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ANUJ TRANSFORM FOR THE SOLUTION OF ABEL'S INTEGRAL EQUATION OF SECOND KIND

Sudhanshu Aggarwal[1*], Anuj Kumar[2] and Shikha Bansal[3]

[1]Assistant Professor, Department of Mathematics, National Post Graduate College, Barhalganj, Gorakhpur-273402, Uttar Pradesh, India
[2]Research Scholar, SRM Institute of Science and Technology, Delhi-NCR, Campus, Modinagar, Ghaziabad-201204, Uttar Pradesh, India
[3]Assistant Professor, SRM Institute of Science and Technology, Delhi-NCR, Campus, Modinagar, Ghaziabad-201204, Uttar Pradesh, India
sudhanshu30187@gmail.com, anuj.doc84@gmail.com, shikhab@srmist.edu.in

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ABSTRACT
This paper introduces the Anuj transform as a reliable and efficient technique for addressing Abel's integral equation of second kind (AIESK). Through the exploration of this method, the researchers have demonstrated its excellence and effectiveness in tackling such types of equations. The outcomes obtained from utilizing the Anuj transform reflect its robustness and applicability in practical settings. A variety of examples have been included to illustrate the application of the proposed method and verify its capabilities. These examples serve as concrete evidence supporting the assertion of the Anuj transform's reliability and efficiency when solving AIESK. The detailed analysis provided in this paper offers valuable insights into the potential of this approach and its significance in the realm of integral equations.

Keywords: Anuj transform; Convolution; Integral equation; Inverse Anuj transform

MSC2010: 35A22, 45A05, 45D05

1. INTRODUCTION

In recent years, an increasing cohort of scholars and researchers hailing from diverse academic disciplines such as engineering, biology, physics, mathematics, and chemistry have exhibited a growing penchant for delving into the enigmatic realm revolving around integral equations. This intriguing field, nestled within the domain of mathematics, has captured the attention of many due to its myriad practical applications across various scientific and engineering domains. With its ability to tackle complex real-world problems through sophisticated mathematical constructs and models, integral equations have emerged as a focal point of interest, nurturing a vibrant community of academics and experts dedicated to unlocking its intricate secrets and extracting invaluable insights from its intricacies. In addition to tackling challenges like bacteria growth, blood glucose concentration variations, and the dynamics of hanging chains, integral equations play a vital role in unraveling the complexities of various systems and processes [1-3]. Among these equations, Abel's integral equation stands out for its remarkable elegance and practicality, finding extensive application across diverse fields from mathematics and astrophysics to visco-elasticity, classical mechanics, and electrical engineering [4]. Its widespread adoption both in academic research and industrial applications underscores its significance in advancing scientific exploration and technological breakthroughs. The versatility and robustness of Abel's integral equation highlight its enduring relevance in addressing intricate problems and driving innovation in our increasingly interconnected world of knowledge and discovery. Navigating the complexities of AIESK not only demands a profound grasp of intricate mathematical principles and theoretical frameworks but also necessitates hands-on experience across various domains where its applications manifest. The equation's potential is harnessed by scientists, engineers, and researchers alike, enabling them to delve into uncharted territories of knowledge and expand the limits of human comprehension across diverse disciplines. This incredible power not only paves the way for groundbreaking discoveries and innovations but also shapes the course of future scientific and technological advancements, making waves that will ripple through generations to come.

Integral transform methods, such as the Upadhyaya transform [5-6], Rishi transform[7-9], Jafari transform [10], Mohand transform [11], Sawi transform [12], Elzaki transform [13], Laplace-Carson transform [14], Laplace transform [15], Aboodh transform [16], Kamal transform [17], Sumudu transform [18], Sawi transform [19], and Anuj transform [20-22] are widely recognized and frequently employed techniques for tackling intricate mathematical problems. These transformative approaches involve converting the original equation into a different domain, making it more accessible to identify the solution. Given the intricacies of solving AIESK, the requirement for specialized methodology and stringent techniques is evident to derive accurate and dependable solutions. Hence, we are compelled to delve into a promising strategy known as the "Anuj transform" for addressing AIESK effectively. Anuj transform is a newly developed integral transform method. Within the context of this paper, the transformative approach has been effectively leveraged to address the complex problem of AIESK. By utilizing this transformative method, we delve deeper into the intricacies of resolving AIESK, shedding light on the various aspects involved in its solution. The subsequent sections of the paper are thoughtfully structured to facilitate understanding and engagement. Section 2 serves as the foundation, offering key definitions and fundamental properties essential to grasp the subject matter. Moving forward, in Section 3, the innovative Anuj transform specifically designed for AIESK is introduced, paving the way for a detailed exploration. Section 4 equally plays a crucial role, showcasing the practical application and validation of our proposed methodology through a series of numerical instances, highlighting the efficiency and precision of the approach. Finally, in the conclusive Section 5, a succinct summary encapsulates the key findings and implications, wrapping up the paper cohesively.

2. BASIC DEFINITIONS AND PROPERTIES OF ANUJ TRANSFORMS: In this particular section, the focus is directed towards elucidating several fundamental definitions and key properties associated with the Anuj transform, a mathematical concept that plays a significant role in various analytical applications. With a systematic and rigorous approach, this segment aims to equip readers with the necessary knowledge and understanding to navigate the theoretical landscape of the Anuj transform and leverage its transformative capabilities for practical problem-solving and theoretical inquiries.

2.1 DEFINITION OF ANUJ TRANSFORM

A function $u(x) \in \mathcal{C}, x \geq 0$, where $\mathcal{C}$ is the collection of the piecewise continuous exponential order functions, has the Anuj transform and it is given by [21]

$$\dot{\mathcal{A}}\{u(x)\} = q^2 \int_0^\infty u(x) e^{-\left(\frac{x}{q}\right)} dx = f(q),\ q > 0$$

The Anuj's transformations of fundamental mathematical functions are given in Table 1 (see Table 1).

1. **Table-1:** The Anuj's transformations of fundamental mathematical functions [21]

S.N.	$u(x) \in \mathcal{C}, x \geq 0$	$\dot{\mathcal{A}}\{u(x)\} = f(q)$
1	1	q^3
2	e^{ax}	$\left(\frac{q^3}{1 - qa}\right)$
3	$x^a, a \in N$	$a!\, q^{a+3}$
4	$x^a, a > -1, a \in R$	$q^{a+3}\Gamma(a + 1)$
5	$\sin(ax)$	$\left(\frac{a\, q^4}{1 + q^2 a^2}\right)$
6	$\cos(ax)$	$\left(\frac{q^3}{1 + q^2 a^2}\right)$
7	$\sinh(ax)$	$\left(\frac{a q^4}{1 - q^2 a^2}\right)$

8	$\cosh(ax)$	$\left(\frac{q^3}{1-q^2a^2}\right)$

2.2 LINEARITY PROPERTY OF ANUJ TRANSFORMS [2]

If $u_i(x) \in \mathcal{C}, x \geq 0$ and $\dot{\mathcal{A}}\{u_i(x)\} = \mathfrak{f}_i(q)$ then $\dot{\mathcal{A}}\{\sum_{i=1}^{n} a_i u_i(x)\} = \sum_{i=1}^{n} a_i \dot{\mathcal{A}}\{u_i(x)\} = \sum_{i=1}^{n} a_i \mathfrak{f}_i(q)$, where a_i are arbitrary constants.

2.3 TRANSLATION PROPERTY OF ANUJ TRANSFORMS [2]

If $u(x) \in \mathcal{C}, x \geq 0$ and $\dot{\mathcal{A}}\{u(x)\} = \mathfrak{f}(q)$ then $\dot{\mathcal{A}}\{e^{ax}u(x)\} = (1-qa)^2 \mathfrak{f}\left(\frac{q}{1-qa}\right)$, where a is arbitrary constant.

2.4 CHANGE OF SCALE PROPERTY OF ANUJ TRANSFORMS [3]

If $u(x) \in \mathcal{C}, x \geq 0$ and $\dot{\mathcal{A}}\{u(x)\} = \mathfrak{f}(q)$ then $\dot{\mathcal{A}}\{u(ax)\} = \frac{1}{a^3}\mathfrak{f}(aq)$, where a is arbitrary constant.

2.5 CONVOLUTION (FALTUNG) PROPERTY OF ANUJ TRANSFORMS [21]

If $u_i(x) \in \mathcal{C}, x \geq 0, i = 1,2$ and $\dot{\mathcal{A}}\{u_i(x)\} = \mathfrak{f}_i(q), i = 1,2$ then

$\dot{\mathcal{A}}\{u_1(x) * u_2(x)\} = \frac{1}{q^2}\prod_{i=1}^{2} \dot{\mathcal{A}}\{u_i(x)\} = \frac{1}{q^2}\prod_{i=1}^{2} \mathfrak{f}_i(q)$.

2.6 INVERSE ANUJ TRANSFORM [23]

The inverse Anuj transform of $\mathfrak{f}(q)$, assigned by $\dot{\mathcal{A}}^{-1}\{\mathfrak{f}(q)\}$, is another function $u(x)$ having the characteristic that $\dot{\mathcal{A}}\{u(x)\} = \mathfrak{f}(q)$.

3. ANUJ TRANSFORM FOR THE SOLUTION OF ABEL'S INTEGRAL EQUATION OF SECOND KIND: The AIESK is given by [4]

$$u(x) = F(x) + \int_0^x \frac{u(t)}{\sqrt{(x-t)}}dt, \quad (1)$$

where $F(x)$ and $u(x)$ are known and unknown functions respectively.

The use of Anuj transform of both sides of Eq. (1) gives

$$\dot{\mathcal{A}}\{u(x)\} = \dot{\mathcal{A}}\{F(x)\} + \dot{\mathcal{A}}\left\{\int_0^x \frac{u(t)}{\sqrt{(x-t)}}dt\right\}$$

$$\Rightarrow \dot{\mathcal{A}}\{u(x)\} = \dot{\mathcal{A}}\{F(x)\} + \dot{\mathcal{A}}\left\{u(x) * x^{-\left(\frac{1}{2}\right)}\right\} \quad (2)$$

The use of the property 2.4 in Eq. (2) gives

$$\dot{\mathcal{A}}\{u(x)\} = \dot{\mathcal{A}}\{F(x)\} + \frac{1}{q^2}\dot{\mathcal{A}}\{u(x)\}\dot{\mathcal{A}}\left\{x^{-\left(\frac{1}{2}\right)}\right\}$$

$$\Rightarrow \dot{\mathcal{A}}\{u(x)\} = \dot{\mathcal{A}}\{F(x)\} + \frac{1}{q^2}\dot{\mathcal{A}}\{u(x)\}q^{\frac{5}{2}}\Gamma\left(\frac{1}{2}\right)$$

$$\Rightarrow \dot{\mathcal{A}}\{u(x)\} = \left[\frac{1}{1-q^{\frac{1}{2}}\Gamma\left(\frac{1}{2}\right)}\right]\dot{\mathcal{A}}\{F(x)\}$$

$$\Rightarrow \dot{\mathcal{A}}\{u(x)\} = \left[\frac{1}{1-\sqrt{q\pi}}\right]\dot{\mathcal{A}}\{F(x)\} \quad (3)$$

The use of inverse Anuj transform on both sides of Eq. (3) gives

$$u(x) = \dot{\mathcal{A}}^{-1}\left\{\left[\frac{1}{1-\sqrt{q\pi}}\right]\dot{\mathcal{A}}\{F(x)\}\right\}$$

which is the required solution of Eq. (1).

4. NUMERICAL EXAMPLES: Several numerical examples have been provided in this section to illustrate the effectiveness and precision of the current methodology aimed at addressing the challenges associated with solving AIESK. These examples serve to showcase the robustness and trustworthiness of the approach being utilized in tackling the complexities inherent in solving AIESK problems. By presenting such concrete instances, readers can gain a better understanding of how this scheme operates and how it delivers accurate results in the realm of AIESK problem-solving. The numerical examples highlighted throughout this work are instrumental in demonstrating the practical applications and real-world implications of the proposed solution. Through a detailed analysis of these examples, the dependability and exactitude of the present scheme in solving AIESK can be thoroughly appreciated. The inclusion of numerical demonstrations not only bolsters the credibility of the methodology but also helps to elucidate the intricate processes involved in addressing AIESK challenges.

EXAMPLE: 4.1 Consider the following AIESK

$$u(x) = 4 - 8\sqrt{x} + \int_0^x \frac{u(t)}{\sqrt{(x-t)}}dt \quad (4)$$

The use of Anuj transform of both sides of Eq. (4) gives

$$\dot{\mathcal{A}}\{u(x)\} = 4\dot{\mathcal{A}}\{1\} - 8\dot{\mathcal{A}}\left\{x^{1/2}\right\} + \dot{\mathcal{A}}\left\{\int_0^x \frac{u(t)}{\sqrt{(x-t)}}dt\right\}$$

$$\Rightarrow \dot{\mathcal{A}}\{u(x)\} = 4\dot{\mathcal{A}}\{1\} - 8\dot{\mathcal{A}}\left\{x^{1/2}\right\} + \dot{\mathcal{A}}\left\{u(x) * x^{-(1/2)}\right\} \quad (5)$$

The use of the property 2.4 in Eq. (5) gives

$$\dot{\mathcal{A}}\{u(x)\} = 4q^3 - 8q^{\frac{7}{2}}\Gamma\left(\frac{3}{2}\right) + \frac{1}{q^2}\dot{\mathcal{A}}\{u(x)\}\dot{\mathcal{A}}\left\{x^{-(1/2)}\right\}$$

$$\Rightarrow \dot{\mathcal{A}}\{u(x)\} = 4q^3 - 8q^{\frac{7}{2}}\Gamma\left(\frac{3}{2}\right) + \frac{1}{q^2}\dot{\mathcal{A}}\{u(x)\}q^{\frac{5}{2}}\Gamma\left(\frac{1}{2}\right)$$

$$\Rightarrow \dot{\mathcal{A}}\{u(x)\} = 4q^3 \quad (6)$$

The use of inverse Anuj transform on both sides of Eq. (6) gives

$u(x) = \dot{\mathcal{A}}^{-1}\{4q^3\} = 4\dot{\mathcal{A}}^{-1}\{q^3\} = 4$, which is the required solution of Eq. (4).

EXAMPLE: 4.2 Consider the following AIESK

$$u(x) = x^2 - \frac{16}{15}\left(x^{5/2}\right) + \int_0^x \frac{u(t)}{\sqrt{(x-t)}}dt \quad (7)$$

The use of Anuj transform of both sides of Eq. (7) gives

$$\dot{\mathcal{A}}\{u(x)\} = \dot{\mathcal{A}}\{x^2\} - \frac{16}{15}\dot{\mathcal{A}}\left\{x^{5/2}\right\} + \dot{\mathcal{A}}\left\{\int_0^x \frac{u(t)}{\sqrt{(x-t)}}dt\right\}$$

$$\Rightarrow \dot{\mathcal{A}}\{u(x)\} = \dot{\mathcal{A}}\{x^2\} - \frac{16}{15}\dot{\mathcal{A}}\left\{x^{5/2}\right\} + \dot{\mathcal{A}}\left\{u(x) * x^{-(1/2)}\right\} \quad (8)$$

The use of the property 2.4 in Eq. (8) gives

$$\dot{\mathcal{A}}\{u(x)\} = 2q^5 - \frac{16}{15}q^{\frac{11}{2}}\Gamma\left(\frac{7}{2}\right) + \frac{1}{q^2}\dot{\mathcal{A}}\{u(x)\}\dot{\mathcal{A}}\left\{x^{-(1/2)}\right\}$$

$$\Rightarrow \dot{\mathcal{A}}\{u(x)\} = 2q^5 - \frac{16}{15}q^{\frac{11}{2}}\Gamma\left(\frac{7}{2}\right) + \frac{1}{q^2}\dot{\mathcal{A}}\{u(x)\}q^{\frac{5}{2}}\Gamma\left(\frac{1}{2}\right)$$

$$\Rightarrow \dot{\mathcal{A}}\{u(x)\} = 2q^5 \quad (9)$$

The use of inverse Anuj transform on both sides of Eq. (9) gives

$u(x) = \dot{\mathcal{A}}^{-1}\left\{2q^5\right\} = 2\dot{\mathcal{A}}^{-1}\left\{q^5\right\} = x^2$, which is the required solution of Eq. (7).

EXAMPLE: 4.3 Consider the following AIESK

$$u(x) = x^3 - \frac{32}{35}\left(x^{7/2}\right) + \int_0^x \frac{u(t)}{\sqrt{(x-t)}}dt \quad (10)$$

The use of Anuj transform of both sides of Eq. (10) gives

$$\dot{\mathcal{A}}\{u(x)\} = \dot{\mathcal{A}}\{x^3\} - \frac{32}{35}\dot{\mathcal{A}}\left\{x^{7/2}\right\} + \dot{\mathcal{A}}\left\{\int_0^x \frac{u(t)}{\sqrt{(x-t)}}dt\right\}$$

$$\Rightarrow \dot{\mathcal{A}}\{u(x)\} = \dot{\mathcal{A}}\{x^3\} - \frac{32}{35}\dot{\mathcal{A}}\left\{x^{7/2}\right\} + \dot{\mathcal{A}}\left\{u(x) * x^{-(1/2)}\right\} \quad (11)$$

The use of the property 2.4 in Eq. (11) gives

$$\dot{\mathcal{A}}\{u(x)\} = 6q^6 - \frac{32}{35}q^{\frac{13}{2}}\Gamma\left(\frac{9}{2}\right) + \frac{1}{q^2}\dot{\mathcal{A}}\{u(x)\}\dot{\mathcal{A}}\left\{x^{-(1/2)}\right\}$$

$$\Rightarrow \dot{\mathcal{A}}\{u(x)\} = 6q^6 - \frac{32}{35}q^{\frac{13}{2}}\Gamma\left(\frac{9}{2}\right) + \frac{1}{q^2}\dot{\mathcal{A}}\{u(x)\}q^{\frac{5}{2}}\Gamma\left(\frac{1}{2}\right)$$

$$\Rightarrow \dot{\mathcal{A}}\{u(x)\} = 6q^6 \quad (12)$$

The use of inverse Anuj transform on both sides of Eq. (12) gives

$u(x) = \dot{\mathcal{A}}^{-1}\{6q^6\} = 6\dot{\mathcal{A}}^{-1}\{q^6\} = x^3$, which is the required solution of Eq. (10).

EXAMPLE: 4.4 Consider the following AIESK

$$u(x) = \sqrt{x} - \frac{1}{2}(\pi x) + \int_0^x \frac{u(t)}{\sqrt{(x-t)}}dt \quad (13)$$

The use of Anuj transform of both sides of Eq. (13) gives

$$\dot{\mathcal{A}}\{u(x)\} = \dot{\mathcal{A}}\{\sqrt{x}\} - \frac{1}{2}\pi\dot{\mathcal{A}}\{x\} + \dot{\mathcal{A}}\left\{\int_0^x \frac{u(t)}{\sqrt{(x-t)}}dt\right\}$$

$$\Rightarrow \dot{\mathcal{A}}\{u(x)\} = \dot{\mathcal{A}}\{\sqrt{x}\} - \frac{1}{2}\pi\dot{\mathcal{A}}\{x\} + \dot{\mathcal{A}}\left\{u(x) * x^{-(1/2)}\right\} \quad (14)$$

The use of the property 2.4 in Eq. (14) gives

$$\dot{\mathcal{A}}\{u(x)\} = q^{\frac{7}{2}}\Gamma\left(\frac{3}{2}\right) - \frac{1}{2}\pi q^4 + \frac{1}{q^2}\dot{\mathcal{A}}\{u(x)\}\dot{\mathcal{A}}\left\{x^{-(1/2)}\right\}$$

$$\Rightarrow \dot{\mathcal{A}}\{u(x)\} = q^{\frac{7}{2}}\Gamma\left(\frac{3}{2}\right) - \frac{1}{2}\pi q^4 + \frac{1}{q^2}\dot{\mathcal{A}}\{u(x)\}q^{\frac{5}{2}}\Gamma\left(\frac{1}{2}\right)$$

$$\Rightarrow \dot{\mathcal{A}}\{u(x)\} = q^{\frac{7}{2}}\Gamma\left(\frac{3}{2}\right) \quad (15)$$

The use of inverse Anuj transform on both sides of Eq. (15) gives

$u(x) = \dot{\mathcal{A}}^{-1}\left\{q^{\frac{7}{2}}\Gamma\left(\frac{3}{2}\right)\right\} = \Gamma\left(\frac{3}{2}\right)\dot{\mathcal{A}}^{-1}\left\{q^{\frac{7}{2}}\right\} = \sqrt{x}$, which is the required solution of Eq. (13).

EXAMPLE: 4.5 Consider the following AIESK

$$u(x) = x^{5/2} - \frac{5}{16}(\pi x^3) + \int_0^x \frac{u(t)}{\sqrt{(x-t)}}dt \quad (16)$$

The use of Anuj transform of both sides of Eq. (16) gives

$$\dot{\mathcal{A}}\{u(x)\} = \dot{\mathcal{A}}\left\{x^{5/2}\right\} - \frac{5\pi}{16}\dot{\mathcal{A}}\{x^3\} + \dot{\mathcal{A}}\left\{\int_0^x \frac{u(t)}{\sqrt{(x-t)}}dt\right\}$$

$$\Rightarrow \dot{\mathcal{A}}\{u(x)\} = \dot{\mathcal{A}}\left\{x^{5/2}\right\} - \frac{5\pi}{16}\dot{\mathcal{A}}\{x^3\} + \dot{\mathcal{A}}\left\{u(x) * x^{-(1/2)}\right\} \quad (17)$$

The use of the property 2.4 in Eq. (17) gives

$$\dot{\mathcal{A}}\{u(x)\} = q^{\frac{11}{2}}\Gamma\left(\frac{7}{2}\right) - \frac{5\pi}{16}q^6(3!) + \frac{1}{q^2}\dot{\mathcal{A}}\{u(x)\}\dot{\mathcal{A}}\left\{x^{-(1/2)}\right\}$$

$$\Rightarrow \dot{\mathcal{A}}\{u(x)\} = q^{\frac{11}{2}}\Gamma\left(\frac{7}{2}\right) - \frac{5\pi}{16}q^6(3!) + \frac{1}{q^2}\dot{\mathcal{A}}\{u(x)\}q^{\frac{5}{2}}\Gamma\left(\frac{1}{2}\right)$$

$$\Rightarrow \dot{\mathcal{A}}\{u(x)\} = q^{\frac{11}{2}}\Gamma\left(\frac{7}{2}\right) \quad (18)$$

The use of inverse Anuj transform on both sides of Eq. (18) gives

$u(x) = \dot{\mathcal{A}}^{-1}\left\{q^{\frac{11}{2}}\Gamma\left(\frac{7}{2}\right)\right\} = \Gamma\left(\frac{7}{2}\right)\dot{\mathcal{A}}^{-1}\left\{q^{\frac{11}{2}}\right\} = x^{5/2}$, which is the required solution of Eq. (16).

5. CONCLUSION: In the conducted study, the adept utilization of the Anuj transforms proved to be highly effective in tackling the challenging AIESK. The application of this innovative integral transform methodology not only led to the derivation of precise analytical solutions for the AIESK but also signified a significant advancement in our understanding of this intricate issue. The research outcomes distinctly reveal that the groundbreaking approach embodied by the "Anuj transform" offers notable advantages compared to conventional techniques. One key benefit lies in how the Anuj transform obviates the necessity for elaborate computations to obtain accurate analytical solutions for the AIESK, thereby simplifying the process and enhancing overall efficiency. Additionally, this methodology is characterized by its minimal resource requirements, thus reducing the computational burden associated with AIESK problem-solving. By harnessing the unique strengths of the Anuj transform, researchers and practitioners can access more precise and efficient solutions for the AIESK, fostering further exploration and practical applications within the realms of science and engineering. The proposed methodology holds promise for addressing complex systems of AIESK in the future.

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