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Modeling Warfare Conflicts at the Operational Level as a Game under a Social-Learning DeGroot Network

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Abstract
This study investigates the integration of the DeGroot model and game theory to enhance strategic decision-making in military contexts. Traditional models such as the Prisoners' Dilemma and Chicken Game are critiqued for their dependence on assumptions of rationality and complete information. Recent advancements, including Prospect Theory and bounded rationality, are explored for their ability to address real-world complexities, such as misinformation and cognitive biases. The DeGroot model is introduced as an effective tool for iterative belief updates, reflecting the dynamic nature of military operations. The paper proposes a multi-level dynamic game framework incorporating the DeGroot update method to account for evolving payoffs and strategic interactions over time. This integrated framework is exemplified through historical instances such as the Iraq War, where misinformation about weapons of mass destruction led to flawed strategic decisions, and the Vietnam War, where biases distorted perceived payoffs. The model underscores the significance of real-time information processing and the influence of biases and misinformation on decision-making. Future research directions include the integration of machine learning algorithms to enhance data processing, the exploration of specific cognitive biases in decision-making, the application of the model to non-military conflicts, empirical validation with real-time data, extensions to complex multi-agent systems, and the incorporation of humanitarian decision-making. The proposed framework provides a comprehensive approach to understanding and predicting military strategies, with potential applications extending beyond traditional warfare to other strategic domains such as cyber security and economic conflicts.
***Keywords**: Warfare, Game Theory, DeGroot Model, Belief Updates*

Introduction

Game Theory has been extensively used to model strategic interactions in the context of warfare. Military strategy, including deterrence, signaling, dynamics of alliances, behavior of opposition forces, decisions and potential outcomes of the decisions are extensively modelled through game theory (Von Neumann & Morgenstern, 1944; Schelling, 1960). These decisions traditionally revolve around resource allocations, logistics, deterrence strategies and the timing of attacks or ceasefire (Schelling, 1960; Taylor, 1976). For example, if two rival nations both decide to arm or disarm themselves, they both incur high cost or end up cooperating, respectively. With any other strategic combination, one nation is always left vulnerable – an extension of Prisoners' Dilemma (Dixit & Nalebuff, 1991). Or in the 'Chicken Game', mostly in the dynamics of nuclear deterrence, the threat of mutual destruction is at the core of decision making (Bennet & Stam 2004). In these cases, rational actors tend to navigate the uncertainty of the opposition's actions or alternatives. But recent scholars have appreciated the fact that 'rationality', 'complete' information, and 'perfect' information are too theoretical assumptions to frame a substantial model. It has been observed that misinformation and bias often result in skewed decision making (Nisan & Ronen 1999; Bauman 2021). Misinformation often leads to false assessment of rival's capabilities (even intentions)[1] shaping strategic decisions (Baron & Beshears, 2010). Whereas, overconfidence[2] or fear leads to cognitive bias and non-optimal decisions (Tversky & Kahneman, 1974).The basis stands thus that the players are practically irrational. Furthermore, such rationality can also be compromised because of exogenous factors like political or public opinion that may prompt a strategic shift (Fudenberg & Tirole, 1991).

To account for such complexities several models such as prospect theory, bounded rationality etc. have been proposed in the context of warfare (see Harsanyi & Selten, 1988; Wang et al., 2022). For example, models such as Level-K reasoning accounts for several levels of cognition amongst players (Nagel, 1995). A Level-0 thinker is bounded by immediate payoffs – only consideration for actions; whereas, a level-1 thinker would go one step further to anticipate the opponent's actions. Dynamic game theory models have also been introduced to cater to adaptation of strategies pertaining to new information (Fudenberg & Levine, 1998; Holmström & Myerson, 1983). Dynamic programming can account for how strategies may evolve over time, incorporating changes in payoff structures (Bellman, 1957). But it is to be noted that there are certain drawbacks that are yet to be constructively addressed. First of all, when we talk of bounded rationality and choice, we are constrained by the cognitive prejudices of the agents. In the context of war, it is not about a single soldier taking decision at any point in time. It is a force that decides to fight, abort or surrender (of course under a leader). Furthermore, players are assumed to have complete and symmetric information that mostly leads to a static understanding of the game (see Dempster-Shafer Theory, 1976; Aumann 1976). In case of a dynamic game, the involvement is modelled more at the exogenous level; whereas, the endogenous subtleties like how the troop feels about a sudden change in the opposition's logistics, is not accounted for. Most importantly, the aspects of misinformation and bias are completely overlooked during information updates. In other words, game theoretic model of war is yet to be formally developed at the operational level.

Theoretical Motivation

Bayesian Game Theory has had its application in several social and economic fields, including military conflicts. It somewhat takes care of the inherent limitations of the classical models. It encompasses incomplete and asymmetric information catering to belief-updates under uncertainty, based on probabilistic reasoning.

Assume that with a set of random variables $(x_{1,}x_2, \dots, x_n)$ a Directed Acyclic Graph (DAG) that defines dependency structure with a conditional probability distribution $P(x_i|P_a(x_i))$, where $P_a(x_i)$ is the set of initial (Parent) nodes of x_i; the joint probability distribution over the Bayesian network is given by:

$$P(x_1, x_2, \dots, x_n) = \prod_{i=1}^{n} P\{x_i|P_a(x_i)\} \qquad (1)$$

Appendix 1 shows a hypothetical payoff structure of two conflicting nations, and their Bayesian Nash Equilibrium scenario under a 'Missile Defense System'. It is to be noted, however, that the Bayesian structures have the following inherent limitations in modeling iterative processes in multi-agent settings, like that of any conflict at the operational level.

1. They are designed primarily for single-shot updates of beliefs, and hence they cannot model iterative social learnings. In a warfare setting, players would continuously adapt based on new information received from neighboring nodes.
2. As the network grows with larger troops, diversity and more parties joining in; updating the conditional probabilities at each node becomes computationally expensive.
3. Bayesian network ignores the topological structure of relationships between agents. In any complex network like war, information diffusion and opinion formation plays a crucial role.

The Initial DeGroot Model (DeGroot, 1974)

This model, as against the Bayesian Network, captures the process of social learning, and agents continuously update their beliefs by interacting (iteratively) with the neighbors. In strategic settings, such as war or any organized conflict, where players need to update strategies continuously, this model has been proved to be the most efficient (Parhizkar et al., 2022). In Bayesian Model, the belief-update is primarily given by Bayes' Theorem. But in DeGroot model, we use graph theoretic approach with weights (w_{ij}) assigned to every node. The model updates the belief as the weighted average of the neighbors' beliefs:

$$x_i(t+1) = \sum_{j \in N(i)} w_{ij} x_j(t) \qquad (2)$$

Where x_i is the belief of the i^{th} agent, for updated belief at time $(t+1)$. Here w_{ij} reflects the strength of relationship between agents, and the structure of the Graph G, determines how fast the information spreads, and how the consensus is reached (Tahbaz-salehi & Jadbabaie, 2010). In our present context, payoffs generally evolve over time. If w_{ij} is properly adjusted by the agents, it can easily incorporate the dynamic nature too. The linearity of $Eq. 2$ ensures that the network arrives or converges to a consensus, even in large settings, thus reducing the computational complexity to a great extent (Golub & Jackson, 2010).

We advance the scenario in Appendix 1 to three countries – A, B and C and their common adversary. Assume that these countries interact and share intelligence on the missile defense potential of the adversary nation. Each of the country's decision depends on its own acquired intelligence, and also those of its allies. Let the belief of each of the countries at any time t be $x_i(t)$ where$i\epsilon \{A, B, C\}$. Each of them iteratively update their beliefs based on neighbor opinion, with the influence factor $w_{ij}(t)$, of the j th neighbor of the country i, where and t is the current time or round. Then, the model updates the belief $x_i(t+1)$of country i and time $(t+1)$ as –

$$x_i(t+1) = x_i(t). \sum_{j\in N(i)} w_{ij}(t). x_j(t) \qquad (3)$$

s.t. the constraint: $\sum_{j\in N(i)} w_{ij}(t) = 1$ for all i

[Consider $x.(t)$ to be the belief of country i at time t with $N(i)$ as the set of neighboring countries. And let $w_{ij}(t)$ be the influence that country i assigns to country $j's$ opinion at time t.]

Practically, $w_{ij}(t)$ is seldom static. Countries will adjust the degree of trust based on the reliability of past experience. For example, if A finds B's intelligence to be consistently accurate, then $w_{AB}(t)$will go up. Now, let $\delta_{ij}(t)$ represent the trust score that country i assigns to country $j's$ information at time t, then the weights of the next round are updated as follows:

$$w_{ij}(t+1) = \frac{\delta_{ij}(t)}{\sum_{k\epsilon N(i)} \delta_{ik}(t)} \qquad (4)$$

Now, let $\delta_{ij}(t)$ be a function of the accuracy of information (intelligence) provided by country j to country i. If the information is accurate, the trust score $\delta_{ij}(t+1)$ is updated positively, else it is penalized. The update rule can be given as:

$$\delta_{ij}(t+1) = \delta_{ij}(t) + \eta.(Accuracy\ Indicator) - \zeta.(Error\ indicator) \qquad (5)$$

Where η is the positive reinforcement parameter, and ζ is the penalty parameter. Both $\eta\ and\ \zeta$ takes the values of either 1 or 0.

Convergence of Belief

As per the DeGroot Model, it is predicted that individual beliefs of a country will converge to a consensus belief, if the weights are positive and the network remains connected. Thus, the consensus belief $x^*$is the weighted average of the initial beliefs, and weights are determined by the limiting influential weights –

$$x^* = \sum_{i=1}^{n} w_i(\infty). x_i(0)$$

Where $w_i(\infty)$represents the steady-state influence weights for country i and $x_i(0)$ is the initial belief of country i. Therefore, the agent that provides consistently accurate information will have a greater influence will have a greater influence on the consensus.

Multi-level Game with DeGroot network

When we converge our model to the ground level operations, we start by representing the model with two separate directed graphs for each side: one for the attacker and the other for the defender. As the model progresses, we would appreciate the fact that the stake for each of the players in the war are significantly different. Each node in each graph can represent a unit, or an ally, or even a passive party (or even a proxy), but all are capable of providing intelligence or strategic advice. Consider the attacker's graph $G_A(V_A, E_A)$ and the defender's graph $G_D(V_D, E_D)$, where Vdenotes the set of vertices or units, and Edenotes the intelligence sharing links. Let $x_i^A(t)$ be the belief of unit i in $G_A$at time t, and $x_j^D(t)$ for unit j in G_D. The elements of belief can be enemy strength, optimal strategy, battlefield conditions etc. Then the DeGroot update rule for the attacker and the defender goes as –

$$x_i^A(t+1) = x_i^A(t). \sum_{k\in \mathrm{N}_A(i)} w_{ik}^A(t). x_k^A(t) \qquad (6.1)$$

and

$$x_j^D(t+1) = x_j^D(t). \sum_{l\in \mathrm{N}_D(j)} w_{jl}^D(t). x_l^D(t) \qquad (6.2)$$

Where, $\mathrm{N}_A(i)$ and $\mathrm{N}_D(j)$ are set of neighbors for the two parties respectively; and $w_{ik}^A(t)$ and $w_{jl}^D(t)$ satisfies the conditions:

$$\sum_{k\in \mathrm{N}_A(i)} w_{ik}^A(t) = 1 \ \& \sum_{l\in \mathrm{N}_D(j)} w_{jl}^D(t) = 1$$

Now, let $\delta_{ik}^A(t)$ be the trust score that unit i assigns to unit k at any given time t, the weights are adjusted as: $w_{ik}^A(t+1) = \delta_{ik}^A(t) \Big/ \sum_{k'\epsilon \mathrm{N}_A(i)} \delta_{ik'}^A(t)$ (for the attacker), and $w_{jl}^D(t+1) = \delta_{jl}^D(t) \Big/ \sum_{l'\epsilon \mathrm{N}_D(j)} \delta_{jl'}^D(t)$ (for the defender).

The trust score evolves based on the accuracy of past intelligence or the effectiveness of the past strategic advice. Extending Eq. 5 we thus get -

$$\delta_{ik}^{A}(t+1) = \delta_{ik}^{A}(t) + \eta.(Accuracy_{ik}^{A}(t)) - \zeta.(Error_{ik}^{A}(t)) \quad (7)$$

Each unit now uses its update belief $x_i^A(t)$ for attacker or $x_j^D(t)$ for defender for strategy at different levels of the game. For example, the probability of choosing strategy s by unit i in the attacker's network is:

$$P_i^A(s,t) = f_s^A(x_i^A t)$$

Where, the function $f_s^A(\cdot)$ may be perceived as a logistic function reflecting decision under uncertainty. Then the payoff for each unit depends on its own strategy as well as that of the others:

$$u_i^A(t) = \sum_{s\in S}\sum_{s'\in S'} P_i^A(s,t).P_{-i}^D(s',t).\pi_i^A(s,s',t) \quad (8)$$

Where, S and $S'$are the set of strategies for the attacker and the defender respectively. Assuming the payoff is from the perspective of the attacker, $P_{-i}^D(s',t)$ is the probability distribution over the defender's strategies from the perspective of unit i. And, $\pi_i^A(s,s',t)$ represents the payoff of unit i, when it chooses strategy s and the other chooses strategy s'. Therefore, let $X^A(t)$ and $X^D(t)$be the vectors of beliefs (of size n x 1), and $W^A(t)$ and $W^D(t)$ be the influence matrices (of size n x n where all the diagonal elements are 0), and $D^A(t)$ is a diagonal matrix with element(ij) is equal to $x_i^A(t)$ for every $i = j$ and zero if $i\ not\ equal\ to\ j$ then the matrix representation of the belief update is: $X^A(t+1) = D^A(t).W^A(t).X^A(t)$ with the influence weight update as: $W^A(t+1) = \text{Normalize}\{\Delta^A(t)\}$; where $\Delta^A(t)$is the trust score matrix[3].

Lagrange Optimization Framework

We now formally integrate DeGroot update beliefs and strategies into the Lagrange Optimization Problem, where each party aims to maximize its total expected payoff over a specific time horizon T under resource constraints.

Objective Function: $\max_{\{s_i^A(t)\}} \sum_{t=0}^{T} \beta^t \{\sum_{i\epsilon V_A} u_i^A(t)\}$,

s.t. $\sum_{t=0}^{T} \beta^t [\sum_{i\epsilon\ A} c_i^A\{s_i^A(t)\}] \leq R_A$

where $\beta\epsilon(0,1]$ is the discount factor; $c_i^A\{s_i^A(t)\}$is the cost associated with strategies $s_i^A(t)$for unit i; and R_A is the total available resource. Therefore, for the attacker, the Lagrangian becomes:

$$\mathcal{L}^A = \sum_{t=0}^{T} \beta^t \{\sum_{i\epsilon V_A} u_i^A(t)\} - \lambda_A [\sum_{t=0}^{T} \beta^t \{\sum_{i\epsilon V_A} c_i^A(s_i^A t)\} - R_A] \quad (9)$$

Taking derivative of the Lagrangian w.r.t. each $S_i^A(t)$ and setting to 0, we get:

$$\frac{\partial \mathcal{L}^A}{\partial s_i^A(t)} = \beta^t \left[\frac{\partial u_i^A(t)}{\partial s_i^A(t)} - \lambda_A \frac{\partial c_i^A\{s_i^A(t)\}}{\partial s_i^A(t)}\right] = 0$$

That yields the optimality condition: $\frac{\partial u_i^A(t)}{\partial s_i^A(t)} = \lambda_A \frac{\partial c_i^A\{s_i^A(t)\}}{\partial s_i^A(t)}$ (10)

The condition is: each troop unit, i.e. each node should choose a strategy where marginal benefit equals marginal cost, weighted by the Lagrange Multiplier λ_A.

Incorporating belief updates

Since $u_i^A(t)$ [Eq.10], directly depends on the belief $x_i^A(t)$ that in turn depends on the strategies over the DeGroot model, we account for the dependency as the derivative:

$\frac{\partial u_i^A(t)}{\partial s_i^A(t)} = \frac{\partial u_i^A(t)}{\partial s_i^A(t)} + \sum_{k\in N_A(i)} \frac{\partial u_i^A(t)}{\partial x_k^A(t)} . \frac{\partial x_k^A(t)}{\partial s_i^A(t)}$ The RHS is practically free of belief [4].

As units adjusts their trust scores $\delta_{ik}^A(t)$ based on observed strategies, i.e. choosing strategies that are deemed to be more trust worthy, then –

$$\partial \delta_{ik}^A(t) \Big/ \partial s_i^A(t) \neq 0$$

Now solving for the optimization for $s_i^A(t)$ for all i & t, i.e. for all units at all time, we get

$$\beta^t \left[\frac{\partial u_i^A(t)}{\partial s_i^A(t)} + \sum \frac{\partial u_i^A(t)}{\partial w_{ik}^A(t)} . \frac{\partial w_{ik}^A(t)}{\partial s_i^A(t)}\right] - \lambda_A \beta^t \frac{\partial c_i^A s_i^A(t)}{\partial s_i^A(t)} \quad (11)$$

s.t. $\sum_{t=0}^{T} \beta^t [\sum_{i\in V_A} c_i^A\{s_i^A(t)\}] = R_A$

Endogenous Bias and Perceived Payoffs: A Practical Extension

In warfare, the consensus decisions that are made by units or commanders often contains endogenous biases in form of irrationality, prejudices or even misinformation processing. We here assume that 'exogenous' misinformation is already taken care of in previous setups while agents modify their trusts based on the reliability of past intelligence. In this section we only extend the model to endogenous errors. When a payoff is perceived incorrectly, it leads to incorrect (non-optimal) decisions at the operational level, even though the network is reliable. Such decisions may be because of biases like historical animosities, overconfidence, mutual distrust and the like.

As we incorporate this element in the proposed DeGroot game, we introduce a bias factor that affects the weighting of information from different sources. So let $\mathrm{b}_i^A(t)$ and $\mathrm{b}_j^D(t)$ be the bias level of unit i in the graph representing the attacking force, and unit j of the defending force. Therefore, the 'Bias Belief Update' becomes:

$$x_i^A(t+1) = \mathrm{b}_i^A(t).x_i^A(t) + \left(1 - \mathrm{b}_i^A(t)\right).x_i^A(t).\sum_{k\in \mathrm{N}_A(i)} w_{ik}^A(t).x_k^A(t) \qquad (12.1)$$

And,

$$x_j^D(t+1) = \mathrm{b}_j^D(t).x_j^D(t) + \left(1 - \mathrm{b}_j^D(t)\right).x_j^D(t)\sum_{l\in \mathrm{N}_D(j)} w_{jl}^D(t).x_l^D(t) \qquad (12.2)$$

For the attacking and the defending units respectively, where $\mathrm{b}_i^A(t)$ & $\mathrm{b}_j^D(t) \in [0,1]$ represents the extent of bias at time t. A higher $\mathrm{b}_i(t)$ means i reduces the influence of external intelligence. The bias factor too should ideally evolve over the outcomes of previous decisions. For instance, if a unit makes a 'perfect mistake', the prejudices will be positively reinforced leading to an increase in bias. The evolutionary process can be modelled as:

$$\mathrm{b}_i(t+1) = \mathrm{b}_i(t) + \alpha.Success_i^A(t) - \beta.Failure_i^A(t) \qquad (13)$$

Where α & β are adjustment factors and $Success_i^A(t)$ and $Failure_i^A(t)$ are binary indicators of strategic success or failure respectively. Now, this success and failure is often influenced by the perceived payoff that happens to be skewed by the endogenous bias, as we model this in the next section.

Misinformation and Perceived Payoffs

There can be cases where units may receive faulty or deliberately deceptive intelligence from external sources. This kind of information may also affect the perceived payoffs adversely. We account for this discrepancy as follows:

Let $\hat{\pi}_i^A(s,s',t)$ e the perceived payoff of i, when the attackers choose strategy 's' and defenders choose s'. Let the true payoff e $\pi_i^A(s,s',t)$. Therefore,

$$\hat{\pi}_i^A(s,s',t) = \pi_i^A(s,s',t) + \epsilon_i^A t$$

where ϵ is the misinformation factor. The overall utility then is give as:

$$\hat{\mathrm{u}}_i^A(t) = \sum_{s\in S}\sum_{s'\in S'} P_{-i}^D(s',t).\hat{\pi}_i^A(s,s',t) \qquad (14.1)$$

Then from Eq. 12.1, the DeGroot update process with the noise element becomes:

$$x_i^A(t+1) = x_i^A(t).\sum_{k\in \mathrm{N}_A(i)} w_{ik}^A(t).[x_k^A(t) + v_{ik}^A(t)] \qquad (14.2)$$

Where, $v_{ik}^A(t)$is the misinformation or noise in the intelligence y unit i from unit k.

The distribution followed by the noise term can be considered as the Gaussian: $v_{ik}^A(t) \sim \mathcal{N}(0, \sigma_{ik}^2)$ where σ_{ik}^2 is the inherent variance. This distribution should be practically verified in future research.

Belief Updates on Misinformation: Iraq War (2003)

In this case, we provide a conceptual framework of the application of the proposed model. Both the coalition forces, one led by the United States and the other by the Iraqi regime under Saddam Hussein, demonstrates the impact of misinformation and bias on how updates happen on beliefs, payoffs and subsequent military actions. The misinformation pertains to the intelligence regarding Iraq's possession of weapons of mass destruction (WMD). It was believed that Saddam Hussein had access to, or owned WMDs, and this belief was crucial behind the decision to launch the invasion (Pillar 2006; Ricks 2006; Stiglitz & Bilmes 2008). The belief emerged from faulty intelligence reports, political bias as well as historical animosities (Gause 2005).

Now, let the graph of the coalition force be G_e and it consists of intelligence agencies like CIA, MI6 etc. each providing updates on Iraq's capabilities. Let each agency or military unit be the nodes, and let the edges be the flow of intelligence and strategic recommendations.

Bias Factor: Figures like Dick Cheney, George W. Bush and various other elements in the US Government had a predisposition that viewed Iraq as a direct threat. One reason being the past conflicts like the Gulf war. Such biases increased the weight $b_i^A(t)$and skewed the belief updates towards 'attack'. For example, the biased belief update (for instance, the US department of Defense) regarding Iraq's possession of WMDs can be:

$$x_{DoD}(t+1) = b_{DoD}(t).x_{DoD}(t) + \left(1 - b_{DoD}(t)\right).x_{DoD}(t)\sum_{k\in N_C(DoD)} w_{DoD,k}(t).x_k(t)$$

[DoD refers to Department of defense]

Assumption: The decision maker node is being constantly fed by its own intelligence gathering mechanism in addition to information coming from all its neighboring nodes

Here, $b_{DoD}(t)$is high because of pre-conceived notions, although it was evident that many agencies did not support this view.

Misinformation: Iraqi defector 'Curveball' provided false information regarding mobile WMD labs, that led to erroneous intelligence updates. Now, let this distorted information be $v_{ik}(t)$- the beliefs of agencies like CIA and MI6. The belief update can be represented as –

$$x_{CIA}(t+1) = x_{CIA}(t). \sum_{k \in N_C(CIA)} w_{CIA,k}(t).(x_k(t) + v_{CIA,k}(t))$$

Where $v_{CIA,k}(t)$is the misinformation factor creating noise to the intelligence assessments.

Now, if we look into biases in perceived payoffs, the US administration believed that a sudden invasion would give them a decisive victory, with a relatively lower cost. This bias arises from the prior experience in Gulf War leading to underestimation of Iraq's resistance. Based on our model, this bias would then be –

$\hat{\pi}_{US}(invade, defend, t) = \pi_{US}(invade, defend, t) + \varepsilon_{US}(t)$ where ε is the bias perception of the expected payoff from an invasion.

Hence, the perceived benefits of invasion were higher than the actual benefits – underestimation of the post-war insurgency and the failure to find WMDs.

On the other side, Iraqi's regime under Saddam Hussein exhibited its own biases, particularly with respect to the threat posed by coalition forces. The Government seemed to have operated under a high level of distrust towards its own intelligence agencies; even limited by Hussein's distrust of his inner circle – reducing the effectiveness of the DeGroot Belief updates. Furthermore, the regime placed disproportionate weight on its own capabilities and downplayed the intelligence indicating that the coalition forces were preparing for a full-scale invasion. This belief update can be thus modeled as-

$$x_{commander}(t+1) = b_{commander}(t).\, x_{commander(t)+(1-b_{commander}t).} x_{commander}(t). \sum_{l \in N_l Commander} w_{commander,l}(t).\, x_l(t)$$

The high bias factor $b_{commander}(t)$reflects the regime's overconfidence that reduced the influence of external intelligence updates.

Conclusion and Future Scope

The exploration of advanced frameworks, particularly the integration of the DeGroot graph network with game theory, would potentially provide a robust method for understanding complex decision-making processes in warfare, particularly at the operational level. By introducing dynamic payoffs, belief updates, and endogenous biases into the model, we have offered a more comprehensive view of how misinformation, irrationality, and strategic interactions evolve over time in military conflicts. This approach not only enhances our understanding of historical conflicts but also provides valuable insights for future military strategies and conflict resolution. Our multi-level dynamic game structure with various strategies such as attack, ceasefire, and negotiation, the model allows for the exploration of optimal strategies under conditions of incomplete information. This is critical in modern warfare, where decision-makers are rarely in possession of full intelligence, and strategies must evolve as new information becomes available.

By incorporating the DeGroot update method into a dynamic game-theoretic framework, we can model how payoffs evolve over time as new information is received and processed by military units. This is particularly relevant in real-time warfare scenarios where strategies must adapt rapidly to changing circumstances. The incorporation of biases and misinformation into the belief-updating process reflects how irrationalities, political predispositions, or false intelligence reports can distort decision-making. This aligns well with real-world military actions, such as the U.S.-led invasion of Iraq in 2003, where misinformation regarding weapons of mass destruction led to flawed strategic choices (Pillar, 2006; Ricks, 2006). Additionally, the proposed model provides a more network-centric view of military decision-making, reflecting the interconnectedness of modern military units and intelligence agencies.

For instance, misinformation about Iraq's possession of weapons of mass destruction led to a flawed strategic decision to invade. Our model demonstrates how this misinformation could have been incorporated into a belief-updating framework, allowing decision-makers to adjust their strategies based on more accurate information as the conflict unfolded (Mearsheimer, 2011). Throughout the Vietnam War, the U.S. overestimated the effectiveness of its military strategies and underestimated the resolve of the North Vietnamese forces. This can be modeled as a case where endogenous biases and irrationality (such as underestimating the role of guerrilla warfare) skewed the perceived payoffs of U.S. military actions

(Stiglitz & Bilmes, 2008). The ongoing conflict between Russia and Ukraine highlights the importance of dynamic belief updates and misinformation in modern warfare. Both sides have employed propaganda and misinformation campaigns, which affect not only military strategies but also international diplomatic efforts. In such a scenario, the integration of game theory and DeGroot belief updates could help model how misinformation distorts payoffs and leads to suboptimal strategies, as well as how updated intelligence reshapes these payoffs over time (Gause, 2005).

Future Scope of Research

While the developed model offers a significant step forward in understanding military decision-making under uncertainty, several areas remain for future research. These areas of extension are critical for further refining and expanding the model's applicability:

1. Integration with Machine Learning Algorithms: One promising direction for future research is the integration of machine learning algorithms to enhance the model's ability to process vast amounts of data. In modern warfare, information flows from a wide variety of sources, including satellite imagery, social media, and electronic communications. Incorporating machine learning techniques into the belief-update mechanism could allow for more accurate predictions of enemy movements and strategies (Ricks, 2006).
2. Cognitive Biases in Decision-Making: While this paper has modeled biases mathematically, future research could explore how specific cognitive biases (such as confirmation bias or availability heuristic) affect military decision-making. This would involve a more detailed psychological modeling of the decision-makers involved in the game, and how their personal experiences, political pressures, and previous outcomes shape their strategic choices.
3. Applications to Non-Military Conflicts: Beyond traditional warfare, the model could be applied to other types of strategic conflicts, such as cyber warfare or economic warfare. In these domains, the flow of information and misinformation plays an even greater role, and the dynamic nature of payoffs and belief updates is crucial. Extending the model to non-military conflicts would provide broader applicability to areas like trade negotiations, sanctions, and cybersecurity.
4. Empirical Validation with Real-Time Data: A key next step would be the empirical validation of the model using real-time conflict data. This could involve tracking the evolution of strategies in ongoing conflicts and comparing them with the model's predictions. In particular, using real-time data from conflicts such as the ongoing war in Ukraine could provide valuable insights into the effectiveness of the model in predicting shifts in strategy based on belief updates and payoff changes (Gause, 2005).
5. Extensions to Multi-Agent Systems: In future research, the model could be extended to consider more complex multi-agent systems, where multiple actors (such as allied nations or rebel factions) interact with each other. These extensions would allow for a more comprehensive understanding of coalition dynamics, power-sharing agreements, and shifting alliances, which are common in modern conflict scenarios.
6. Incorporating Game Theory with Humanitarian Decision-Making: Lastly, another future research avenue would be incorporating humanitarian decision-making into the game-theoretic model. This is particularly relevant in the context of ceasefires, temporary truces, and humanitarian corridors, which were modeled as second-level strategies in this paper. Understanding how military decisions interact with humanitarian concerns, such as civilian safety or access to resources, would provide a more holistic approach to conflict resolution.

In conclusion, the integration of game theory, graph theory, and the DeGroot model in this paper provides a sophisticated and practical framework for understanding the complex decision-making processes that characterize modern warfare. By accounting for dynamic payoffs, biases, and misinformation, this model offers a more realistic approach to predicting and analyzing military strategies. Future research can further refine this model by incorporating machine learning techniques, exploring cognitive biases, and validating the framework with real-time conflict data. The potential applications of this model extend beyond military conflicts to other strategic domains, making it a valuable tool for decision-makers in a wide range of scenarios.

Footnotes:

1. It was evident in Gulf war, how misleading intelligence led to flawed decision making and resulted in costly military engagements.
2. Overconfidence is a primary factor enhancing bias that can lead to aggressive stance and miscalculations (see Maoz 1990).
3. Normalization ensures the sum of each row of the matrix to be 1.
4. In DeGroot model, beliefs are updated linearly, and generally do not depend directly on strategies unless they influence the trust score. In the RHS, it is observed that the belief terms are cancelled out.

Appendix 1: Applying Bayesian Game in Missile Defense system and deterrence

Countries always face a decision regarding their investment in missile defense – engage in preemptive strikes, or rely on deterrence. This happens mostly without the complete knowledge of the opposition's missile strengths, even the willingness to retaliate. In this context –

- Each nation's true missile defense potential is a private information, denoted as θ_i
- Each has three potential strategic options: invest in missile, launch a preemptive strike, or depend on deterrence. Each countries belief about the other's true missile potential is given by the probability distribution $P\left(\theta_j\middle|\theta_i\right)$
- Payoff depends on whether a nation's defense can successfully intercept a missile, with the associated cost of preemptive strike and the potential risk of retaliation.

Assume $\theta_A \epsilon \{Weak, Strong\}$ and $\theta_B \in \{Weak, Strong\}$, i.e. Country A's and B's missile defense capacities respectively. Let $P_A(\theta_B|\theta_A)$ be Country A's belief about B, given A's type and similarly, $P_B(\theta_A|\theta_B)$ be Country B's belief about A, give B's type.

Utility Functions for A and B with assumed payoffs:

$$U_A(a_A, a_B;\ \theta_A, \theta_B) = \begin{cases} 10\ if\ a_A = D_A, a_B = D_B \\ 5\ if\ a_A = D_A, a_B = S_B\ ;\ \theta_A = Strong \\ -10\ if\ a_A = D_A, a_B = S_B\ ;\ \theta_A = Weak \\ 20\ if\ a_A = S_A, a_B = D_B\ ;\ \theta_A = Weak \\ -20\ if\ a_A = S_A, a_B = D_B\ ;\ \theta_B = Strong \\ 0\ if\ a_A = S_A, a_B = S_B \end{cases}$$

$$U_B(a_A, a_B;\ \theta_A, \theta_B) = \begin{cases} 10\ if\ a_A = D_A, a_B = D_B \\ 5\ if\ a_A = S_A, a_B = D_B\ ;\ \theta_B = Strong \\ -10\ if\ a_A = S_A, a_B = D_B\ ;\ \theta_B = Weak \\ 20\ if\ a_A = D_A, a_B = S_B\ ;\ \theta_A = Weak \\ -20\ if\ a_A = D_A, a_B = S_B\ ;\ \theta_A = Strong \\ 0\ if\ a_A = S_A, a_B = S_B \end{cases}$$

Positive payoffs represent favorable outcomes (eg. Successful defense), while negative payoffs represent unfavorable outcomes (eg. launching an unsuccessful strike).

Belief Updates ad Expected Utility

Assume Country A holds a belief $P_A(\theta_B|\theta_A)$. This belief is updated as the game advances, Based on observed actions as:

- Belief for Country A:

$$P_A(\theta_B = Weak|\theta_A) = p\ \&\ P_A(\theta_B = Strong|\theta_A) = 1 - p$$

- Belief for Country B:

$$P_B(\theta_A = Weak|\theta_B) = q\ \&\ P_B(\theta_A = Strong|\theta_B) = 1 - q$$

The expected utility for Country A, given its belief about Country B's type is given as:

$$EU_A(a_A|\theta_A) = \sum_{\theta_B \epsilon (weak, strog} P_A(\theta_B|\theta_A) U_A(a_A, a_B; \theta_A, \theta_B)$$

Similarly, the expected utility for Country B is:

$$EU_B(a_B|\theta_B) = \sum_{\theta_A \epsilon (weak, strog} P_B(\theta_A|\theta_B) U_B(a_A, a_B; \theta_A, \theta_B)$$

Bayesian Nash Equilibrium:

Optimal Strategy for Country A:

1. If $EU_A(D_A|\theta_A) > EU_A(S_A|\theta_A)$, then the country should invest in missile.
2. If $EU_A(S_A|\theta_A) > EU_A(D_A|\theta_A)$, then the country should go for preemptive strike.

Optimal Strategy for Country B:

1. If $EU_B(D_B|\theta_B) > EU_B(S_B|\theta_B)$, then the country should invest in missile.
2. If $EU_B(S_B|\theta_B) > EU_B(D_B|\theta_B)$, then the country should go for preemptive strike.

Now let us assume the following probabilities & utility values:

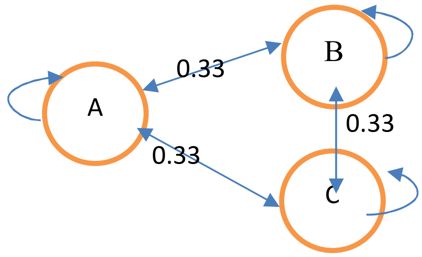
$$P_A(\theta_B = Weak|\theta_A) = 0.6\ \&\ P_A(\theta_B = Strong|\theta_A) = 0.4$$

$$P_B(\theta_A = Weak|\theta_B) = 0.7\ \&\ P_B(\theta_A = Strong|\theta_B) = 0.3$$

Hence, $EU_A(D_A|\theta_A)$ will be $0.6 \times (-10) + 0.4 \times 5 = -4$ [if A invests in missiles], and $EU_A(S_A|\theta_A)$ will be $0.6 \times 20 + 0.4 \times (-20) = 4$ [if A launches preemptive strikes]. Now since, $EU_A(S_A|\theta_A) > EU_A(D_A|\theta_A)$, Country A should go for preemptive strike. Similarly, we can calculate the optimal strategy for Country B.

Appendix 2: Graph Theory Interpretation

The diagram below shows a directed graph with odes being Country A, B and C. The initial influence weights are assumed to be equal to 0.33.



We represent the system as a directed graph $G(V,E)$, where $V = \{A, \mathrm{B}, \mathrm{C}\}$ are the vertices ad $E \subseteq V \times V$ is the set of directed edges representing the influence relations. The weight w_{ij}^{t}at each edge $(i,j) \in E$ represents the degree of trust of country 'i' o the information of 'j'. At time $t = 0$, the influence matrix $W(0)$ is given as:

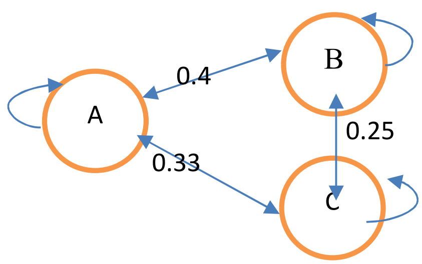
$$W(0) = \begin{bmatrix} w_{AA}(0) & w_{AB}(0) & w_{AC}(0) \\ w_{BA}(0) & w_{BB}(0) & w_{BC}(0) \\ w_{CA}(0) & w_{CB}(0) & w_{CC}(0) \end{bmatrix} = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}$$

The weights evolve based on the accuracy of information. For example, after the first round, if Country B provides more accurate intelligence, its influence weight on country A will rise; while country C's influence weight will fall. Mathematically, this can be represented as:

$w_{ij}(t+1) = {\delta_{ij}(t)}\Big/{\sum_{K \epsilon \mathrm{N(i)}} \delta_{ik}(t)}$ where $\delta_{ij}(t)$is the trust score that Country i assigns to Country j at time t. Therefore, after the first round, the weight matrix will be:

$$W(0) = \begin{bmatrix} 0.371 & 0.483 & 0.146 \\ 0.4 & 0.35 & 0.25 \\ 0.33 & 0.25 & 0.42 \end{bmatrix}$$

Graphically, it can be represented as:



The dynamic updation process reflects the real-world evolution of trust and influence during conflicts.

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