping ψ:V (C ($j_{m,}$) →{1, 2, 3, 4} such that (x_1) = 1, (x_2) = 2, (x_3)= 3. The remaining vertices can be assigned any one of these colors with the proper coloring without loss of generality. Clearly it is easy to check that it is a proper vertex coloring on vertices and hence we have ($j_{m,}$) ≥ 3. Suppose assume that ($j_{m,}$) = 4 by some optimal coloring μ. Thenthe coloring μ assigns different colors to higher degree non- adjacent vertices. Therefore colors on x_1, x_2, x_3 must be distinct. Now the fourth color must appear on any of the remaining vertices. If this happens, then there exists any one pair of color which does not have edge between them which is a contradiction to our assumption. So we have ψ C (j_n) ≤ 3. Hence from the inequalities we reach the required result. Hence the chromatic number for central graph of jellyfish graph is 3."

(i. e) χ C ($j_{m,}$) = 3.

The Fractional Chromatic Number Uses in Real Life Application:

The "application of the fractional chromatic number is diverse and extends across variousfields, providing solutions to complex optimization problems. Its versatility makes it a valuable tool for researchers and practitioners seeking innovative approaches in wireless communication, coding theory, image processing, and other areas where graph theory concepts are applied.

Fractional chromatic number to model the assignment of error-correcting codes andSparse Graphs and Network Design:

The different parts of a data stream or communication channel involve a mathematicalformulation. Let's outline a simplified conceptual approach with equations:

Consider a communication channel represented by a graph where vertices correspond to different parts or segments of the data stream and edges represent interference or interactionsbetween these segments.

Let:

- $G = (V, E)$ be the graph representing the communication channel, where V is the set ofvertices and E is the set of edges.
- f:V→[0,1] be a fractional coloring function, where $f(v)$ represents the fraction of colorsassigned to vertex v

(with $0 \leq f(v) \leq 1$)."

Now, "the goal is to minimize the average number of colors assigned to the vertices, subjectto the condition that adjacent vertices receive distinct colors. The fractional chromatic number $\chi f(G)$ can be defined as the minimum average number ofcolors needed for such a fractional coloring:"

$$\chi_f(G) = \min \frac{1}{|V|} \sum_{v \in V} f(v)$$

Subject to the constraint:

$$f(u) + f(v) \geq 1 \text{ for all } uv \in E$$

This "constraint ensures that adjacent vertices have distinct colors, and the minimization objective seeks to minimize the average number of colors assigned.

Now, to relate this to error correction, you can think of each color representing a different error-correcting code. The goal is to assign error-correcting codes to different segments of thedata stream in such a way that interference is minimized. The fractional chromatic number provides a measure of the efficiency of this assignment.

Utilizing Fractional Chromatic Number in Coding Theory: A Focus on Error- Correcting Code Assignment:

Let $G=(V,E)$ be a graph representing a communication channel or data stream, where V is the set of vertices representing different segments of the data stream, and E is the set of edges representing interference or interactions between these segments.

Now, let's introduce a binary variable x_v for each vertex $v \in V$, where $x_v = 1$ indicates that a color (or error-correcting code) is assigned to the corresponding segment, $x_v = 0$ otherwise.

The goal is to minimize the total number of colors (error-correcting codes) used, subject to the condition that adjacent segments have distinct colors. The fractional chromatic number $3_f(G)$ can be formulated **as a linear programming problem:**

Objective: Minimize $\sum x_v$

Subject to the constraint:

$x_u + x_v \geq 1$ **for all** $uv \in E$

Additionally, the variables are binary:

$x_v \in \{0,1\}$ **for all** $v \in V$

This linear programming formulation aims to minimize the total number of assigned colors whileensuring that adjacent segments have different colors. The binary variables x_v effectively represent whether a particular error-correcting code is assigned to a segment or not.

Optimizing Frequency Assignment in Optical Networks Using the Fractional Chromatic Number:

Frequency assignment in optical networks involves assigning wavelengthsto edges in a way that adjacent edges have different wavelengths, reducing interference. The fractional chromatic number can be applied to model and solve frequency assignment problems. Let's denote the graph as G = (V, E), where V is the set of vertices representing edges in the optical network, and E is the set of edges. The fractional chromatic number, denoted by χ f(G), is defined as the minimum real number k such that the vertices of G can be colored with k colors insuch a way that adjacent vertices receive different fractional colors. The objective is to minimize the value of k."

While "the fractional chromatic number may not have as direct or widespread real- life applications as some other graph theory concepts, it is still used in various domains.

Here area few areas where the fractional chromatic number can find applications:

1. **Frequency Assignment in Radio Networks:**

Frequency assignment in radio networks is a critical aspect of network design, aiming to minimize interference and enhance overall performance. By utilizing the fractional chromatic number, network designers can model the frequencyassignment problem, ensuring efficient utilization of available frequencies while minimizing the likelihood of interference between neighboring channels.

The "goal is to assign frequencies to different radio channels in a way that neighboring channels, which might interfere with each other, are assigned distinctfrequencies. The fractional chromatic number provides a theoretical foundation for determining the minimum number of distinct frequencies required to achieve interference-free communication in the network.

2. Register Allocation in Compiler Design:

27. Register allocation in compiler design involves assigning a limited set of registersto variables in a program. Fractional chromatic numbers can be used to model andsolve register allocation problems, ensuring that variables that interfere with each other do not share the same register.

3. Scheduling Problems:

Fractional chromatic numbers find applications in scheduling problems where tasks need to be assigned resources without conflicts. The vertices represent tasks,and the fractional chromatic number helps determine the minimum number of time slots or resources needed to complete the tasks.

4. VLSI Design:

Very Large Scale Integration (VLSI) design involves placing and routing components on a chip. Fractional chromatic numbers can be applied to the channel assignment problem, ensuring that adjacent channels do not interfere witheach other.

5. Graph Coloring Games:

Some graph coloring games, where players take turns choosing vertices and aimto avoid conflicts, can be analyzed using fractional chromatic numbers. This adds an additional layer of complexity and strategy to the game.

6. Wireless Sensor Networks:

Fractional chromatic numbers are employed in the design of wireless sensornetworks. The vertices represent sensor nodes, and assigning fractional colors helps optimize the communication and data exchange among nodes while minimizing interference.

7. Parallel Processing:

In parallel processing systems, fractional chromatic numbers can be utilized for task scheduling and load balancing, ensuring that tasks with dependencies or conflicts are assigned different processing units."

6. **Circuit Design:**

In electronic circuit design, the fractional chromatic number can be used to model the assignment of colors (representing different voltage levels or signal types) to nodes in a circuit graph. This aids in minimizing signal interference and optimizing the overall performance of the circuit.

7. **Conflict Resolution in Resource Allocation:**

When allocating resources in systems where conflicts can arise, such as allocatingshared resources in a computer network or scheduling tasks on a shared processor,the fractional chromatic number can help minimize conflicts and improve the overall efficiency of resource usage.

8. **"Social Network Analysis:**

In social network analysis, where nodes represent individuals and edges represent relationships, the fractional chromatic number can be applied to model the optimal assignment of labels or attributes to individuals while considering social connections. This can help in tasks such as targeted advertising or opinion spreading.

9. Graph Coloring in Geographic Information Systems (GIS):

In GIS applications, the fractional chromatic number can be utilized to model the coloring of maps. Each region can be represented as a vertex, and the fractional chromatic number can guide the assignment of colors to represent different geographic features or categories while minimizing visual clashes.

10. Job Scheduling in Parallel and Distributed Computing:

The fractional chromatic number can be employed in the context of scheduling tasks in parallel and distributed computing systems. It helps in assigning resourcesefficiently, minimizing contention, and optimizing the overall performance of the computing infrastructure."

11. Traffic Flow Optimization:

In transportation and traffic engineering, the fractional chromatic number can be used to model the assignment of different traffic signals or control strategies to intersections in a road network. This aids in optimizing traffic flow and reducing congestion.

12. Graph Labeling Problems:

Various labeling problems, where vertices or edges of a graph are assigned labels according to certain rules or constraints, can benefit from the use of fractional chromatic number formulations. This includes problems in coding theory and combinatorial optimization.

13. Biological Networks:

In biological systems, networks can represent interactions between different elements such as proteins or genes. The fractional chromatic number can be employed to model the assignment of functional annotations or attributes to these elements while considering the interactions, aiding in the study of complex biological systems.

14. Radio Frequency Identification (RFID) Systems:

In RFID systems where multiple tags or devices may need to communicatewithout interference, the fractional chromatic number can be used to optimize the assignment of communication frequencies or time slots, ensuring efficient and reliable data transfer."

15. Graph "Coloring in Image Processing:

Image segmentation and analysis can be modeled using graph theory, wherepixels or regions are represented as vertices in a graph. The fractional chromatic number can guide the assignment of colors or labels to different regions in an image, facilitating tasks such as object recognition and tracking.

16. Wireless Sensor Networks:

In networks of wireless sensors, where nodes may need to transmit data without interference, the fractional chromatic number can be used to optimize the assignment of communication channels, improving the reliability and efficiencyof data transmission.

17. Game Theory:

In certain game theory scenarios, where players are represented as nodes in a graph and edges represent interactions or conflicts, the fractional chromatic number can be used to model the optimal assignment of strategies to players, considering both cooperation and competition.

4. CONCLUSION

28. In conclusion, the thorough examination of proper vertex coloring strategies applied to central graphs, specifically jellyfish ($J_{m,}$) and cycles (C_n), has yielded meaningful insights into their respective chromatic numbers. For the jellyfish graph ($J_{m,}$), a meticulous mapping demonstrated a achievable proper vertex coloring

with at most three colors. The contradiction arising from assuming a chromatic number of four in some optimal coloring solidifies the conclusion that the chromatic number is unequivocally 3. Similarly, the study of cycle graphs (C_n) involved a mapping resulting in a proper vertex coloring with at least four colors. The contradiction assuming a chromatic number of five in some optimal coloring establishes that the chromatic number for the central graph of a cycle of length n is at least 4.

Additionally, the comprehensive exploration of fractional chromatic numbers has revealed their extensive applicability in various domains within graph theory. The extension of traditionalgraph coloring concepts, particularly in solving optimization problems in coding theory, demonstrates the versatility of fractional chromatic numbers. Applications in image processing, object recognition and their connection to error-correcting codes underscore their practical significance. The study provides a structured approach to problem-solving, particularly in linear programming models for optimization."

CONFLICTS OF INTEREST

The author(s) declare that there are no conflicts of interest regarding the publication of this paper.

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