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Using Estimation Methods for the Transformed kappa Distribution

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Abstract:
The researcher used the three-parameter Kappa distribution to build a new probability distribution using the quadratic transformation formula, so it becomes the four-parameter Kappa distribution. The distribution properties were studied, extracted, and the distribution was applied to five traditional estimation methods. Among these methods used are the Cramer-Von Mises Minimum method, the Anderson-Darling method, the Anderson-Darling Right-Tail method, and the Anderson-Darling Left-Tail method. In order to find the best method among the estimation methods, the Monte Carlo simulation method was used using the Mathematica12.2 program. The researcher used four models for different sample sizes (small, medium, large). Different values were chosen for the distribution parameters, in order to study the behavior of the measures using the mean square error (MSE). The preference in estimating the parameters was Cramer-Von Mises For small, medium and large sample sizes. The distribution was applied to real data for heart patients. The sample size was (104) observations in weeks representing the patient's survival time until death. Using goodness of fit criteria, the superiority of the transformed distribution was proven compared to the distribution before transformation and other distributions under study.
Keywords: kappa,timation,AN, Survival Function

1. Introduction

Researchers have discussed distributions extensively and frequently in experimental statistical data to choose the appropriate model and related issues in applied sciences such as environment, medicine, engineering, modeling and analysis of experimental data, there are many distributions that can be used in this type of experimental data, the necessity of the procedures used in such statistical analysis relied heavily on the assumed probability model or distribution.

In this study we use the transformation formula approach for a new model that generalizes the kappa distribution and this distribution is called the transformed kappa distribution according to the rank transformation formula.

The researchers have developed a new class of transformed distributions, namely the cubic rank transformation function and the quadratic transformation function (second degree), this new parametric family provides solvable distributions that are able to fit a set of complex data

Where two types of estimation methods were studied, the (Cramer-Von Mises Minimum)and Anderson Darling method

In (2017), (Venegas et al.) proposed a new univariate three-parameter distribution, the transformed exponential Maxwell distribution. This new univariate distribution is a generalization of the Maxwell and exponential distributions. The probabilistic properties were studied, the functions used in reliability studies were derived, the maximum likelihood function and Bayesian estimation of the parameters were estimated, the formula was studied and applied to real data. In the same year (2017), (Deka et al.) derived the transformed Gumball distribution (TEGD) using the exponential Gumball distribution (EGD) and the quadratic rank transformation map (QRTM).

The expressions, analytical aspects and shapes of the proposed distribution function, probability density function, risk rate function and coefficient function were studied, and the TEGD parameter estimation was applied using the maximum likelihood method. TEGD was implemented on a set of real data for the water quality parameter and was found to be better suited than the exponential Gumbel distribution and the (Gumbel) distribution. In (2018), (Umar and Terna) proposed a new distribution called the transformed (exponential Lomax) distribution as an extension of the (Lomax) distribution known in the form of (exponential Lomax) using the quadratic transformation map that was studied in a previous research using the transformation map and extracting the probability density function and the cumulative distribution function of the transformed distribution, as some properties of the new distribution were studied after derivation and the distribution parameters were estimated using the maximum likelihood method, and the proposed probability distribution tool was reviewed by comparison with some other generalizations of the (Lomax) distribution using three sets of real data. The results obtained indicated that the effectiveness of TELD is better than other distributions that contain Lomax power. In the same year, (Nofal) et al. presented a generalization of the (Weibull) distribution using the quadratic order transformation formula or what is called the (Exponentiated Additive Weibull) distribution. It is more flexible and capable of analyzing more complex data and includes 27 sub-models as special cases. Many of the computational properties of the new distribution were given as closed bodies for the normal moment and the quantitative function, the moment-generating function and the use of the maximum likelihood method to estimate the model parameters. The model is shown using two sets of real data. In (2019), (Khan, M.S.) proposed and studied the modified inverse Weibull distribution consisting of four parameters. The theoretical properties of the modified inverse Weibull distribution were studied. Which includes quantitative deviations, median, entropy, geometric mean and harmonic mean. The parameter estimates are obtained using the maximum likelihood method and an application to a real data set is provided to demonstrate the best fit of the transformed inverse modified Weibull distribution and to estimate the elasticity of the transformed distribution by implementing automated data.

(The Survival Function)

$$S(t) = 1 - F(t)$$

The Hazard Function

$$h(t) = \frac{F(t + \Delta t) - F(t)}{s(t)}$$

The probability density function of the Kappa distribution can be written as follows:

$$f(x)= \begin{cases} \frac{\alpha\theta}{\beta}\left(\frac{x}{\beta}\right)^{\theta-1}\left[\alpha+\left(\frac{x}{\beta}\right)^{\theta\alpha}\right]^{-\left(\frac{\alpha+1}{\alpha}\right)} & , \quad (\text{if } x>0\ ) \\ 0 & , \quad \text{otherwise} \end{cases}$$

Thus, the survival function for the Kappa distribution is as follows:

$$S(t) = 1 - \left[\frac{\left(\frac{x}{\beta}\right)^{\theta a}}{\alpha + \left(\frac{x}{\beta}\right)^{\theta a}}\right]^{\left(\frac{1}{a}\right)}$$

Transmuted kappa Distribution

Pdf (TKP)

$$f(x,\alpha,\theta,\beta,\gamma) = \frac{\alpha\theta}{\beta}\left(\frac{X}{\beta}\right)^{\theta-1}\left[\alpha + \left(\left(\frac{X}{\beta}\right)^{\theta\alpha}\right)\right]^{-\left(\frac{\alpha+1}{\alpha}\right)}\left(1 + \gamma - 2\gamma\left[\frac{\left(\left(\frac{X}{\beta}\right)^{\theta\alpha}\right)}{\alpha+\left(\frac{X}{\beta}\right)^{\alpha\theta}}\right]^{\frac{1}{\alpha}}\right) \quad \ldots (16-2)$$

$$\mathbf{f(x,\alpha,\theta,\beta,\gamma)} = \frac{\boldsymbol{\alpha\theta}}{\boldsymbol{\beta}}\left(\frac{\boldsymbol{X}}{\boldsymbol{\beta}}\right)^{\boldsymbol{\theta-1}}\left[\boldsymbol{\alpha} + \left(\left(\frac{\boldsymbol{X}}{\boldsymbol{\beta}}\right)^{\boldsymbol{\theta\alpha}}\right)\right]^{-\left(\frac{\boldsymbol{\alpha+1}}{\boldsymbol{\alpha}}\right)}\left(\mathbf{1} + \boldsymbol{\gamma} - \mathbf{2}\boldsymbol{\gamma}\left[\frac{\left(\left(\frac{\boldsymbol{X}}{\boldsymbol{\beta}}\right)^{\boldsymbol{\theta\alpha}}\right)}{\boldsymbol{\alpha}+\left(\frac{\mathbf{x}}{\boldsymbol{\beta}}\right)^{\boldsymbol{\alpha\theta}}}\right]^{\frac{1}{\boldsymbol{\alpha}}}\right)$$

$$\int_0^\infty f(x,\alpha,\theta,\beta,\gamma).dx = \begin{Bmatrix} \int_0^\infty \frac{\alpha\theta}{\beta}\left(\frac{x}{\beta}\right)^{\theta-1}\left[\alpha+\left(\frac{x}{\beta}\right)^{\theta\alpha}\right]^{-\left(\frac{\alpha+1}{\alpha}\right)}.dx \\ +\int_0^\infty \gamma\frac{\alpha\theta}{\beta}\left(\frac{x}{\beta}\right)^{\theta-1}\left[\alpha+\left(\frac{x}{\beta}\right)^{\theta\alpha}\right]^{-\left(\frac{\alpha+1}{\alpha}\right)}.dx \\ -2\gamma\int_0^\infty \frac{\alpha\theta}{\beta}\left(\frac{x}{\beta}\right)^{\theta-1}\left[\alpha+\left(\frac{x}{\beta}\right)^{\theta\alpha}\right]^{-\left(\frac{\alpha+1}{\alpha}\right)}\left[\frac{\left(\frac{x}{\beta}\right)^{\alpha\theta}}{\alpha+\left(\frac{x}{\beta}\right)^{\alpha\theta}}\right]^{\frac{1}{\alpha}}.dx \end{Bmatrix}$$

Let $c_1 = \frac{\alpha\theta}{\beta}\left(\frac{x}{\beta}\right)^{\theta-1}\left[\alpha+\left(\frac{x}{\beta}\right)^{\theta\alpha}\right]^{-\left(\frac{\alpha+1}{\alpha}\right)}$ ...

$$\int_0^\infty c_1 dx = \int_0^\infty \frac{\alpha\theta}{\beta}\left(\frac{x}{\beta}\right)^{\theta-1}\left[\alpha+\left(\frac{x}{\beta}\right)^{\theta\alpha}\right]^{-\left(\frac{\alpha+1}{\alpha}\right)}.dx$$

Let $= \frac{x}{\beta}$, $x = u\beta$, $dx = \beta du$

$$= \int_0^\infty \frac{\alpha\theta}{\beta}(u)^{\theta-1}\left[\alpha+(u)^{\theta\alpha}\right]^{-\left(\frac{\alpha+1}{\alpha}\right)}\beta du$$

Let $z = (u)^{\theta\alpha}$ $u = (z)^{\frac{1}{\theta\alpha}}$, $du = \frac{1}{\theta\alpha}(z)^{\frac{1}{\theta\alpha}-1}dz$

$[1+y]^{-\left(\frac{\alpha+1}{\alpha}\right)}\alpha dy$

$$= \alpha^{-\left(\frac{\alpha+1}{\alpha}\right)}(\alpha)^{-1+\frac{1}{\alpha}}\alpha\int_0^\infty (y)^{-1+\frac{1}{\alpha}}[1+y]^{-\left(\frac{\alpha+1}{\alpha}\right)}dy$$

$$= \frac{1}{\alpha}\int_0^\infty (y)^{-1+\frac{1}{\alpha}}[1+y]^{-\left(\frac{\alpha+1}{\alpha}\right)}dy$$

$$= \frac{1}{\alpha}\int_0^\infty \frac{(y)^{-1+\frac{1}{\alpha}}}{[1+y]^{\left(\frac{\alpha+1}{\alpha}\right)}}dy$$

$$\beta(\alpha,\beta) = \int_0^\infty \frac{x^{\alpha-1}}{(1+x)^{\alpha+\beta}}dx = \frac{\Gamma\alpha\Gamma\beta}{\Gamma\alpha+\beta}$$

$$\beta = (\alpha+\beta)-\alpha$$

$$\alpha = \frac{1}{\alpha}, \beta = 1$$

$$= \frac{1}{\alpha}\frac{\Gamma\frac{1}{\alpha}\Gamma 1}{\Gamma\frac{1}{\alpha}+1}$$

$$= \frac{1}{\alpha}\frac{\left(\frac{1}{\alpha}-1\right)!\quad(1-1)!}{\left(\frac{1}{\alpha}\right)!}$$

$$= \frac{1}{\alpha}\frac{\left(\frac{1}{\alpha}-1\right)!\quad(0)!}{\left(\frac{1}{\alpha}\right)\left(\frac{1}{\alpha}-1\right)!}$$

$$= \frac{1}{\alpha}\frac{1}{\left(\frac{1}{\alpha}\right)}$$

$=1$

Let $c_2 = \gamma\frac{\alpha\theta}{\beta}\left(\frac{x}{\beta}\right)^{\theta-1}\left[\alpha+\left(\frac{x}{\beta}\right)^{\theta\alpha}\right]^{-\left(\frac{\alpha+1}{\alpha}\right)}$

$\longrightarrow \quad c_2 = \gamma c_1$

Let $c_3=2\gamma\frac{\alpha\theta}{\beta}(\frac{x}{\beta})^{\theta-1}[\alpha+(\frac{x}{\beta})^{\theta\alpha}]^{-(\frac{\alpha+1}{\alpha})}[\frac{(\frac{x}{\beta})^{\alpha\theta}}{\alpha+(\frac{x}{\beta})^{\alpha\theta}}]^{\frac{1}{\alpha}}$ ... (18-2)

$=\int_0^\infty 2\gamma\frac{\alpha\theta}{\beta}(\frac{x}{\beta})^{\theta-1}[\alpha+(\frac{x}{\beta})^{\theta\alpha}]^{-(\frac{\alpha+1}{\alpha})}[\frac{(\frac{x}{\beta})^{\alpha\theta}}{\alpha+(\frac{x}{\beta})^{\alpha\theta}}]^{\frac{1}{\alpha}}.dx$

Let $=\frac{x}{\beta}$, $x = u\beta$, $dx = \beta du$

$= 2\gamma\int_0^\infty \frac{a\theta}{\beta}(u)^{\theta-1}[a + (u)^{\theta a}]^{-(\frac{a+1}{a})}[\frac{(u)^{a\theta}}{a + (u)^{a\theta}}]^{\frac{1}{a}}\beta du$

$= 2\gamma\int_0^\infty (z)^{\frac{2}{\theta a}-1}[a + z]^{-(\frac{a+2}{a})}dz$

$= a^{-(\frac{a+2}{a})}\int_0^\infty (z)^{\frac{2}{\theta a}-1}[1 + \frac{z}{a}]^{-(\frac{a+2}{a})}dz$

Let $y = \frac{z}{a}$, $z = ya$, $dz = a\,dy$

$= 2\gamma a^{-(\frac{a+2}{a})}\int_0^\infty (ya)^{\frac{2}{\theta a}-1}[1 + \frac{z}{a}]^{-(\frac{a+2}{a})}a\,dy$

$= 2\gamma a^{-(\frac{a+2}{a})}a^{\frac{2}{\theta a}-1}a\int_0^\infty (y)^{\frac{2}{\theta a}-1}[1 + y]^{-(\frac{a+2}{a})}a\,dy$

$= 2\gamma\frac{1}{a}\int^\infty (y)^{\frac{2}{\theta a}-1}[1 + y]^{-(\frac{a+2}{a})}dy$

$= 2\gamma\frac{1}{a}\int^\infty (y)^{\frac{2}{\theta a}-1}[1 + y]^{-(\frac{a+2}{a})}dy$

$\beta(\alpha,\beta)=\int_0^\infty\frac{x^{\alpha-1}}{(1+x)^{\alpha+\beta}}dx=\frac{\Gamma\alpha\Gamma\beta}{\Gamma\alpha+\beta}$

$\boldsymbol{\beta} = (\boldsymbol{\alpha} + \boldsymbol{\beta}) - \boldsymbol{\alpha}$

$\boldsymbol{\alpha} = \frac{2}{\alpha}$, $\boldsymbol{\beta} = 1$

$= \frac{1}{\alpha}\frac{\Gamma\frac{2}{\alpha}\Gamma 1}{\Gamma\frac{2}{\alpha}+1}$

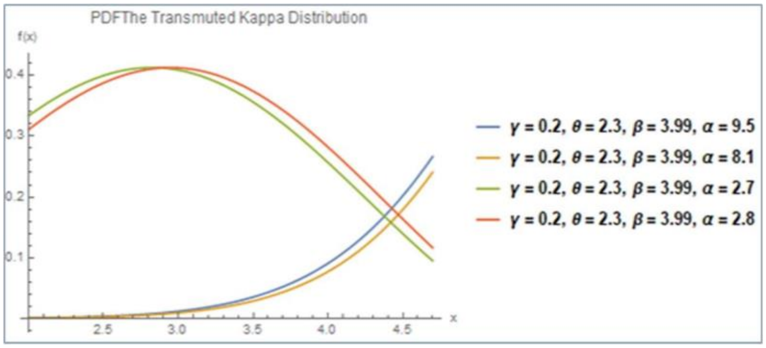
$= 2\gamma\frac{1}{2}$

$= \gamma$

$$\int_0^\infty g(x)dx = \begin{pmatrix} \int_0^\infty \frac{\alpha\theta}{\beta}(\frac{x}{\beta})^{\theta-1}[\alpha + (\frac{x}{\beta})^{\theta\alpha}]^{-(\frac{\alpha+1}{\alpha})}.dx \\ +\int_0^\infty \gamma\frac{\alpha\theta}{\beta}(\frac{x}{\beta})^{\theta-1}[\alpha + (\frac{x}{\beta})^{\theta\alpha}]^{-(\frac{\alpha+1}{\alpha})}.dx \\ -2\gamma\int_0^\infty \frac{\alpha\theta}{\beta}(\frac{x}{\beta})^{\theta-1}[\alpha + (\frac{x}{\beta})^{\theta\alpha}]^{-(\frac{\alpha+1}{\alpha})}[\frac{(\frac{x}{\beta})^{\alpha\theta}}{\alpha + (\frac{x}{\beta})^{\alpha\theta}}]^{\frac{1}{\alpha}}.dx \end{pmatrix}$$

$\int_0^\infty g(x)dx = 1 + \gamma - \gamma$

$\int_0^\infty g(x)dx=1$



C.d.f (TKP)

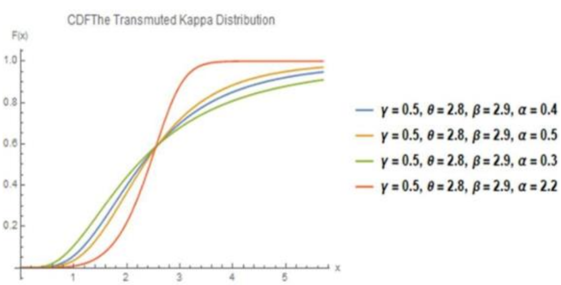
$$F(x,\alpha,\theta,\beta,\gamma) = \int_0^x g(x),\,dx \qquad (19-2)$$

$$= \int_0^x \frac{\alpha\theta}{\beta}\left(\frac{x}{\beta}\right)^{\theta-1}\left[\alpha+\left(\frac{x}{\beta}\right)^{\theta\alpha}\right]^{-\left(\frac{\alpha+1}{\alpha}\right)} .dx + \int_0^x \gamma\frac{\alpha\theta}{\beta}\left(\frac{x}{\beta}\right)^{\theta-1}\left[\alpha+\left(\frac{x}{\beta}\right)^{\theta\alpha}\right]^{-\left(\frac{\alpha+1}{\alpha}\right)} -2\gamma\int_0^x \frac{\alpha\theta}{\beta}\left(\frac{x}{\beta}\right)^{\theta-1}\left[\alpha+\left(\frac{x}{\beta}\right)^{\theta\alpha}\right]^{-\left(\frac{\alpha+1}{\alpha}\right)} dx$$

$$= \left(\alpha+\left(\frac{x}{\beta}\right)^{\theta\alpha}\right)^{\frac{-1}{\alpha}} + \gamma\left(\alpha+\left(\frac{x}{\beta}\right)^{\theta\alpha}\right)^{\frac{-1}{\alpha}} - 2\gamma\left[\frac{\left(\alpha+\left(\frac{x}{\beta}\right)^{\alpha\theta}\right)^{-2/\alpha}\left(\frac{x}{\beta}\right)^{2\theta}}{2}\right]$$

$$= \left(\alpha+\left(\frac{x}{\beta}\right)^{\theta a}\right)^{\frac{-1}{a}}\left(\frac{x}{\beta}\right) + \gamma\left(\alpha+\left(\frac{x}{\beta}\right)^{\theta a}\right)^{\frac{-1}{a}}\left(\frac{x}{\beta}\right) - \gamma\left(\alpha+\left(\frac{x}{\beta}\right)^{a\theta}\right)^{-2/a}\left(\frac{x}{\beta}\right)^{2\theta}$$

$$\mathbf{F(x,\alpha,\theta,\beta,\gamma)} = \left(a+\left(\frac{x}{\beta}\right)^{0a}\right)^{\frac{-1}{a}}\left(\frac{x}{\beta}\right)(1+y) - y\left(a+\left(\frac{x}{\beta}\right)^{a0}\right)^{-2/a}\left(\frac{x}{\beta}\right)^{20} ;x,\alpha,\theta,\beta,\geq 0$$



Estimation Methods

1- Method of Cramer-Von Mises Minimum

Cramer-Von Mises Minimum method is based on minimum distance estimators. We can obtain minimum distance estimates for Cramer-Von Mises Minimum method by minimizing the distance between the function c(γ,θ,β,α,x) for the unknown parameters. We can obtain the estimators by partial differentiation c(γ,θ,β,α,x) for the unknown parameters and setting it equal to zero as follows:

$$c(\gamma,\theta,\beta,x) = \frac{1}{12n} + \sum_{i=1}^{n}\left[F(\gamma,\theta,\beta,x) - \frac{2i-1}{2n}\right]^2$$

$$c(\gamma,\theta,\beta,x) = \frac{1}{12n} + \sum_{i=1}^{n}\left[\left(\frac{\left(\frac{x}{\beta}\right)^{\theta\alpha}}{\alpha+\left(\frac{x}{\beta}\right)^{\theta\alpha}}\right)^{\frac{1}{\alpha}}(1+\gamma) - \gamma\left(\frac{\left(\frac{x}{\beta}\right)^{\theta\alpha}}{\alpha+\left(\frac{x}{\beta}\right)^{\theta\alpha}}\right)^{\frac{2}{\alpha}} - \frac{2i-1}{2n}\right]^2$$

$$\frac{c(\gamma,\theta,\beta,x)}{6\gamma} = 2\sum_{i=2}^{n}\left(\frac{\left(\frac{x}{\beta}\right)^{\theta\alpha}}{\alpha+\left(\frac{x}{\beta}\right)^{\theta\alpha}}\right)^{\frac{1}{\alpha}} - \left(\frac{\left(\frac{x}{\beta}\right)^{\theta\alpha}}{\alpha+\left(\frac{x}{\beta}\right)^{\theta\alpha}}\right)^{\frac{2}{\alpha}}\left[\left(\frac{\left(\frac{x}{\beta}\right)^{\theta\alpha}}{\alpha+\left(\frac{x}{\beta}\right)^{\theta\alpha}}\right)^{\frac{1}{\alpha}}(1+\gamma) - \gamma\left(\frac{\left(\frac{x}{\beta}\right)^{\theta\alpha}}{\alpha+\left(\frac{x}{\beta}\right)^{\theta\alpha}}\right)^{\frac{2}{\alpha}} - \frac{2i-1}{2n}\right] = 0 \qquad (46-2)$$

$$\frac{\partial c(\gamma,\theta,\beta,x)}{\partial \alpha}=\left(2\sum_{i=1}^{n}\left[\left(\frac{(\frac{x}{\beta})^{\theta\alpha}}{\alpha+(\frac{x}{\beta})^{\theta\alpha}}\right)^{\frac{1}{\alpha}}(1+\gamma)-\gamma\left(\frac{(\frac{x}{\beta})^{\theta\alpha}}{\alpha+(\frac{x}{\beta})^{\theta\alpha}}\right)^{\frac{2}{\alpha}}-\frac{2i-1}{2n}\right]\frac{\left(1-\frac{\alpha}{\alpha+(\frac{x}{\beta})^{\alpha\theta}}\right)^{\frac{1}{\alpha}}\left((-1+(-1+2(1-\frac{\alpha}{\alpha+(\frac{x}{\beta})^{\alpha\theta}})^{\frac{1}{\alpha}})\gamma)\right)\left(-(\alpha+(\frac{x}{\beta})^{\alpha\theta})\mathrm{Log}[1-\frac{\alpha}{\alpha+(\frac{x}{\beta})^{\alpha\theta}}]+\alpha(-1+\alpha\theta\,\mathrm{Log}[\frac{x}{\beta}])\right)}{\alpha^{2}(\alpha+(\frac{x}{\beta})^{\alpha\theta})}\right)=0 \quad (47\text{-}2)$$

$$\frac{\partial c(\gamma,\theta,\beta,x)}{\partial \theta}=\left(2\sum_{i=1}^{n}\left[\left(\frac{(\frac{x}{\beta})^{\theta\alpha}}{\alpha+(\frac{x}{\beta})^{\theta\alpha}}\right)^{\frac{1}{\alpha}}(1+\gamma)-\gamma\left(\frac{(\frac{x}{\beta})^{\theta\alpha}}{\alpha+(\frac{x}{\beta})^{\theta\alpha}}\right)^{\frac{2}{\alpha}}-\frac{2i-1}{2n}\right]\left(-\frac{2\left(\frac{(\frac{x}{\beta})^{\alpha\theta}}{\alpha+(\frac{x}{\beta})^{\alpha\theta}}\right)^{-1+\frac{2}{\alpha}}\gamma\left(\frac{\alpha(\frac{x}{\beta})^{\alpha\theta}\mathrm{Log}[\frac{x}{\beta}]}{\alpha+(\frac{x}{\beta})^{\alpha\theta}}-\frac{\alpha(\frac{x}{\beta})^{2\alpha\theta}\mathrm{Log}[\frac{x}{\beta}]}{(\alpha+(\frac{x}{\beta})^{\alpha\theta})^{2}}\right)}{\alpha}+\frac{\left(\frac{(\frac{x}{\beta})^{\alpha\theta}}{\alpha+(\frac{x}{\beta})^{\alpha\theta}}\right)^{-1+\frac{1}{\alpha}}(1+\gamma)\left(\frac{\alpha(\frac{x}{\beta})^{\alpha\theta}\mathrm{Log}[\frac{x}{\beta}]}{\alpha+(\frac{x}{\beta})^{\alpha\theta}}-\frac{\alpha(\frac{x}{\beta})^{2\alpha\theta}\mathrm{Log}[\frac{x}{\beta}]}{(\alpha+(\frac{x}{\beta})^{\alpha\theta})^{2}}\right)}{\alpha}\right)\right)=0 \quad (48\text{-}2)$$

$$\frac{\partial c(\gamma,\theta,\beta,x)}{\partial \beta}=\left(2\sum_{i=1}^{n}\left[\left(\frac{(\frac{x}{\beta})^{\theta\alpha}}{\alpha+(\frac{x}{\beta})^{\theta\alpha}}\right)^{\frac{1}{\alpha}}(1+\gamma)-\gamma\left(\frac{(\frac{x}{\beta})^{\theta\alpha}}{\alpha+(\frac{x}{\beta})^{\theta\alpha}}\right)^{\frac{2}{\alpha}}-\frac{2i-1}{2n}\right]\left(-\frac{2\left(\frac{(\frac{x}{\beta})^{\alpha\theta}}{\alpha+(\frac{x}{\beta})^{\alpha\theta}}\right)^{-1+\frac{2}{\alpha}}\gamma\left(-\frac{x\alpha(\frac{x}{\beta})^{-1+\alpha\theta}\theta}{(\alpha+(\frac{x}{\beta})^{\alpha\theta})\beta^{2}}+\frac{x\alpha(\frac{x}{\beta})^{-1+2\alpha\theta}\theta}{(\alpha+(\frac{x}{\beta})^{\alpha\theta})^{2}\beta^{2}}\right)}{\alpha}+\frac{\left(\frac{(\frac{x}{\beta})^{\alpha\theta}}{\alpha+(\frac{x}{\beta})^{\alpha\theta}}\right)^{-1+\frac{1}{\alpha}}(1+\gamma)\left(-\frac{x\alpha(\frac{x}{\beta})^{-1+\alpha\theta}\theta}{(\alpha+(\frac{x}{\beta})^{\alpha\theta})\beta^{2}}+\frac{x\alpha(\frac{x}{\beta})^{-1+2\alpha\theta}\theta}{(\alpha+(\frac{x}{\beta})^{\alpha\theta})^{2}\beta^{2}}\right)}{\alpha}\right)\right)=0$$

2- Anderson- Darling method

This method was introduced by Anderson and Darling as an alternative to statistical tests. Anderson Darling estimates (ADEs) of (α,β,γ,θ) can be determined by minimizing the Anderson-Darling function with respect to (α,β,γ,θ) and the function is determined by -:

$$AD=-n-\frac{1}{n}\sum_{i=0}^{n}(2i-1)[\log F(x)+\log(1-F(x))]$$

$$AD=\{-n-\frac{1}{n}\sum_{i=0}^{n}(2i-1)[[\log[(\frac{(\frac{x}{\beta})^{\theta a}}{a+(\frac{x}{\beta})^{\theta a}})^{\frac{1}{a}}(1+\gamma)-\gamma(\frac{(\frac{x}{\beta})^{\theta a}}{a+(\frac{x}{\beta})^{\theta a}})^{\frac{2}{a}}]+[\log\{1-(\frac{(\frac{x}{\beta})^{\theta a}}{a+(\frac{x}{\beta})^{\theta a}})^{\frac{1}{a}}(1+\gamma)-\gamma(\frac{(\frac{x}{\beta})^{\theta a}}{a+(\frac{x}{\beta})^{\theta a}})^{\frac{2}{a}}\}]]\}$$

$$\frac{\partial AD}{\partial \alpha}=\{-n-\frac{1}{n}\sum_{i=0}^{n}(2i-1)[[\log[(\frac{(\frac{x}{\beta})^{\theta\alpha}}{\alpha+(\frac{x}{\beta})^{\theta\alpha}})^{\frac{1}{\alpha}}(1+\gamma)-\gamma(\frac{(\frac{x}{\beta})^{\theta\alpha}}{\alpha+(\frac{x}{\beta})^{\theta\alpha}})^{\frac{2}{\alpha}}]+[\log\{1-(\frac{(\frac{x}{\beta})^{\theta\alpha}}{\alpha+(\frac{x}{\beta})^{\theta\alpha}})^{\frac{1}{\alpha}}(1+\gamma)-$$

$$\gamma\left(\frac{(\frac{x}{\beta})^{\theta\alpha}}{\alpha+(\frac{x}{\beta})^{\theta\alpha}}\right)^{\frac{2}{\alpha}}\}]]\frac{\partial F(x_i)}{\partial\alpha}\}=0\quad\ldots(50-2)$$

$$\frac{\partial AD}{\partial\beta}=\{-n-\frac{1}{n}\sum_{i=0}^{n}(2i-1)[[log[(\frac{(\frac{x}{\beta})^{\theta\alpha}}{\alpha+(\frac{x}{\beta})^{\theta\alpha}})^{\frac{1}{\alpha}}(1+\gamma)-\gamma(\frac{(\frac{x}{\beta})^{\theta\alpha}}{\alpha+(\frac{x}{\beta})^{\theta\alpha}})^{\frac{2}{\alpha}}]+[log\{1-(\frac{(\frac{x}{\beta})^{\theta\alpha}}{\alpha+(\frac{x}{\beta})^{\theta\alpha}})^{\frac{1}{\alpha}}(1+\gamma)-$$

$$\gamma\left(\frac{(\frac{x}{\beta})^{\theta\alpha}}{\alpha+(\frac{x}{\beta})^{\theta\alpha}}\right)^{\frac{2}{\alpha}}\}]]\frac{\partial F(x_i)}{\partial\beta}\}=0\quad\ldots(51-2)$$

$$\frac{\partial AD}{\partial\gamma}=\left(-n-\frac{1}{n}\sum_{i=0}^{n}(2i-1)[[log\begin{array}{c}[(\frac{(\frac{x}{\beta})^{\theta\alpha}}{\alpha+(\frac{x}{\beta})^{\theta\alpha}})^{\frac{1}{\alpha}}(1+\gamma)-\gamma(\frac{(\frac{x}{\beta})^{\theta\alpha}}{\alpha+(\frac{x}{\beta})^{\theta\alpha}})^{\frac{2}{\alpha}}]+\\ [log\{1-(\frac{(\frac{x}{\beta})^{\theta\alpha}}{\alpha+(\frac{x}{\beta})^{\theta\alpha}})^{\frac{1}{\alpha}}(1+\gamma)-\gamma(\frac{(\frac{x}{\beta})^{\theta\alpha}}{\alpha+(\frac{x}{\beta})^{\theta\alpha}})^{\frac{2}{\alpha}}\}]]\end{array}[\frac{\partial F(x_i)}{\partial\alpha}]\right)=0\ldots(52-2)$$

$$\frac{\partial AD}{\partial\theta}=\{-n-\frac{1}{n}\sum_{i=0}^{n}(2i-1)[[log[(\frac{(\frac{x}{\beta})^{\theta\alpha}}{\alpha+(\frac{x}{\beta})^{\theta\alpha}})^{\frac{1}{\alpha}}(1+\gamma)-\gamma(\frac{(\frac{x}{\beta})^{\theta\alpha}}{\alpha+(\frac{x}{\beta})^{\theta\alpha}})^{\frac{2}{\alpha}}]+[log\{1-(\frac{(\frac{x}{\beta})^{\theta\alpha}}{\alpha+(\frac{x}{\beta})^{\theta\alpha}})^{\frac{1}{\alpha}}(1+\gamma)-$$

$$\gamma\left(\frac{(\frac{x}{\beta})^{\theta\alpha}}{\alpha+(\frac{x}{\beta})^{\theta\alpha}}\right)^{\frac{2}{\alpha}}\}]]\frac{\partial F(x_i)}{\partial\theta}\}=0$$

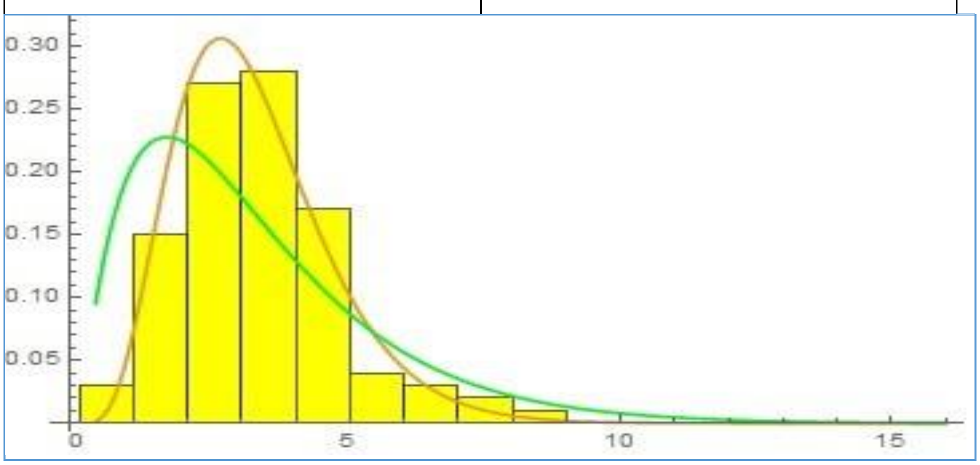
APPLICATION OF The Transformed Kappa Distribution (TKD)

the Flexibility and performance of The Transformed Kappa Distribution are Evaluated on Competing models viz One parameter Exponential Distribution (ED), Three Parameter lindely Distribution(TPLD), Gamma Distribution(G.D), and Three Parameter kappa Distribution (KD). and Weibull Distribution(WD). Here, the distribution is fitted to data set for the number of weeks The Patients people with heart diseasewere in hospital before death For AL hussein Educational Hospital in Karbala, for sample size (n=104) (see table 1.), the performance of the distribution was compared with exponential, Exponential Pareto, Lindely Three parametric, weibel distribution and weibel Pareto distribution for the data set using akaike information Criterion (AIC), (BIC), Akaike Information Criterion Corrected (AICC). Distribution with the lowest AIC, AICC considered the most Flexible and Superior Distribution For a Given Data Set. The Results are Presented in The Tables (2).

TABLE 1. Data set for the number of weeks The Patients people with heart

0.1	0.3	1.2	1.4
0.1	0.3	1.2	1.4
0.1	0.4	1.2	1.5
0.2	0.4	1.3	1.5
0.2	0.4	1.3	1.5
0.2	0.4	1.3	1.5
0.2	0.4	1.3	1.5
0.3	1	1.3	1.6
0.3	1	1.4	1.6
0.3	1.2	1.4	1.6

Index	Value
Mean	2.2980769
Min	0.1
Median	2.15
Skewness	0.16056
Standard Deviation	1.392000779
Variance	1.93767
Kurtosis	2.1504
Maximum	4.9



i	ti	f(ti)	F(ti)	S(ti)	H(ti)
	0.1	**0.170365**	**0.0151385**	**0.984861**	**0.172984**
	0.1	**0.170365**	**0.0151385**	**0.984861**	**0.172984**
	0.1	**0.170365**	**0.0151385**	**0.984861**	**0.172984**
	0.2	**0.202742**	**0.0338469**	**0.966153**	**0.209844**
	0.2	**0.202742**	**0.0338469**	**0.966153**	**0.209844**
	0.2	**0.202742**	**0.0338469**	**0.966153**	**0.209844**
	0.2	**0.202742**	**0.0338469**	**0.966153**	**0.209844**
	0.3	**0.229143**	**0.0554878**	**0.944512**	**0.242605**
	0.3	**0.229143**	**0.0554878**	**0.944512**	**0.242605**
	0.3	**0.229143**	**0.0554878**	**0.944512**	**0.242605**
	0.3	**0.229143**	**0.0554878**	**0.944512**	**0.242605**
	0.3	**0.229143**	**0.0554878**	**0.944512**	**0.242605**
	0.4	**0.250295**	**0.0795006**	**0.920499**	**0.271912**
	0.4	**0.250295**	**0.0795006**	**0.920499**	**0.271912**
	0.4	**0.250295**	**0.0795006**	**0.920499**	**0.271912**

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