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Fractional Thermoelasticity Problem of infinite Solid Disk - boundary condition value problem

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ABSTRACT:

The two-dimensional problem for an infinite solid disk is examined in this paper within the framework of the Fractional Thermoelasticity Problem of an Infinite Solid Disk and the boundary condition value problem. It uses the Caputo fractional derivative of order in the heat conduction equation. It is assumed that the cylinder's curved surface is in contact with a rigid surface and is constantly being heated. The issue is resolved using the Laplace transform, Fourier, transform, and Hankel transform and its inverses. The distributions of temperature, displacement, and stress are computed numerically, and visually shown, and the findings are thoroughly analyzed.

Keywords: *Infinite solid disk, Fractional, thermoelasticity problem, Laplace transform, Hankel transform, Fourier transform*

1. INTRODUCTION

The theories related to generalized thermoelasticity for the dynamical system at the single relaxation for isotropic bodies were first presented by Lord et al. (1967). The behavior of thermoelastic materials without energy dissipation was proposed by Green & Naghdi., (1993) using linear and nonlinear theories. The deformation of theoretical Thermo elasticity for the circular plate has been studied for heat supply which has been partially dispersed as presented by Ishihara et al. (1997). The equation for fractional heat conduction in single and 2-dimensional problems is studied, Povstenko., (2005) proposed stresses using the Caputo fractional derivative that correspond to the fundamental Cauchy problem solutions. Povstenko., (2009) studied theories related to thermal stress based on the equation adding the heat conduction with special and time fractional derivatives. The linked thermoelasticity theory and the generalized thermoelasticity theory with one relaxation period were used by Sherief et al. (2010) to construct the novel theory of thermoelasticity.

Using the integral transform method, the mathematical model for quasi-static thermoplastic problems is studied in their indefinite solid long cylinder (Gaikwad et al., 2010). Sur et al. (2012)

new theory of generalized thermoelasticity at two temperatures was first forth for the novel analysis related to heat conduction and heat flux related to fractional orders thermoelasticity. A thin hollow circular disk deformed thermo elastically as a result of a partially distributed heat source (Gaikwad & Ghadle., 2012). The thermal deflection and heat conduction problem of non-homogeneous materials due to the generation of internal heat inside the thin hollow circular disc were studied by Gaikwad & Ghadle., (2012). A newmathematical model of the thermoelasticity theory was put forward (Sur et al., 2012). The Green Naghdi model and 3-phase lag thermoelastic model are regarded to be subject to a regularly varied heat source in the setting of an unbounded medium, isotropic, and functionally graded medium. For a 1D problem involving an infinitely long cylinder, Raslan (2014) investigated the theories related to fractional thermoelasticity problems. The problem of 2 dimensions is studied in a thick plate with traction-free upper and lower surfaces that are being subjected to the specified axisymmetric temperature distribution was introduced by Raslan., (2015) using the fractional thermoelasticity problem theories. The thermoelastic problems of the mathematical model and the disk prone to heat generation were studied in circular sector disk (Gaikwad., 2015)

The postulates of 2-dimensional distribution of steady-state temperature in thin circular plates due to the consistent nature of the generation of internal energy (Gaikwad., 2016). A thin circular plate's axisymmetric thermoelastic stress analysis owing to heat generation was covered by Gaikwad., (2019).In this study, the fractional thermoelasticity problem is formulated for solving the temperature and stress distribution of radial and circumference of thin circular discs. We expect beginning conditions to be zero. The boundary surfaces at $(r = a)$; $(\varphi = 0)$; $(\varphi = \varphi_0)$; $(z = 0)\ and\ (z = h)$ are maintained under the specified heat fluxes of $f_1\,(\varphi, z, t), f_2\,(r, z, t);\ \ f_3\,(r, z, t),\ \ f_4(r, \varphi, t) and\ f_5(r, \varphi, t)$ respectively.

The generalized finite Fourier transforms and the finite Hankel transform, as well as their inverses, have been used to solve the governing heat conduction equation with the aid of Mittag-Leffler functions. The mathematical model is built specifically with the pure Aluminium circular sector disk in mind. Using Mathcad software, the findings for thermal stress, displacement, and temperature have been estimated numerically and graphically shown.

2. LITERATURE REVIEW

Abouelregal et al. (2020) numerous efforts are made to better understand the Fourier classical heat transfer and several changes have been made. When some of these models are unsuccessful, therefore based on the Moore-Gibson-Thompson equation novel thermoelasticity model has been proposed. Combining the equation of hyperbolic partial differentiation for change in displacement field and the parabolic differential equation for the increase in temperature, this thermomechanical model was built. The investigated wave propagation in an infinite, isotropic body is subjected to a continuous thermal line source using the proposed model.

Adhe & Ghadle., (2023) internal heat generation on inhomogeneous materials is the focus of this paper's analysis of thermoelasticity problems for plane elasticity and thermal stresses. Here, the method of direct integration is used to condense the original issues and establish the governing and boundary equations. The governing equation is then transformed into integral equations by applying more iteration techniques. The iterative method was used to execute the numerical calculations, resulting in speedy convergence. On a graph, the distribution of the Shear and Young's moduli, as well as the dimensionless stresses, are displayed.

Singh et al. (2019) the current study examined the thermoelastic interaction in the memorydependant derivative of the three-phase lag model for the material of partial infinite elastic things with a heat source. The differential equation form of vector-matrix in the domain of Laplace transform is used to define the coupled governing equations, which involve time delay and kernel functions. The eigenvalue technique has been used to resolve the analytical formulations of the issue. The Laplace transformation is reversed using the Honig-Hirdes numerical approach. By selecting several forms of time delay settings, graphical representation and kernel function have been carried out to produce numerical results.

Sherief & Hussain., (2020) The fractional order thermoelasticity theory is applied to two-dimensional axisymmetric issues. In the Laplace transform domain, the general solution is reached by taking a straight route without employing potential functions. The two issues of a solid sphere and an endless space with a spherical cavity are solved using the resulting formulation. Every time, a particular axisymmetric temperature distribution is applied to a surface that is assumed to be traction-free. Utilizing the transform's inversion formula as well as Fourier expansion methods, the Laplace transforms are inverted. The distributions of temperature, displacement, and stress in the physical domain are obtained by accelerating the convergence of the resulting series using numerical techniques. Graphical representations and discussions accompany the numerical results.

3. FORMULATION OF THE PROBLEM

A two-dimensional problem is considered for a circular solid disk that occupies the space of $0 \leq r \leq a;\ 0 \leq \varphi \leq \varphi_0 (< 2\pi), 0 \leq z \leq h$. Initial conditions are assumed for the problem. The boundary surfaces $(r = a);\ (\varphi = 0);\ (\varphi = \varphi_0);\ (z = 0)\ and\ (z = h)$ for time t > 0 kept for the heat-flux $f_1\,(\varphi, z, t), f_2\,(r, z, t);\ f_3\,(r, z, t),\ f_4(r, \varphi, t) and\ f_5(r, \varphi, t)$ correspondingly.The nonlocal Caputo type temporal fractional heat conduction equation of order for a thin circular disk is taken into account when creating a mathematical model.

The time-fractional differential equationat time t with the temperature of the circular disk is given below;

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \varphi^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{k}\frac{\partial^\alpha T}{\partial t^\alpha} \qquad (3.1)$$

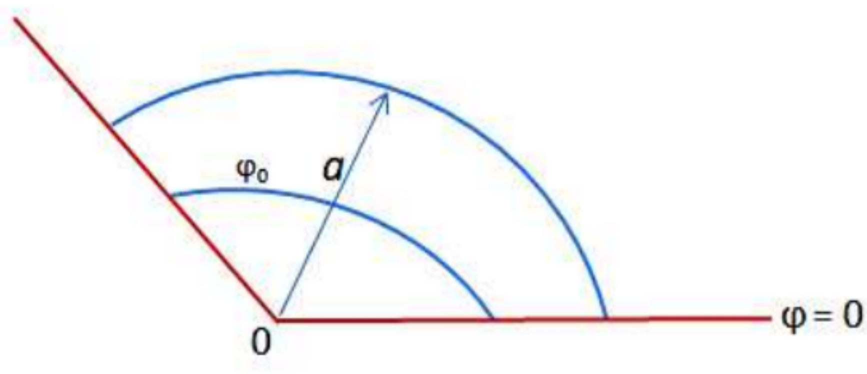

Figure 3.1Representation of fractional thermoelastic problem

In $0 \leq r \leq a;\ 0 \leq \varphi \leq \varphi_0 (< 2\pi), 0 \leq z \leq h$, for t > 0. The boundary value problems using the physical conditions, we can get the following equation;

$$k_t D_{RL}^{1-\alpha} \frac{\partial T}{\partial r} = f_1\,(\varphi, z, t);\ at\ r = a;\ for\ t > 0; \qquad (3.2)$$

$$k_t D_{RL}^{1-\alpha} \frac{\partial T}{\partial \varphi} = f_2\,(r, z, t);\; at\; \varphi = 0;\; for\; t > 0; \tag{3.3}$$

$$k_t D_{RL}^{1-\alpha} \frac{\partial T}{\partial \varphi} = f_3\,(r, z, t);\; at\; \varphi = \varphi_0;\; for\; t > 0; \tag{3.4}$$

$$k_t D_{RL}^{1-\alpha} \frac{\partial T}{\partial z} = f_4\,(r, \varphi, t);\; at\; z = 0;\; for\; t > 0; \tag{3.5}$$

$$k_t D_{RL}^{1-\alpha} \frac{\partial T}{\partial z} = f_5\,(r, \varphi, t);\; at\; z = h;\; for\; t > 0; \tag{3.6}$$

Then the initial conditions are;

$$T\; =\; 0;\; at\; t = 0, 0 < \alpha < 2; \tag{3.7}$$

$$\frac{\partial T}{\partial t} = 0;\; at\; t = 0, 1 < \alpha < 2; \tag{3.8}$$

We made the assumption that the circular disk is in a planar state of tension for thin h in accordance with Gaikwad [32]. In actuality, "the closer to a plane state of stress is the actual state, the smaller the thickness of the hollow disk compared to its diameter."

The displacement equation is;

$$U_{i,kk} + \left(\frac{1+\nu}{1\,-\,\nu}\right) e_{,i} = 2\left(\frac{1+\nu}{1\,-\,\nu}\right) \alpha_t T_{,i} \tag{3.9}$$

$$e\; =\; U_{k,k};\; k, i\; =\; 1,2,$$

$U_i =\; \psi_{,i}$, i =1,2,

Wecan have;

$$\nabla^2 \psi = (1+\nu)\alpha_t T$$

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}$$

$$\sigma_{ij} = 2\mu(\psi_{,ij}\; -\; \delta_{ij}\psi_{,kk}), i, j, k\; = 1,2.., \tag{3.10}$$

The potential function of displacement $\psi\;(r, \varphi, z, t)$ can be written as;

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r}\frac{\partial \psi}{\partial r} = (1+\nu)\alpha_t T \tag{3.11}$$

Using $\frac{\partial \psi}{\partial r}\; = 0\; at\; r = a\; for\; time\; t$

At initial stage$T = \psi = 0;\; at\; t = 0$

The stress function $\sigma_{\theta\theta}$and σ_{rr} are;

$$\sigma_{\theta\theta}\; = -2\mu \frac{\partial^2 \psi}{\partial r^2} \tag{3.12}$$

$$\sigma_{rr}\; = \frac{-2\mu}{r}\frac{\partial \psi}{\partial r} \tag{3.13}$$

The boundary conditions are given below;

$$\sigma_{rr} = \sigma_{r\varphi} = 0;\; for\; r\; = a; 0 \le \varphi < \varphi_0; t > 0; \tag{3.14}$$

$$\sigma_{\varphi\varphi} = \sigma_{r\varphi} = 0; for\varphi = 0; 0 \le r < \mathrm{a}; t > 0; \tag{3.15}$$

$$\sigma_{\varphi\varphi} = \sigma_{r\varphi} = 0; for\varphi = \varphi_0; 0 \le r < \mathrm{a}; t > 0; \tag{3.16}$$

The problem formulation is considered from equation (3.1) to (3.16).

4. PROBLEM SOLUTION

DETERMINING THE TEMPERATURE FIELD

Inverse transform and Fourier transform are defined for obtaining temperature function $T\;(r, \varphi, z, t)$ using z variable in range $0 \le z <$ h as follows;

$$\bar{T}\,(r,\varphi,z,t) \;=\; \int_{z\prime=0}^{h} K(\eta_p, z')T(r,\varphi,z',t')dz' \; (4.1)$$

$$\bar{T}\,(r,\varphi,z,t) \;=\; \sum_{n=1}^{\infty} K(\eta_p, z)\bar{T}(r,\varphi,\eta_p,t) \; (4.2)$$

where,

$$K(\eta_p, z) = \sqrt{\frac{2}{h}}cos(\eta_p z)$$

$\eta_{1,} and\ \eta_2$are considered positive roots of the equation;

$$sin(\eta_p, z) = 0;\ p = 1,2,3$$

where,

$$\eta_p = \frac{p\pi}{h};\ p = 1,2,3,..$$

Fourier transform is applied to equation (3.1)can be defined inequation (4.1) by using boundary conditions (3.2) to (3.8); we can obtain;

$$\frac{\partial^2 \bar{T}}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{T}}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \bar{T}}{\partial \varphi^2} + \frac{\partial^2 T}{\partial z^2} - \eta_p^2 \bar{T} = \frac{1}{k}\frac{\partial^\alpha \bar{T}}{\partial t^\alpha} \; (4.3)$$

The boundary conditions are;

$$k_t D_{RL}^{1-\alpha}\frac{\partial \bar{T}}{\partial r} = \overline{f_1}\,(\varphi,\eta_p,t);\ at\ r = a;\ for\ t > 0; \quad (4.4)$$

$$k_t D_{RL}^{1-\alpha}\frac{\partial \bar{T}}{\partial \varphi} = \overline{f_2}\,(r,\eta_p,t);\ at\ \varphi = 0;\ for\ t > 0; \quad (4.5)$$

$$k_t D_{RL}^{1-\alpha}\frac{\partial \bar{T}}{\partial \varphi} = \overline{f_3}\,(r,\eta_p,t);\ at\ \varphi = \varphi_0;\ for\ t > 0; \quad (4.6)$$

Using,

$$\bar{T} = 0;\ at\ t = 0;\ 0 < \alpha < 2; \quad (4.7)$$

$$\frac{\partial \bar{T}}{\partial t} = 0;\ at\ t = 0, 1 < \alpha < 2; \quad (4.8)$$

Inverse transform and Fourier transform over φ variable in range $0 \le \varphi \le \varphi_0$ can be defined as below;

$$\bar{\bar{T}}\,(r,\nu_n,\eta_p,t) \;=\; \int_{\varphi\prime=0}^{\varphi_0} K_0(\nu_n,\varphi')\bar{T}(r,\varphi',\eta_p,t)d\varphi' \; (4.9)$$

$$\bar{T}\,(r,\varphi,\eta_p,t) \;=\; \sum_{n=1}^{\infty} K_0(\nu_n,\varphi)\bar{\bar{T}}(r,\nu_n,\eta_p,t) \; (4.10)$$

Where,

$$K(\nu_n,\varphi) = \sqrt{\frac{2}{\varphi_0}}cos(\nu_n\varphi)$$

Where Eigenvalues $\nu_n$are considered as positive roots of equation;

$$sin(\nu_n\varphi_0) = 0;\ \nu_n = \frac{n\pi}{\varphi_0};\ n = 1,2,3..$$

Fourier transform is applied to equations (4.3) and (4.9) by using the conditions (4.4) to (4.8), we can obtain the following equation;

$$\frac{\partial^2 \bar{\bar{T}}}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{\bar{T}}}{\partial r} - \frac{v_n^2}{r^2}\bar{\bar{T}} - \eta_p^2 \bar{\bar{T}} = \frac{1}{k}\frac{\partial^\alpha \bar{\bar{T}}}{\partial t^\alpha} \quad (4.11)$$

Using

$$k_t D_{RL}^{1-\alpha}\frac{\partial \bar{\bar{T}}}{\partial r} = \overline{f_1}(v_n, \eta_p, t);\ at\ r = a;\ for\ t > 0; \quad (4.12)$$

$$\bar{\bar{T}} = 0;\ at\ t = 0;\ 0 < \alpha < 2; \quad (4.13)$$

$$\frac{\partial \bar{\bar{T}}}{\partial t} = 0;\ at\ t = 0, 1 < \alpha < 2; \quad (4.14)$$

Inverse transform and Hankel transform over r variable at range $0 \leq r < a$can be defined as below;

$$\bar{\bar{\bar{T}}}\,(\beta_m, v_n, \eta_p, t) = \int_{r'=0}^{\mathrm{a}} r'.K_1(\beta_m, r')\bar{T}(r, v_n, \eta_p, t)dr' \quad (4.15)$$

$$\bar{T}\,(r, v_n, \eta_p, t) = \sum_{m=1}^{\infty} K_1(\beta_m, r)\bar{\bar{\bar{T}}}(\beta_m, v_n, \eta_p, t) \quad (4.16)$$

Where,

$$K(\beta_m, r) = \sqrt{\frac{2}{\mathrm{a}}}\frac{1}{\left[1 - \frac{v_n^2}{\beta_m^2 a^2}\right]^{1/2}}\frac{J_0(\beta_m r)}{J_0(\beta_m a)}$$

$\beta_{1,} and\ \beta_2$ are considered positive roots;

$$J_1(\beta_m a) = 0;\ m = 1,2,3\ldots$$

Hankel transform is applied in equations 4.11 and (4.15) by using the conditions (4.12) to (4.14), we can obtain the following equation

$$\frac{\partial^\alpha \bar{\bar{\bar{T}}}(\beta_m, v_n, \eta_p, t)}{\partial t^\alpha} + k(\beta_m^2 + \frac{v_n^2}{r^2} + \eta_p^2)\bar{\bar{\bar{T}}}(\beta_m, v_n, \eta_p, t) = A(\beta_m, v_n, \eta_p, t) \quad (4.17)$$

And,

$$\bar{\bar{\bar{T}}}(\beta_m, \eta_p, t) = 0;\ at\ t = 0;\ 0 < \alpha < 2; \quad (4.18)$$

$$\frac{\partial \bar{\bar{T}}(\beta_m, \eta_p, t)}{\partial t} = 0;\ at\ t = 0, 1 < \alpha < 2; \quad (4.19)$$

Where,

$$A(\beta_m, v_n, \eta_p, t) = kaK_1(\beta_m, a)\overline{\overline{f_1}}(v_n, \eta_p, t)\ \Big\{a\frac{dK_0(v_n, \varphi)}{d\varphi}\overline{\overline{f_2}}(\beta_m, \eta_p, t)|_{\varphi=0} - \frac{dK_0(v_n, \varphi)}{d\varphi}\overline{\overline{f_3}}(\beta_m, \eta_p, t)|_{\varphi=\varphi_0} + \frac{dK(v_n, \varphi)}{dz}\overline{\overline{f_4}}\,((\beta_m, v_n, t)|_{z=0} + \frac{dK(v_n, \varphi)}{dz}\overline{\overline{f_5}}(\beta_m, \eta_p, t)|_{z=h}\Big\} \quad (4.20)$$

Laplace and inverse transform is applied to equation (3.17), we can obtain;

$$\bar{\bar{\bar{T}}}(\beta_m, v_n, \eta_p, t) = \frac{A(\beta_m, v_n, \eta_p, t)}{k(\beta_m^2 + \frac{v_n^2}{r^2} + \eta_p^2)}[1 - E_\alpha(-k(\beta_m^2 + \frac{v_n^2}{r^2} + \eta_p^2)t^\alpha)] \quad (4.21)$$

Finally, the required temperature is obtained by defining the inverse in equations (4.16), (4.10), and (4.2);

$$T\ (r,\varphi,z,t)\ =\ \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{p=1}^{\infty}K_1(\beta_m,r)\,K_0(\nu_n,\varphi)K(\eta_p,z)\frac{1}{k(\beta_m^2+\frac{\upsilon_n^2}{r^2}+\eta_p^2)}\quad[1-E_\alpha(-k(\beta_m^2+\frac{\upsilon_n^2}{r^2}+\eta_p^2)t^\alpha)]\ x\ b_{mnp} \qquad (4.22)$$