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# A study on Simulated Annealing method for reducing the transportation cost using the hybrid MODI method

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## Abstract

In this study, the goal is to reduce the transportation cost by integrating the MODI method with Simulated Annealing (SA). The process begins with a description of the procedure to produce an initial feasible solution by incorporating heuristic methods, such as Vogel's Approximation Method (VAM), thereby ensuring that these solutions follow supply, demand, and capacity constraints. The MODI method can now be implemented in refining this solution for verification purposes and to identify if it is optimal or not, using dual variables and opportunity costs. At this stage, Simulated Annealing is applied to further improve the solution with the generation and evaluation of neighbouring solutions iteratively so that the values of transportation flows are at their minimum values and there may be a need to minimize transportation costs to reduce them to their minimum values. In this way, the integrated approach effectively and efficiently optimizes tasks.

**Keywords**: Transportation, Simulated Annealing (SA), Dual variables, Neighbouring solutions.

## Introduction

### Transportation

Transportation problems are one of the significant concerns studied in operation management to optimize the transportation cost of the particular system. The prime concern of the transportation method is that it minimizes the overall transportation time or the price required. The transportation problem is a subclass of linear programming problems related to the supply of several units of a homogeneous product produced at some origins to various destinations while satisfying supply and demand linear constraints. The transportation model is frequently used in industries with multiple production and demand centre or multiple warehouses to reduce transportation costs. Transport problems have broad applications in solving network optimization problems, production-inventory control, communication networks, employment scheduling, etc.

### MODI Method

MODI stands for the "Modified Distribution Method," an algorithm in Operations Research to solve transportation problems by finding the optimal distribution of goods from more origins to numerous destinations in order to minimize the whole cost of transportation. It calculates "opportunity costs" that are filled in unoccupied cells while making allocations iteratively, till no possible cost reduction, which is the cheapest distribution plan.

### Simulated Annealing

Simulated Annealing is an optimization algorithm that seeks to find the best or a nearly best solution. In this approach, "heat" relates to the level of randomness in the search, which diminishes over time through a cooling schedule to improve the solution. This technique is commonly applied in combinatorial optimization, where many problems have multiple local optima that traditional methods such as gradient descent may fail to escape from.

The formula is $P = \exp(-\Delta E / T)$, where:

P: represents the likelihood of adopting the new solution.

$\Delta E$: refers to the change in the objective function value between the new and current solution (it is positive if the new solution is less favourable).

T: denotes the present "temperature" parameter, which influences the probability that the algorithm will embrace less optimal solutions (a higher temperature results in an increased acceptance probability).

### Algorithm

### Initial Feasible Solution

Use a heuristic method (e.g., **Northwest Corner Rule**, **Least Cost Method**, or **VAM**) to generate an initial **feasible solution** $X_0$ (which respects supply, demand, and capacities for bounded problems).

If the problem is **bounded**, ensure that the initial allocation respects the **capacity constraints** for all routes:

$$x_{kp} \leq \text{capacity}_{kp}$$

Let C be the total transportation cost

Objective function

To minimize the transportation cost

$C = \sum_{k=1}^{i} \sum_{p=1}^{j} g_{kp}\, a_{kp}$ (1)

where:

$g_{kp}$ – Cost of goods transportation from source k to destination p.

$a_{kp}$ – transportation amount from source k to destination p.

i – number of sources

j – number of destinations

Supply constraints for each source are defined as

$$\sum_{k=1}^{n} a_{kp} \leq S_p \quad \forall\, p = 1,2 \ldots.,m$$

$S_p$ is the supply source at p.

Demand constraints for each destination are defined as

$$\sum_{p=1}^{m} a_{kp} \leq D_k \quad \forall\, k = 1,2 \ldots.,n$$

$D_k$ is the supply destination at k.

Let non negative constraints be

$$a_{kp} > 0$$

After obtaining an initial feasible solution, the MODI method is applied to check if the current solution is optimal. The MODI method works by calculating **dual variables** (or potentials) for the rows and columns, and checking for optimality based on the following procedure:

Calculate the potential $u_k$ for the suppliers and $v_p$for the consumers using the below equation. These potentials represent the opportunity cost associated with each supplier and consumer:

$$u_k + v_p = g_{kp} \quad \forall\, (k,p) \text{in the solution}$$

For a basic feasible solution, you can choose $u_1 = 0$and solve for the remaining potentials using the above equation. For every non-basic route (k, p) compute the opportunity cost:

To obtain the initial solution northwest corner method is utilized. For each unused route (k,p), the opportunity cost is calculate by

$$\text{opportunity cost } \Delta_{kp} = g_{kp} - (u_k + v_p)$$

If all $\Delta_{kp} > 0$ solution is optimal. If $\Delta_{kp} < 0$, proceed to adjust the solution.Then, make the feasible solution obtain from MODI method as a initial solution. Continue applying the MODI method until the solution is optimal or sufficiently close to optimal. Let $T_{init}$ be the initial temperature and alpha be the cooling rate.(set $T_{init}$ =

100, alpha = 0.95)

$$T_{iter} = T_{init} * alpha^{iter}$$

Where iter is the number of iterations

Generate a neighbouring solution by shifting a unit from one route to another to the shipment quantities $a_{kp}$.

**Neighbourhood Generation (Perturbation)**:

- Generate a **neighbour solution**Xnew by making a small change in the transportation flow. This change could involve:
- **Shifting flow** from one route to another.

**For Bounded Problems**: Ensure that the perturbed solution respects the upper bounds on each route,

$$x_{kp} \leq capacity_{kp}$$

If the perturbation violates this constraint, reject or penalize the move.

For new solution a', calculate cost $C' \; using(1)$.

Compute the change in cost

$$\Delta_c = C' - C$$

Accept the solution if $\Delta_c \leq 0$(i. e. new solution has lower cost)

else if $\Delta_c \geq 0$, then accept with its probability

$$P = \exp\left(-\frac{\Delta_c}{T_{iter}}\right)$$

**For Bounded Problems**: If the perturbation violates capacity constraints, accept it with a very low probability or **reject** it.

Reduce the temperature using:

$T_{final}=\alpha T_{old}$

Repeat the process for a predefined number of iterations or until the temperature drops below a threshold.Repeat the above process, cooling down the temperature until:

$$T_{iter} < T_{final}$$

## NUMERICAL EXAMPLE

| **Source/Destination** | **$D_1$** | **$D_2$** | **$D_3$** | **$D_4$** | **Supply** |
|---|---|---|---|---|---|
| **$S_1$** | 17 | 13 | 11 | 14 | 400 |
| **$S_2$** | 16 | 18 | 14 | 10 | 300 |
| **$S_3$** | 21 | 24 | 13 | 10 | 250 |
| **Demand** | 275 | 225 | 200 | 250 | **950** |

**Solution:**

The given transportation is balanced.

S1, S2, $S_3$ are the source.

$D_1$, $D_2$, $D_3$are the destination.

Total number of supply constraints: 3

Total number of demand constraints: 4

Using Vogel's appropriation method

| **Source/Destination** | **$D_1$** | **$D_2$** | **$D_3$** | **$D_4$** | **Supply** |
|---|---|---|---|---|---|
| **$S_1$** | 17 | 13 | 11 | 14 | 400 |
| **$S_2$** | 16 | 18 | 14 | 10 | 300 |

| | | | | | |
|---|---|---|---|---|---|
| $S_3$ | 21 | 24 | 13 | 10 | 250 |
| **Demand** | 275 | 225 | 200 | 250 | |

From this table, it can be seen that the no. of. independent allocations are (m + n - 1) = (3 + 4 -1) = 6

The initial feasible solution is

| **Source/Destination** | $D_1$ | $D_2$ | $D_3$ | $D_4$ | **Supply** |
|---|---|---|---|---|---|
| $S_1$ | 17 **(175)** | 13 **(225)** | 11 | 14 | 400 |
| $S_2$ | 16 **(50)** | 18 | 14 | 10 **(250)** | 300 |
| $S_3$ | 21 **(50)** | 24 | 13 **(200)** | 10 | 250 |
| **Demand** | 275 | 225 | 200 | 250 | |

The transportation cost= (17 x 175) + (13 x 225) + (16 x 50) + (10 x 250) + (21 x 50) + (13 x 200).
= 12850rs

Here, the number of allocated cells is 6 which is equal to (m + n - 1) = (3 + 4 -1) = 6.
**Assessing the solution for optimality through Modi method**
Next, we have to find optimality test using Modi method
Allocation Table is
Table 1:

| **Source/Destination** | $D_1$ | $D_2$ | $D_3$ | $D_4$ | **Supply** |
|---|---|---|---|---|---|
| $S_1$ | 17 **(175)** | 13 **(225)** | 11 | 14 | 400 |
| $S_2$ | 16 **(50)** | 18 | 14 | 10 **(250)** | 300 |
| $S_3$ | 21 **(50)** | 24 | 13 **(200)** | 10 | 250 |
| **Demand** | 275 | 225 | 200 | 250 | |

Table 2:

| **Source/Destination** | $D_1$ | $D_2$ | $D_3$ | $D_4$ | **Supply** | $U_i$ |
|---|---|---|---|---|---|---|
| $S_1$ | 17 **(3)** | 13 **(225)** | 11 **(175)** | 14 **(6)** | 400 | $U_1 = 0$ |
| $S_2$ | 16 **(275)** | 18 **(3)** | 14 **(1)** | 10 **(25)** | 300 | $U_2 = 2$ |
| $S_3$ | 21 **(5)** | 24 **(9)** | 13 **(25)** | 10 **(225)** | 250 | $U_3 = 2$ |
| **Demand** | 275<br>$V_1 = 14$ | 225<br>$V_2 = 13$ | 200<br>$V_3 = 11$ | 250<br>$V_4 = 8$ | | |

Since all $d_{ij} \geq 0$.
So, the final optimal solution is arrived.
Table 3:

| **Source/Destination** | $D_1$ | $D_2$ | $D_3$ | $D_4$ | **Supply** |
|---|---|---|---|---|---|
| $S_1$ | 17 | 13 **(225)** | 11 **(175)** | 14 | 400 |

| | | | | | |
|---|---|---|---|---|---|
| **$S_2$** | 16 **(275)** | 18 | 14 | 10 **(25)** | 300 |
| **$S_3$** | 21 | 24 | 13 **(25)** | 10 **(225)** | 250 |
| **Demand** | 275 | 225 | 200 | 250 | |

The minimum total transportation cost = 13 x 225 + 11 x 175 + 16 x 275 +10 x 25 +10 x225
= 12075

**By using Modi method, the transportation cost is 12075.**

Next, going to apply Simulated Annealing,

| **Source/Destination** | **$D_1$** | **$D_2$** | **$D_3$** | **$D_4$** | **Supply** |
|---|---|---|---|---|---|
| **$S_1$** | 0 | **225** | **175** | 0 | 400 |
| **$S_2$** | **275** | 0 | 0 | **25** | 300 |
| **$S_3$** | 0 | 0 | **25** | **225** | 250 |
| **Demand** | 275 | 225 | 200 | 250 | |

The cost matrix is

$$\begin{array}{rrrr} & 17 & 13 & 1114 \\ c = [ & 16 & 18 & 1410] \\ & 21 & 24 & 1310 \end{array}$$

Now, calculate the total transportation cost

$Z_{initial}$ = (0 x 17) + (225 x 13) + (175 x 11) + (0 x 14) + (275 x 16) + (0 x 18) + (0 x 14) + (25 x 10) + (0 x 21) + (0 x 24) + (25 x 13) + (225 x 10)

=12,075

**The value of $Z_{initial}$ is 12,075.**

Simulated annealing steps,

Let temperature T0=100

Alpha=0.95

Now perturb the solution by shifting some allocations

Move 50 units fromS$_1$ $D_2 \rightarrow$ to $S_2D_1$ $\longrightarrow$

| **Source/Destination** | **$D_1$** | **$D_2$** | **$D_3$** | **$D_4$** | **Supply** |
|---|---|---|---|---|---|
| **$S_1$** | 50 | **175** | 175 | 0 | 400 |
| **$S_2$** | **225** | 0 | 0 | 25 | 300 |
| **$S_3$** | 0 | 0 | 25 | 225 | 250 |
| **Demand** | 275 | 225 | 200 | 250 | |

Calculate the $Z_{new}$ value,

$Z_{new}$ = (50 x 17) + (175 x 13) + (175 x 11) + (0 x 14) + (225 x 16) + (0 x 18) + (0 x 14) + (25 x 10) + (0 x 21) + (0 x 24) + (25 x 13) + (225 x 10)

=11475

**The $Z_{new}$value is 11475.**

Calculate the change in cost

$Del_z$=$Z_{new}$ - $Z_{initial}$

=11475-12075

=-600

$Del_z$ value is negative.

Change in cost

P=exp(-$del_z$/T)

=exp(-(-600)/100)
=403.4288
P is very greater so accept the solution.

## CONCLUSION

In this study, the MODI Method, a well-known transportation method, was employed to obtain an optimal solution. Initially, the MODI method reduced the transportation cost. Furthermore, applying Simulated Annealing optimized the cost down by making allocation adjustments. The combined use of the MODI method and Simulated Annealing demonstrated effectiveness in reducing transportation expenses, indicating a robust strategy for addressing transportation challenges. The final optimal solution revealed considerable cost reductions, emphasizing the efficiency of this combined approach in logistics and supply chain management. Notably, the Simulated Annealing method proved to be the most effective approach in achieving the minimum transportation cost.

## REFERENCE

1) Raigar, S., & Modi, G. (2017). An effective methodology for solving transportation problem. *International Journal of Scientific and Innovative Mathematical Research (IJSIMR)*, *5*(10), 23-27.
2) Babu, M. A., Tabassum, J., & Hassan, M. N. (2016). A heuristic on risk management system in goods transportation model using multi-optimality by MODI method. *Open Journal of Applied Sciences*, *6*(8), 539-551.
3) Hasan, M. K. (2012). Direct methods for finding optimal solution of a transportation problem are not always reliable. *International Refereed Journal of Engineering and Science (IRJES)*, *1*(2), 46-52.
4) Yang, H. (2024). Balance of mixed flow assembly line based on industrial engineering mathematics and simulated annealing improved algorithm. *Results in Engineering*, *22*, 102071.
5) Xiao, S., Peng, P., Zheng, P., & Wu, Z. (2024). A Hybrid Adaptive Simulated Annealing and Tempering Algorithm for Solving the Half-Open Multi-Depot Vehicle Routing Problem. *Mathematics*, *12*(7), 947.
6) Yu, V. F., Jodiawan, P., Lin, S. W., Nadira, W. F., Asih, A. M. S., & Vinh, L. N. H. (2024). Using Simulated Annealing to Solve the Multi-Depot Waste Collection Vehicle Routing Problem with Time Window and Self-Delivery Option. *Mathematics*, *12*(3), 501.
7) Zanazzo, E., Ceschia, S., Dovier, A., &Schaerf, A. (2024). Solving the medical student scheduling problem using simulated annealing. *Journal of Scheduling*, 1-14.
8) Hamdy, T. Operations Research- An Introduction (Sixth Edition). Pearson Education Inc, 2002.
9) Taha. H.A., Operations Research- Introduction, Prentice Hall of India (PVT), New Delhi, 2004
10) S. Rekha, B. Srividhya & S. Vidya, ‘Transportation Cost Minimization: Max Min Penalty Approach’ IOSR Journal of Mathematics Vol. 10 Issue 2 pp 6-8, (2014).
11) J. E. Reeb and S. Leavengood, Transportation problem: A special case for inear programming, Oregon : Oregon State University Extension Service Publications EM 8779, 2002.
12) Moses, E. P., & Hamidu, B. A. (2024). Development of Optimal Route Method to Obtain the Optimal Solution of Transportation Problem. *Sch J Phys Math Stat*, *7*, 74-82.
13) Victor-Edema, U. A., &Akehwe, O. E. (2024). APPLICATION OF LINEAR PROGRAMMING IN THE MINIMIZATION OF TRANSPORTATION COST IN DANGOTE CEMENT, PORT HARCOURT.
14) Samson, A. (2024). Transportation Model: Review of Its Practical Application in Kogi State, Nigeria.
15) Sharma, G., Abbas, S., & Gupta, V. (2012). Solving transportation problem with the various method of linear programming problem. *Asian journal of current engineering and maths*, *1*(3), 81-83.
16) Hlayel, A. A., & Alia, M. A. (2012). Solving transportation problems using the best candidates method. *Computer Science & Engineering*, *2*(5), 23.
17) Mondal, R. N., Hossain, M. R., & Uddin, M. K. (2012). Solving transportation problem with mixed constraints. In *Proceedings of the 2012 International Conference on Industrial Engineering and Operations Management Istanbul, Turkey, July* (Vol. 3, No. 6).
18) Korukoğlu, S., &Ballı, S. (2011). An improved Vogel's approximation method for the transportation problem. *Mathematical and computational Applications*, *16*(2), 370-381.