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SOME NEW DEVELOPMENTS IN CHAIN TYPE ESTIMATORS FOR ESTIMATING POPULATION MEAN IN CLUSTER SAMPLING

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Abstract: In this paper we suggest a ratio type exponential estimator, a chain ratio-type estimator, a chain ratio-type exponential estimator and a chain ratio-ratio-type exponential estimator in cluster sampling. Further we consider a generalized class of chain ratio-ratio-type exponential estimator. The properties of the suggested class of estimators are studied under large sample approximation. Conditions are obtained under which these estimators are more efficient than the usual unbiased estimators.

Keywords: Main and Subsidiary variable, Bias, MSE, Efficiency, Cluster sampling.
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1. INTRODUCTION

In random sampling, it is presumed that the population has been divided into a finite number of distinct and identifiable units defined as sampling units. The smallest units into which the population can be divided is called an element of the population. A group of such elements is known as a cluster. When the sampling unit is a cluster, the procedure is called cluster sampling. Usually the cluster sampling is extensively used in large scale sample surveys because it not only reduces the cost of the surveys, but also ensures that the investigator can have a longer sample size than if he or she was using the technique of simple random sampling (*SRS*).

Besides, it may also be used when it is either difficult or costly or empirical to prepare sampling frame of the units, to select a sample. It is well known that the use of auxiliary information improves the efficiency of the estimates of the population parameter. Ratio and regression estimators are good examples in this context. In practice, a ratio estimator is commonly used when the study variable y is highly correlated with the auxiliary variable x. When the population mean $\overline{X}$ of the auxiliary variable x is known, a number of modified versions of ratio estimators have already been suggested by various authors, Bahl and Tuteja [1], Kadilar and Cingi [2], Singh [3], Singh et al.[6,7], Solanki et al.[8] etc.

Further, many authors such as Singh et al. [9], Sisodia and Dwivedi [10], Upadhyaya and Singh [12], and others have used some population parameters of the auxiliary variable to improve the precision of ratio estimators. In addition to these studies, Kadilar and Cingi [2] developed a chain ratio estimator along with its properties in

SRS. We consider a finite population of size N, cluster each of size M from which a sample of size n cluster is drawn according to the simple random sampling without replacement. Consider a finite population U of size N identifiable distinct unit $(U_1, U_2, ..., U_n)$. It is assumed that the study variable y and the auxiliary variable x defined on U.

Let (y_{ij}, x_{ij}) be the values of the (study, auxiliary) variables respectively on the i^{th} cluster $(i = 1, 2, \ldots, n)$ and j^{th} unit $(j = 1, 2, \ldots, n)$.

The following notations will be used throughout this work:

$y_i = \sum_{j=1}^{M} y_{ij}$: total of all the units in the cluster of y,

$\overline{Y}_i = \frac{1}{N} \sum_{i=1}^{N} y_i$: the population mean of the i^{th} cluster,

$\overline{Y} = \overline{\overline{Y}} = \frac{1}{M} \overline{Y}_j = \frac{1}{NM} \sum_{i=1}^{N} y_i = \frac{1}{NM} \sum_{i=1}^{N} \sum_{j=1}^{M} y_{ij}$: the population mean of y,

$\overline{Y}_j = \frac{1}{M} \sum_{j=1}^{M} y_{ij}$: the mean per element of y of the i^{th} cluster,

$\overline{\overline{Y}} = \frac{1}{N} \sum_{i=1}^{N} \overline{Y}_{i.}$: the mean of cluster means of y whole population,

$S_y^2 = \frac{1}{(NM-1)} \sum_{i=1}^{N} \sum_{j=1}^{M} (y_{ij} - \overline{\overline{Y}})^2$: the population mean square of y,

$S_x^2 = \frac{1}{(NM-1)} \sum_{i=1}^{N} \sum_{j=1}^{M} (x_{ij} - \overline{\overline{X}})^2$: the population mean square of x,

$S_{yi}^2 = \frac{1}{(M-1)} \sum_{j=1}^{M} (y_{ij} - \overline{Y}_i)^2$: the mean square of y between elements in the i^{th} cluster,

$S_{xi}^2 = \frac{1}{(M-1)} \sum_{j=1}^{M} (x_{ij} - \overline{X}_i)^2$: the mean square of x between elements in the i^{th} cluster,

$S_{yw}^2 = \frac{1}{N} \sum_{i=1}^{N} S_{yi}^2$: the mean square of y within clusters,

$S_{xw}^2 = \frac{1}{N} \sum_{i=1}^{N} S_{xi}^2$: the mean square of x within cluster,

$S_{yb}^2 = \frac{1}{(N-1)} \sum_{i=1}^{N} (\overline{Y}_{i.} - \overline{\overline{Y}})$: the mean square of y between cluster means,

$S_{xb}^2 = \frac{1}{(N-1)} \sum_{i=1}^{N} (\overline{X}_i - \overline{\overline{X}})^2$: the mean square of x between cluster means,

$x_i = \sum_{j=1}^{M} x_{ij}$: total of all the units in the cluster of x,

$\overline{X}_{.j} = \frac{1}{N} \sum_{i=1}^{N} x_i = \frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{M} x_{ij}$,

$\overline{X} = \overline{\overline{X}} = \frac{1}{NM}\sum_{i=1}^{N}\sum_{j=1}^{M} x_{ij}$: the population mean of x ,

$\overline{X}_i = \frac{1}{M}\sum_{j=1}^{M} x_{ij}$: the mean per element of x of the i^{th} cluster,

$\overline{\overline{X}} = \frac{1}{N}\sum_{i=1}^{N}\overline{X}_{i.}$: the mean of cluster means of x whole population,

$S_{xy} = \frac{1}{(NM-1)}\sum_{i=1}^{N}\sum_{j=1}^{M}(y_{ij}-\overline{\overline{Y}})(x_{ij}-\overline{\overline{X}})$: the covariance between x and y .

Let a simple random sample without replacement (*SRSWOR*) of size *n* (clusters) be drawn from finite population U of size N (clusters).

$\overline{Y}_{.j} = \frac{1}{N}\sum_{j=1}^{n} y_i$, $y_j = \sum_{j=1}^{M} y_{ij}$,

$\overline{y} = \frac{1}{M}\overline{Y}_{.j} = \frac{1}{nM}\sum_{i=1}^{N}\sum_{j=1}^{M} y_{ij}$: the mean of cluster means of y in the sample,

$\overline{x}_j = \frac{1}{n}\sum_{j=1}^{n} x_i$, $x_i = \sum_{j=1}^{m} x_{ij}$,

$\overline{\overline{x}} = \frac{1}{M}\overline{x}_{.j} = \frac{1}{nm}\sum_{i=1}^{n}\sum_{j=1}^{m} x_{ij}$: the mean of cluster means of x in the sample.

We note that the sample means $(\overline{\overline{x}}, \overline{\overline{y}},)$ are unbiased estimates of population means $\overline{Y} = \overline{\overline{Y}}, \overline{X} = \overline{\overline{X}}$ respectively.

$f = \frac{n}{N}, \gamma = \frac{(1-f)}{n}\frac{(NM-1)}{M^2(N-1)}$,

$\rho_y = \frac{\sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{k\neq j}^{M}(y_{ij}-\overline{\overline{Y}})(y_{ik}-\overline{\overline{Y}})}{(M-1)(NM-1)S_y^2}$ is the intra class correlation coefficient between pair of units of *y*,

$\rho_x = \frac{\sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{k\neq j}^{M}(x_{ij}-\overline{\overline{X}})(x_{ik}-\overline{\overline{X}})}{(M-1)(NM-1)S_x^2}$ is the intra class correlation coefficient between pair of units of *x*,

$\rho_{xy} = \frac{\sum_{i=1}^{N}\sum_{j=1}^{M}(x_{ij}-\overline{\overline{X}})(y_{ij}-\overline{\overline{X}})}{(NM-1)S_xS_y}$ is the population correlation coefficient between x and y ,

$\rho'_{xy} = \frac{\sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{k\neq j}^{M}(y_{ij}-\overline{\overline{Y}})(x_{ik}-\overline{\overline{X}})}{(M-1)(NM-1)S_xS_y}$ is the intra class correlation coefficient between x and y .

Further, we have

$$Var(\overline{\overline{y}}) = \gamma S_y^2\{1+(M-1)\rho_y\} = \gamma\overline{Y}^2C_y^2\{1+(M-1)\rho_y\}. \quad (1.1)$$

Similarly

$$MSE(\overline{\overline{x}}) = Var(\overline{\overline{y}}) = \gamma S_y^2\{1+(M-1)\rho_x\} \quad (1.2)$$

and

$$Cov\ (\bar{\bar{y}},\bar{\bar{x}})=\gamma S_x S_y\{\rho_{xy}+(M-1)\rho'_{xy}\}. \tag{1.3}$$

1.1. Ratio estimator in cluster sampling

The usual ratio estimator for population's mean $\bar{\bar{Y}}$ of the study variable y in cluster sampling is defined by

$$\bar{y}_{RC}=\left(\frac{\bar{\bar{y}}}{\bar{\bar{x}}}\right)\bar{X}, \tag{1.4}$$

where the population mean $\bar{X}$ of the auxiliary variable x is know in advance.

To the first degree of approximation (FDA), the bias and mean squared error (MSE) of the ratio estimator $\bar{y}_{RC}$ are respectively given by

$$B(\bar{y}_{RC})=\gamma\bar{\bar{Y}}C_y^2\{1+(M-1)\rho_x\}\left[1-\left\{\frac{\rho_{xy}+(M-1)\rho'_{xy}}{\rho_{xy}+(M-1)\rho_{xy}}\right\}\frac{C_y}{C_x}\right],$$

$$=\gamma\bar{\bar{Y}}C_y^2\{1+(M-1)\rho_x\}(1-k), \tag{1.5}$$

$$MSE(\bar{y}_{RC})=\gamma\bar{\bar{Y}}^2[C_y^2\{1+(M-1)\rho_y\}+C_x^2\{1+(M-1)\rho_x\}-2\{\rho_{xy}+(M-1)\rho'_{xy}\}C_yC_x]$$

$$=\gamma\bar{\bar{Y}}^2[C_y^2\{1+(M-1)\rho_y\}+C_x^2\{1+(M-1)\rho_x\}(1-2k)]. \tag{1.6}$$

where $k=\left[1-\left\{\frac{\rho_{xy}+(M-1)\rho'_{xy}}{\rho_{xy}+(M-1)\rho_{xy}}\right\}\frac{C_y}{C_x}\right], C_y=\frac{S_y}{\bar{\bar{Y}}}$ and $C_x=\frac{S_x}{\bar{\bar{X}}}$.

From (1.1) and (1.3) we have

$$MSE(\bar{\bar{Y}})-MSE(\bar{y}_{RC})=-\gamma\bar{\bar{Y}}^2[C_x^2\{1+(M-1)\rho_x\}(1-2k)]>0 \text{ if}$$

$$k>\frac{1}{2},\ \rho_x>-\frac{1}{M-1} \tag{1.7}$$

The ratio estimator $\bar{y}_{RC}$ is more efficient than the usual unbiased estimator $\bar{\bar{Y}}$ as long as the condition $k>(1/2)$ is satisfied.

In this paper we have suggested ratio type exponential estimator, chain ratio-type estimator, chain ratio-type exponential estimator and chain-ratio-ratio-type exponential estimator in cluster sampling. Further a generalized class of chain ratio-ratio-type exponential estimators has been considered. Properties of the suggested class of estimators have been studied under large sample approximation. Conditions are obtained under which the suggested estimators are more efficient than the usual unbiased estimator and ratio estimator.

2. SOME ESTIMATORS BASED ON AUXILIARY INFORMATION IN CLUSTER SAMPLING

2.1. Ratio-type exponential estimator

Motivated by the work of Bahl and Tuteja [1] we define a ratio-type exponential estimator for population mean $\bar{Y}$ as

$$d_1=\bar{\bar{y}}\exp\left(\frac{\bar{\bar{X}}-\bar{\bar{x}}}{\bar{\bar{X}}+\bar{\bar{x}}}\right). \tag{2.1}$$

To obtain the bias and mean squared error (MSE) of the estimator d_1, we write

$\bar{\bar{y}}=\bar{\bar{Y}}(1+e_y)$, $\bar{\bar{x}}=\bar{\bar{X}}(1+e_x)$, such that $E(e_y) = E(e_x) = 0$

and $E(e_y^2) = \gamma C_y^2 \{1 + (M-1)\rho_y\}, E(e_x^2) = \gamma C_x^2 \{1 + (M-1)\rho_x\}$

$E(e_y e_x) = \gamma C_x C_y \{\rho_{xy} + (M-1)\rho'_{xy}\}.$

Putting $\bar{\bar{y}} = \bar{\bar{Y}}(1+e_y)$ and $\bar{\bar{x}} = \bar{\bar{X}}(1+e_x)$ in (2.1) we have

$$d_1 = \bar{\bar{Y}}(1+e_y)\exp\left\{\frac{-e_x}{2}\left(1+\frac{e_x}{2}\right)^{-1}\right\}. \tag{2.2}$$

Expanding the right hand side of (2.2) and multiplying out we have

$$d_1 = \bar{\bar{Y}}(1+e_y)\left[1 - \frac{e_x}{2}\left(1+\frac{e_x}{2}\right)^{-1} + \frac{e_x^2}{8}\left(1+\frac{e_x}{2}\right)^{-2} - \ldots\right]$$

$$= \bar{\bar{Y}}\left[1 + e_y - \frac{e_x}{2} - \frac{e_x e_y}{2} + \frac{3e_x^2}{8} + \frac{3e_y e_x^2}{8} - \ldots\right]$$

Ignoring the terms of e 's having power greater than two we have

$$d_1 \cong \bar{\bar{Y}}\left[1 + e_y - \frac{e_x}{2} - \frac{e_x e_y}{2} + \frac{3e_x^2}{8}\right]$$

or

$$(d_1 - \bar{\bar{Y}}) \cong \bar{\bar{Y}}\left[e_y - \frac{e_x}{2} + 3e_x^2 - \frac{e_x e_y}{2}\right]. \tag{2.3}$$

So bias of d_1 to the first degree of approximation is

$$B(d_1) = \gamma(\bar{\bar{Y}}/8)C_y^2\{1 + (M-1)\rho_x\}(3-4k). \tag{2.4}$$

Squaring both side of (2.3) and ignoring terms of e 's having power greater than two we have

$$(d_1 - \bar{\bar{Y}})^2 \cong \bar{\bar{Y}}^2\left[e_y^2 - e_x e_y + \frac{e_x^2}{4}\right], \tag{2.5}$$

Taking the expectation of both side of (2.5) we get the MSE of d_1 to the FDA as

$$MSE(d_1) = \gamma\bar{\bar{Y}}^2\left[C_y^2\{1 + (M-1)\rho_y\} + \frac{C_x^2\{1 + (M-1)\rho_x\}}{4}(1-4k)\right], \tag{2.6}$$

2.2. Bias Comparisons of $\bar{y}_{RC}$ and d_1

From (1.5) and (2.4) we have $|B(d_1)| < |B(\bar{y}_{RC})|$ if

$$48k^2 - 104k + 55 > 0. \tag{2.7}$$

Thus proposed ratio-type exponential estimator d_1 is absolutely less biased than the ratio estimator $\bar{y}_{RC}$ as long as the condition (2.7) is satisfied and the intra class correlation coefficient $\rho_x > \frac{-1}{M-1}$.

2.3. MSE comparison of $\bar{\bar{y}}$ and $\bar{y}_{RC}$

From (1.1) and (1.6) we have

$$MSE(\bar{\bar{Y}}) - MSE(\bar{y}_{RC}) = \gamma\left\{\frac{C_x^2\{1+(M-1)\rho_x\}}{4}\right\}(4k-1) > 0 \text{ if}$$

$$k > \frac{1}{4}, \rho_x > \frac{-1}{M-1}. \qquad (2.8)$$

Thus d_1 is more efficient than usual unbiased estimator $\bar{\bar{y}}$ as long as the condition (2.8) is satisfied.

From (1.7) and (2.6) we have $MSE(d_1) < MSE(\bar{Y}_{RC})$ if

$$(1-k) < 4(1-2k),\ \rho_x > \frac{1}{M-1}$$

i.e. if

$$k < \frac{3}{4}, \rho_x > -\frac{1}{M-1}. \qquad (2.9)$$

So the proposed ratio-type exponential estimator d_1 is more efficient than ratio estimator $\bar{y}_{RC}$ if the condition (2.9) is satisfied.

Combining the inequalities (2.8) and (2.9) we get that the ratio-type exponential estimator d_1 is more efficient than $\bar{\bar{y}}$ and $\bar{y}_{RC}$ if the

$$\frac{1}{4} < k < \frac{3}{4}, \rho_x > -\frac{1}{M-1}\ , \qquad (2.10)$$

holds good.

2.4. Estimator based on square root transformation

Motivated by Swain [11] we define the following ratio type estimator for $\bar{\bar{Y}}$ in cluster sampling as

$$d_2 = \bar{\bar{y}}\left(\frac{\bar{\bar{X}}}{\bar{\bar{x}}}\right)^{\frac{1}{2}} \qquad (2.11)$$

Putting $\bar{\bar{y}} = \bar{\bar{Y}}(1+e_y)$ and $\bar{\bar{x}} = \bar{\bar{X}}(1+e_x)$ in (2.11) we have

$$d_2 = \bar{\bar{y}}(1+e_y)(1+e_x)^{\frac{-1}{2}}$$

Now expanding the right hand size of (2.11) and multiplying out we have

$$d_2 = \bar{\bar{Y}}\left(1+e_y - \frac{1}{2}e_x - \frac{1}{2}e_xe_y + \frac{3}{8}e_x^2 + \frac{3}{8}e_ye_x^2 - \ldots\right).$$

Ignoring $e^{\upsilon}(\upsilon > 2)$ we have

$$(d_2 - \bar{\bar{Y}}) = \bar{\bar{Y}}\left[e_y - \frac{1}{2}e_x - \frac{1}{2}e_xe_y + \frac{3}{8}e_x^2\right]. \qquad (2.12)$$

Taking expectation of both sides of (2.12) we get the bias of d_2 to the FDA as

$$B(d_2) = \frac{\gamma\bar{\bar{Y}}^2C_x^2\{1+(M-1)\rho_x\}}{8}(3-4k), \qquad (2.13)$$

$= B(d_1)$ (from (2.4)) .

Squaring both sides of (2.12) and neglecting terms of e 's having power greater than two we have

$$(d_2 - \bar{\bar{Y}})^2 = \bar{\bar{Y}}^2\left[e_y^2 - e_x e_y + \frac{1}{4}e_x^2\right] \tag{2.14}$$

So the MSE of d_2 is given as

$$MSE(d_2) = \gamma\bar{\bar{Y}}^2\left[C_y^2\{1+(M-1)\rho_y\} + \frac{C_x^2\{1+(M-1)\rho_x\}}{4}(1-4k)\right] \tag{2.15}$$

$= MSE(d_1)$ (from (2.6)).

It can be easily shown from (1.1), (1.6), and (2.15) that d_2 is more efficient than:

(i) $\bar{\bar{y}}$ if $k > \frac{1}{4}, \rho_x > \frac{-1}{M-1}$.

(ii) $\bar{y}_{RC}$, if , $k > \frac{3}{4}$, $\rho_x > \frac{-1}{M-1}$.

(iii) $\bar{\bar{y}}$ and $\bar{y}_{RC}$ if $\frac{1}{4} < k < \frac{3}{4}$, $\rho_x > \frac{-1}{M-1}$.

Further from (2.6) and (2.13) it is concluded that both the estimators d_1 and d_2 are equally efficient at least up to the first degree of approximation.

2.5. Chain ratio type estimator

Replacing $\bar{\bar{y}}$ by $\bar{y}_{RC}$ in (1.4) we get the chain ratio-type estimator for population mean $\bar{\bar{Y}}$ as

$$\bar{y}_{CRC} = \bar{y}_{RC}\left(\frac{\bar{\bar{X}}}{\bar{\bar{x}}}\right)$$

$$= \bar{\bar{y}}\left(\frac{\bar{\bar{X}}}{\bar{\bar{x}}}\right)\left(\frac{\bar{\bar{X}}}{\bar{\bar{x}}}\right) = \bar{\bar{y}}\left(\frac{\bar{\bar{X}}^2}{\bar{\bar{x}}^2}\right). \tag{2.16}$$

Writing (2.16) in terms of e 's we have

$$\bar{y}_{CRC} = \bar{\bar{Y}}(1+e_y)(1+e_x)^{-2}$$

$$= \bar{\bar{Y}}(1+e_y)(1-2e_x+3e_x^2-...)$$

$$= \bar{\bar{Y}}(1+e_y-2e_x-2e_xe_y+3e_x^2+3e_ye_x^2...)$$

which is approximated as

$$\bar{y}_{CRC} \cong \bar{\bar{Y}}(1+e_y-2e_x-2e_xe_y+3e_x^2)$$

or

$$(\bar{y}_{CRC} - \bar{\bar{Y}}) \cong \bar{\bar{Y}}(e_y-2e_x-2e_xe_y+3e_x^2) \tag{2.17}$$

So that bias of $\bar{y}_{CRC}$ to the FDA is given by

$$B(\bar{y}_{CRC}) = \gamma\bar{\bar{Y}}C_x^2\{1+(M-1)\rho_x\}(3-2k) . \tag{2.18}$$

The approximate value of $(\bar{y}_{CRC} - \bar{\bar{Y}})^2$ given by

$$(\bar{y}_{CRC}-\bar{\bar{Y}})^2=\bar{Y}^2\left(e_y^2-4e_xe_y+4e_x^2\right). \quad (2.19)$$

So the MSE of $\bar{y}_{CRC}$ to the FDA is given by

$$MSE(\bar{y}_{CRC})=\gamma\bar{\bar{Y}}^2[C_y^2\{1+(M-1)\rho_y\}+4C_x^2\{1+(M-1)\rho_x\}(1-k)]. \quad (2.20)$$

From (1.3) and (2.20) it follows that

$|B(\bar{y}_{CRC})|<|B(\bar{y}_{RC})|$ if

$$\frac{4}{3}<k<2, \quad (2.21)$$

Thus $\bar{y}_{CRC}$ is absolutely less biased than $\bar{y}_{RC}$ if the condition (2.4) is satisfied.

From (2.4) and (2.21) we have

$|B(\bar{y}_{CRC})|<|B(d_1)|$ if

$$|(3-2k)|<\frac{1}{8}|(3-4k)|,\ \rho_x>\frac{-1}{M-1}. \quad (2.22)$$

2.6. Efficiency compression

From (1.1) and (1.6) we have $MSE(\bar{y}_{CRC})<MSE(\bar{\bar{y}})$ if

$$k>0,\ \rho_x>\frac{-1}{M-1}. \quad (2.23)$$

From (1.6) and (2.20) we have

$MSE(\bar{y}_{CRC})<MSE(\bar{y}_{RC})$ if

$$k>(3/2)=1.5. \quad (2.24)$$

Further from (2.6) and (2.20) we have

$MSE(\bar{y}_{CRC})<MSE(d_1)$ if

$$k>(5/2)=1.25, \quad (2.25)$$

From (2.23), (2.24) and (2.25) we note that the sufficient condition for the proposed chain ratio-type estimator $\bar{y}_{CRC}$ to be more efficient than $\bar{\bar{y}}$, $\bar{y}_{RC}$ and d_1 is that $k>1.50$.

2.7. Chain ratio-type exponential estimator

Replacing $\bar{\bar{y}}$ by d_1 in (2.1) we get a chain ratio-type exponential estimator for $\bar{\bar{Y}}$ as

$$d_{C1e}=\bar{\bar{Y}}\exp\left\{\frac{2\left(\bar{\bar{X}}-\bar{\bar{x}}\right)}{\left(\bar{\bar{X}}+\bar{\bar{x}}\right)}\right\}. \quad (2.26)$$

Putting $\bar{\bar{y}}=\bar{\bar{Y}}(1+e_y)$ and $\bar{\bar{x}}=\bar{\bar{X}}(1+e_x)$ in (2.26) we have

$$d_{C1e}=\bar{\bar{Y}}(1+e_y)\exp\left\{\frac{2(\bar{X}-\bar{X}-\bar{X}e_x)}{(\bar{X}+\bar{X}+\bar{X}e_x)}\right\}$$

$$=\bar{\bar{Y}}(1+e_y)\left[1-e_x\left(1+\frac{e_x}{2}\right)^{-1}+\frac{e_x^2}{2}\left(1+\frac{e_x}{2}\right)^{-2}-\ldots\right]$$

$$= \bar{\bar{Y}}(1 + e_y - e_x - e_x e_y + e_x^2 + e_y e_x^2 - \dots)$$

Ignoring $e^{\upsilon}(\upsilon > 2)$ we have

$$d_{C1e} \cong \bar{\bar{Y}}(1 + e_y - e_x - e_x e_y + e_x^2)$$

or

$$(d_{C1e} - \bar{\bar{Y}}) \cong \bar{\bar{Y}}(e_y - e_x - e_x e_y + e_x^2) \quad (2.27)$$

So the bias of d_{C1e} to the FDA is given by

$$B(d_{C1e}) = \gamma \bar{\bar{Y}} C_x^2 \{1 + (M-1)\rho_x\}(1-k) \quad (2.28)$$

$$= B(\bar{y}_{RC})$$

The approximate value $\left(d_{Cle} - \bar{\bar{Y}}\right)^2$ is given by

$$(d_{C1e} - \bar{\bar{Y}})^2 = \bar{\bar{Y}}^2(e_y^2 - 2e_x e_y + e_x^2) \quad (2.29)$$

So the *MSE* of d_{C1e} to the FDA is given by

$$MSE(d_{C1e}) = \gamma \bar{\bar{Y}}^2 [C_y^2 \{1 + (M-1)\rho_y\} + C_x^2 \{1 + (M-1)\rho_x\}(1-2k)]$$

$$= MSE(\bar{y}_{RC}) \quad (2.30)$$

It is observed from (2.28) and (2.30) that the bias and MSE of d_{C1e} are the same as that of $\bar{y}_{RC}$. Thus $\bar{y}_{CRC}$ is less biased and more efficient than d_{C1e} if the conditions (2.22) and (2.24) are respectively satisfied.

2.8. Chain ratio-ratio-type exponential estimator

If we replace $\bar{\bar{y}}$ in (2.1) by

$$\bar{y}_{RC} = \bar{\bar{y}}\left(\frac{\bar{\bar{X}}}{\bar{x}}\right),$$

then we obtain a chain ratio-ratio-type exponential estimator for $\bar{\bar{Y}}$ as

$$\bar{\bar{y}}_{CR\mathrm{Re}} = \bar{y}_{RC} \exp\left(\frac{\bar{\bar{X}} - \bar{\bar{x}}}{\bar{\bar{X}} + \bar{\bar{x}}}\right)$$

$$= \bar{\bar{y}}\left(\frac{\bar{\bar{X}}}{\bar{\bar{x}}}\right) \exp\left(\frac{\bar{\bar{X}} - \bar{\bar{x}}}{\bar{\bar{X}} + \bar{\bar{x}}}\right) \quad (2.31)$$

Writing (2.31) in terms of e 's we have

$$\bar{\bar{y}}_{CR\mathrm{Re}} = \bar{\bar{Y}}(1 + e_y)(1 + e_x)^{-1} \exp\left\{\frac{-e_x}{2 + e_x}\right\}$$

$$= \bar{\bar{Y}}(1 + e_y)(1 + e_x)^{-1}\left[1 - \frac{e_x}{2}\left(1 + \frac{e_x}{2}\right)^{-1} + \frac{e_x^2}{8}\left(1 + \frac{e_x}{2}\right)^{-2} - \dots\right]$$

$$= \bar{\bar{Y}}\left(1 + e_y - \frac{3}{2}e_x - \frac{3}{2}e_x e_y + \frac{15}{8}e_x^2 + \frac{11}{8}e_y e_x^2 - \dots\right)$$

Ignoring $e^{\upsilon}(\upsilon > 2)$ in the above expression we have

$$\bar{\bar{y}}_{CR\,\mathrm{Re}} \cong \bar{\bar{Y}}\left(1+e_y-\frac{3}{2}e_x-\frac{3}{2}e_xe_y+\frac{15}{8}e_x^2\right)$$

or

$$\left(\bar{\bar{y}}_{CR\,\mathrm{Re}}-\bar{\bar{Y}}\right)\cong \bar{\bar{Y}}\left(e_y-\frac{3}{2}e_x-\frac{3}{2}e_xe_y+\frac{15}{8}e_x^2\right) \tag{2.32}$$

So the bias of $\bar{\bar{y}}_{CR\,\mathrm{Re}}$ to the FDA is given by

$$B\left(\bar{\bar{y}}_{CR\,\mathrm{Re}}\right)=\frac{3}{8}\gamma\bar{\bar{Y}}C_x^2\{1+(M-1)\rho_x\}(5-4k) \tag{2.33}$$

The approximate value of $(\bar{y}_{CR\,\mathrm{Re}}-\bar{\bar{Y}})^2$ is given by

$$(\bar{y}_{CR\,\mathrm{Re}}-\bar{\bar{Y}})^2 \cong \bar{\bar{Y}}^2\left[e_y^2-3e_xe_y+\frac{9}{4}e_x^2\right] \tag{2.34}$$

So the MSE of $\bar{y}_{CR\,\mathrm{Re}}$ to the FDA is given by

$$MSE(\bar{y}_{CR\,\mathrm{Re}})=\gamma\bar{\bar{Y}}^2\left[C_y^2\{1+(M-1)\rho_y\}+\frac{3}{4}C_y^2\{1+(M-1)\rho_x\}(3-4k)\right]. \tag{2.35}$$

2.9. Efficiency comparison

From (1.1) and (2.35) we have

$$MSE(\bar{\bar{y}}_{CR\,\mathrm{Re}})-MSE(\bar{\bar{y}})=\frac{3\gamma\bar{\bar{Y}}^2}{4}C_x^2\{1+(M-1)\rho_x\}(3-4k)<0 \text{ if}$$

$$(3-4k)<0,\ \rho_x>\frac{-1}{M-1} \text{ if}$$

$$k>\frac{3}{4}=0.75,\ \rho_x>\frac{-1}{M-1}. \tag{2.36}$$

From(1.6),(2.30) and(2.35) we have

$$MSE(\bar{\bar{y}}_{CR\,\mathrm{Re}})-MSE(\bar{\bar{y}}_{RC})=\gamma\bar{\bar{Y}}^2C_x^2\{1+(M-1)\rho_x\}\left(\frac{9}{4}-3k-1+2k\right)$$

$$=2\gamma\bar{\bar{Y}}^2C_x^2\{1+(M-1)\rho_x\}\left(\frac{5}{4}-k\right)$$

which is negative if

$$\left(\frac{5}{4}-k\right)<0,\ \rho_x>\frac{-1}{M-1} \text{ i.e. if}$$

$$k>\frac{5}{4}=1.25,\ \rho_x>\frac{-1}{M-1} \tag{2.37}$$

From (2.6), (2.15) and (2.35) we have

$$MSE(\bar{\bar{y}}_{CR\,\mathrm{Re}})-MSE(d_1ord_2)=\gamma\bar{\bar{Y}}^2C_x^2\{1+(M-1)\rho_x\}\left(\frac{9}{4}-3k-\frac{1}{4}+k\right)$$

$$=2\gamma\bar{\bar{Y}}^2C_x^2\{1+(M-1)\rho_x\}(1-k)<0 \text{ if}$$

$$k>1,\ \rho_x>\frac{-1}{M-1}. \tag{2.38}$$

Further from (2.20) and (2.35) we have

$$MSE(\bar{\bar{y}}_{CR\mathrm{Re}}) - MSE(\bar{y}_{CRC}) = \gamma \bar{\bar{Y}}^2 C_x^2 \{1+(M-1)\rho_x\}\left(\frac{9}{4} - 3k - 4 + 4k\right)$$

$$= \gamma \bar{\bar{Y}}^2 C_x^2 \{1+(M-1)\rho_x\}\left(k - \frac{7}{4}\right) < 0 \text{ if}$$

$$k > \frac{7}{4}, \rho_x > \frac{-1}{M-1} \ . \tag{2.39}$$

Thus from (2.36 - (2.39) we conclude that the proposed chain ratio-ratio-type exponential estimator $\bar{\bar{y}}_{CR\mathrm{Re}}$ is more efficient than:

(i) $\bar{\bar{y}}$ if $k > \frac{3}{4}, \rho_x > -\frac{1}{M-1}$;

(ii) the ordinary ratio estimator $\bar{\bar{y}}_{RC}$ and chain ratio-type exponential estimator d_{C1e} if $k > \frac{5}{4}, \rho_x > -\frac{1}{M-1}$;

(iii) the chain ratio-type exponential estimator d_1 and d_2 if $k > 1, \rho_x > -\frac{1}{M-1}$;

and

(iv) the chain ratio-type exponential estimator $\bar{y}_{CRR}$ if $k > \frac{7}{4}, \rho_x > -\frac{1}{M-1}$.

Form the above it is concluded that the $\bar{\bar{y}}_{CR\mathrm{Re}}$ is more efficient than the estimators $\bar{\bar{y}}$, $\bar{\bar{y}}_{RC}$, d_{C1e}, d_1, d_2 and $\bar{y}_{CRC}$ if the following condition is satisfied:

$$\frac{5}{4} < k < \frac{7}{4}, \rho_x > -\frac{1}{M-1} \tag{2.40}$$

3. A MORE GENERAL CLASS OF CHAIN-TYPE ESTIMATORS FOR $\bar{\bar{Y}}$ IN CLUSTER SAMPLING

Let α and δ be constants or functions of known parameters associated with (study, auxiliary) variates, for instance, see Sisodia and Dwivedi [10], Upadhyaya and Singh [12], Singh and Tailor [4], and Kadilar and Cingi [2]. Then we define the class of chain type estimators for population mean $\bar{\bar{Y}}$ as

$$\bar{\bar{y}}_{(p,q)} = \bar{\bar{y}}\left(\frac{\alpha\bar{\bar{X}}+\delta}{\alpha\bar{\bar{x}}+\delta}\right)^p \exp\left\{\frac{q\alpha(\bar{\bar{X}}-\bar{\bar{x}})}{\alpha(\bar{\bar{X}}+\bar{\bar{x}})+2\delta}\right\} , \tag{3.1}$$

where (p, q) are design parameters which take real values . We mention that for $(p,q) = (1,1)$, $\bar{\bar{y}}_{(p,q)}$ yields chain ratio-ratio-type exponential estimator while for $(p,q) = (-1,-1)$, $\bar{\bar{y}}_{(p,q)}$ generates chain product-product-type exponential estimator for $\bar{\bar{Y}}$. Expressing (3.1) in terms of e 's we have

$$\bar{\bar{y}}_{(p,q)} = \bar{\bar{Y}}(1+e_y)(1+\tau e_x)^{-p} \exp\left\{-\frac{q\tau e_x}{2}\left(1+\frac{\tau}{2}e_x\right)^{-1}\right\}, \tag{3.2}$$

where $\tau = \frac{\alpha\bar{X}}{(\alpha\bar{X}+\delta)}$.

Now expanding the right hand side of (3.2) and multiplying out we have

$$\bar{\bar{y}}_{(p,q)}=\bar{\bar{Y}}(1+e_y)\left(1-p\tau e_x+\frac{p(p+1)\tau^2}{2}e_x^2-\ldots\right)\left\{1-\frac{q\tau e_x}{2}\left(1+\frac{\tau e_x}{2}\right)^{-1}+\frac{q^2\tau^2e_x^2}{8}\left(1+\frac{\tau e_x}{2}\right)^{-2}\right\}$$

$$=\bar{\bar{Y}}(1+e_y)\left\{1-\frac{(2p+q)\tau e_x}{2}+\frac{pq\tau^2}{2}e_x^2+\frac{p(p+1)\tau^2}{2}e_x^2+\frac{q(q+2)\tau^2}{8}e_x^2-\ldots\right\}$$

$$=\bar{\bar{Y}}\left(1+e_y-\frac{(2p+q)\tau e_x}{2}-\frac{(2p+q)\tau e_xe_y}{2}+\frac{(2p+q)(2p+q+2)\tau^2e_x^2}{8}+\frac{(2p+q)(2p+q+2)\tau^2e_x^2e_y}{2}-\ldots\right)$$

Ignoring $e^{\upsilon}(\upsilon>2)$ in the above expression we have

$$\bar{\bar{y}}_{(p,q)}\cong\bar{\bar{Y}}\left(1+e_y-\frac{(2p+q)\tau e_x}{2}-\frac{(2p+q)\tau e_xe_y}{2}+\frac{(2p+q)(2p+q+2)\tau^2e_x^2}{8}\right)$$

or

$$\left(\bar{\bar{y}}_{(p,q)}-\bar{\bar{Y}}\right)=\bar{\bar{Y}}\left(e_y-\frac{(2p+q)\tau e_x}{2}-\frac{(2p+q)\tau e_xe_y}{2}+\frac{(2p+q)(2p+q+2)\tau^2e_x^2}{8}\right). \qquad (3.3)$$

So the bias of $\bar{\bar{y}}_{(p,q)}$ to the FDA is given by

$$B\left(\bar{\bar{y}}_{(p,q)}\right)=\frac{\gamma\bar{\bar{Y}}C_x^2}{8}\{1+(M-1)\rho_x\}\left[\tau^2(2p+q)(2p+q+2)-4\tau(2p+q)k\right]$$

$$=\frac{\gamma(2p+q)\bar{\bar{Y}}C_x^2}{8}\{1+(M-1)\rho_x\}\tau\left[\tau(2p+q+2)-4k\right]. \qquad (3.4)$$

The approximate value of $(\bar{\bar{y}}_{(p,q)}-\bar{\bar{Y}})^2$ is given by

$$(\bar{\bar{y}}_{(p,q)}-\bar{\bar{Y}})^2\cong\bar{\bar{Y}}^2\left[e_y^2-\tau(2p+q)e_xe_y+\frac{\tau^2(2p+q)^2e_x^2}{4}\right]. \qquad (3.5)$$

So the MSE of $\bar{\bar{y}}_{(p,q)}$ to the FDA is given by

$$MSE(\bar{\bar{y}}_{(p,q)})=\gamma\bar{\bar{Y}}^2\left[C_y^2\{1+(M-1)\rho_y\}+\frac{\tau(2p+q)}{4}C_x^2\{1+(M-1)\rho_x\}\{\tau(2p+q)-4k\}\right]. \qquad (3.6)$$

3.1. Efficiency compression

Subtracting (2.1) from (3.6) we have

$$MSE(\bar{\bar{y}}_{(p,q)})-MSE(\bar{\bar{y}})=\frac{\gamma\bar{\bar{Y}}^2\tau(2p+q)}{4}C_c^2\{1+(M-1)\rho_x\}\{\tau(2p+q)-4k\}<0 \text{ if}$$

$$k<\frac{\tau(2p+q)}{4},\frac{\tau(2p+q)}{4}<0,\rho_x>\frac{-1}{M-1}. \qquad (3.7)$$

3.2. Special cases

We consider another class of estimators $\bar{\bar{Y}}$ as

$$t_{p^*}=\bar{\bar{y}}\left(\frac{\alpha\bar{\bar{X}}+\delta}{\alpha\bar{\bar{x}}+\delta}\right)^{p^*}, \qquad (3.8)$$

where p^* being a suitable chosen scalar.

To the FDA the bias and MSE of t_{p*} are respectively given by

$$B(t_{p*}) = \frac{\gamma \bar{\bar{Y}} p^*}{2} \tau C_x^2 \{1+(M-1)\rho_x\}\{\tau(p^*+1)-2k\}, \tag{3.9}$$

$$MSE(t_{p*}) = \gamma \bar{\bar{Y}}^2 \left[C_y^2\{1+(M-1)\rho_x\} + \tau p^* C_x^2\{1+(M-1)\rho_x\}(\tau p^* - 2k)\right]$$

$$= \gamma \bar{\bar{Y}}^2 \tau C_x^2 \{1+(M-1)\rho_x\}\left[\tau\left\{\frac{(2p+q)}{2} - p^*\right\}\left\{\frac{(2p+q)}{2} + p^*\right\} - 2k\left\{\frac{(2p+q)}{2} - p^*\right\}\right] \tag{3.10}$$

Subtracting (3.10) from (3.6) we have

$$MSE(\bar{\bar{y}}_{(p,q)}) - MSE(t_{p*}) = \gamma \bar{\bar{Y}}^2 \tau C_x^2 \{1+(M-1)\rho_x\}\left[\frac{\tau(2p+q)^2}{4} - k(2p+q) - \tau p^{*2} + 2kp^*\right]$$

$$= \gamma \bar{\bar{Y}}^2 \tau C_x^2 \{1+(M-1)\rho_x\}\left\{\frac{(2p+q)}{2} - p^*\right\}\left[\tau\left\{\frac{(2p+q)}{2} + p^*\right\} - 2k\right]$$

which is less than zero if

$$\begin{cases} either \;\; k > \dfrac{\tau}{2}\left\{\dfrac{(2p+q)}{2} + p^*\right\}, \left\{\dfrac{(2p+q)}{2} - p^*\right\} > 0, p_x > -\dfrac{1}{(M-1)} \\ or \quad k < \dfrac{\tau}{2}\left\{\dfrac{(2p+q)}{2} + p^*\right\}, \left\{\dfrac{(2p+q)}{2} - p^*\right\} < 0, p_x > -\dfrac{1}{(M-1)} \end{cases} \tag{3.11}$$

Further we consider a family of exponential-type estimators for $\bar{\bar{Y}}$ as

$$t_{q*} = \bar{\bar{y}} \exp\left\{\frac{q^* \alpha(\bar{\bar{X}} - \bar{\bar{x}})}{\alpha(\bar{\bar{X}} + \bar{\bar{x}}) + 2\delta}\right\}, \tag{3.12}$$

where q^* is a suitably chosen scalar.

To the FDA the bias and MSE of t_{q*} are respectively given by

$$B(t_{q*}) = \frac{\gamma \bar{\bar{Y}} q^*}{8} \tau C_x^2 \{1+(M-1)\rho_x\}\{\tau(q^*+1)-4k\}. \tag{3.13}$$

and

$$MSE(t_{q*}) = \gamma \bar{\bar{Y}}^2 \left[C_x^2\{1+(M-1)\rho_x\} + \frac{\tau p^*}{4} C_x^2\{1+(M-1)\rho_x\}(\tau q^* - 4k)\right]. \tag{3.14}$$

From (3.6) and (3.13) we have

$$MSE(\bar{\bar{y}}_{(p,q)}) - MSE(t_{q*}) = \gamma \bar{\bar{Y}}^2 \tau C_x^2 \{1+(M-1)\rho_x\}\left[\frac{\tau(2p+q)^2}{4} - k(2p+q) - \frac{\tau q^{*2}}{4} + kq^*\right]$$

$$= \gamma \bar{\bar{Y}}^2 \tau C_x^2 \{1+(M-1)\rho_x\}(2p+q-q^*)\left[\frac{\tau}{4}(2p+q+q^*) - k\right]$$

which is less than zero if

$$\begin{cases} either \;\; k > \dfrac{(2p+q+q^*)}{4}\tau, (2p+q) > q^*, p_x > \dfrac{-1}{M-1} \\ or \quad k < \dfrac{(2p+q+q^*)}{4}\tau, (2p+q) < q^*, p_x > \dfrac{-1}{M-1}. \end{cases} \tag{3.15}$$

Thus $\bar{\bar{y}}_{(p,q)}$ is more efficient than $\bar{\bar{y}}$, $\bar{y}_p^*$ and $\bar{y}_q^*$ as long as the conditions (3.7), (3.11) and (3.15) are respectively satisfied.

4. IMPROVED CLASS OF ESTIMATORS FOR $\bar{\bar{y}}$ IN CLUSTER SAMPLING

We define an improved version of the suggested class of estimator $\bar{\bar{y}}_{(p,q)}$ as

$$\bar{\bar{y}}_{(p,q)} = G\,\bar{\bar{y}}^*_{(p,q)}$$

$$= G\bar{\bar{y}}\left(\frac{\alpha\bar{\bar{X}}+\delta}{\alpha\bar{\bar{x}}+\delta}\right)^p \exp\left\{\frac{q\alpha\left(\bar{\bar{X}}-\bar{\bar{x}}\right)}{\alpha\left(\bar{\bar{X}}+\bar{\bar{x}}\right)+2\delta}\right\}, \tag{4.1}$$

where G is a suitably chosen constant.
Expressing (4.1) in terms of e 's we have

$$\bar{\bar{y}}^*_{(p,q)} = G\bar{\bar{Y}}\left(1+e_y\right)\left(1+\tau e_x\right)^{-p} \exp\left\{-\frac{q\tau e_x}{2}\left(1+\frac{\tau}{2}e_x\right)^{-1}\right\}. \tag{4.2}$$

Expanding the right hand side of (4.2), multiplying out and ignoring terms of e 's having power greater than two in the above expression we have

$$\bar{\bar{y}}^*_{(p,q)} \cong \bar{\bar{Y}}G\left(1+e_y-\frac{(2p+q)\tau e_x}{2}-\frac{(2p+q)\tau e_x e_y}{2}+\frac{(2p+q)(2p+q+2)\tau^2 e_x^2}{8}\right)$$

or

$$\left(\bar{\bar{y}}^*_{(p,q)}-\bar{\bar{Y}}\right)=\bar{\bar{Y}}\left[G\left\{1+e_y-\frac{(2p+q)\tau e_x}{2}-\frac{(2p+q)\tau e_x e_y}{2}+\frac{(2p+q)(2p+q+2)\tau^2 e_x^2}{8}\right\}-1\right], \tag{4.3}$$

So the bias of $\bar{\bar{y}}^*_{(p,q)}$ to the FDA is given by

$$B\left(\bar{\bar{y}}^*_{(p,q)}\right)=\bar{\bar{Y}}\left[G\left\{1+\frac{\gamma C_x^2\{1+(M-1)\rho_x\}\tau(2p+q)}{8}\left[(2p+q+2)-4k\right]\right\}-1\right]. \tag{4.4}$$

Ignoring $e^{\upsilon}\left(\upsilon>2\right)$ in $(\bar{\bar{y}}^*_{(p,q)}-\bar{\bar{Y}})^2$, we have

$$(\bar{\bar{y}}^*_{(p,q)}-\bar{\bar{Y}})^2 \cong \bar{\bar{Y}}^2\left[1+G^2\left\{1+2e_y-\tau(2p+q)e_x-2\tau(2p+q)e_xe_y+e_y^2+\frac{\tau^2(2p+q)(2p+q+1)}{2}e_x^2\right\}\right.$$
$$\left.-2G\left\{1+e_y-\frac{\tau(2p+q)}{2}e_x-\frac{\tau(2p+q)}{2}e_xe_y+\frac{\tau^2(2p+q)(2p+q+1)}{8}e_x^2\right\}\right]. \tag{4.5}$$

Thus the MSE of $\bar{\bar{y}}^*_{(p,q)}$ to the FDA is given by

$$MSE(\bar{\bar{y}}^*_{(p,q)})=\bar{\bar{Y}}^2\left[1+G^2A-2GB\right], \tag{4.6}$$

where

$$A=\left[1+\gamma\left\{C_y^2\left(1+(M-1)\rho_y\right)+\frac{\tau(2p+q)}{4}C_x^2\left(1+(M-1)\rho_x\right)\left(\tau(2p+q+1)-4k\right)\right\}\right]$$

$$B=\left[1+\gamma\frac{\tau(2p+q)}{8}C_x^2\{1+(M-1)\rho_x\}\{\tau(2p+q+1)-4k\}\right].$$

The $MSE(\bar{\bar{y}}^{*}_{(p,q)})$ in (4.6) is minimized for

$$G=\frac{B}{A}=G_{(opt)} \text{ (say)} \tag{4.7}$$

So the resulting MSE of $\bar{\bar{y}}^{*}_{(p,q)}$ given by

$$MSE_{\min}\left(\bar{\bar{y}}_{(p,q)}\right)=\bar{\bar{Y}}^{2}\left[1-\frac{B^{2}}{A}\right]. \tag{4.8}$$

The MSE $\left(\bar{\bar{y}}_{(p,q)}\right)$ in (3.6) can be expressed as

$$MSE\left(\bar{\bar{y}}_{(p,q)}\right)=\bar{\bar{Y}}^{2}\left(1+A-2B\right). \tag{4.9}$$

So from (4.8) and (4.9) we have

$$MSE\left(\bar{\bar{y}}_{(p,q)}\right)-MSE(\bar{\bar{y}}^{*}_{(p,q)})=\bar{\bar{Y}}^{2}\left(A-2B-\frac{B^{2}}{A}\right)=\bar{\bar{Y}}^{2}\frac{\left(A-B\right)^{2}}{A}\geq 0. \tag{4.10}$$

Thus it follows from (4.10) that the improved family of estimators $\bar{\bar{y}}^{*}_{(p,q)}$ is more efficient than the family of estimators $\bar{\bar{y}}_{(p,q)}$.

5. CONCLUSION

In this paper we have suggested some chain estimators in cluster sampling on the line of Kadilar and Cingi [2]. The bias and mean squared error of the suggested estimators are divided under large sample approximation. The regions of preferences are derived under which the suggested estimators are better than the conventional unbiased estimators. A generalized version of the proposed chain ratio-ratio-type exponential estimators is given along with its properties. An improved version of the generalized classes of chain ratio-ratio-type exponential estimators is also proposed with its properties. Finally it is shown that improved class of estimators $\bar{\bar{y}}^{*}_{(p,q)}$ beats the class of estimators $\bar{\bar{y}}_{(p,q)}$ discussed here.

REFERENCES

[1]. Bahl, S. and Tuteja, R.K. (1991). Ratio and product type exponential estimators, *Jour. Info. Opt. Sci.*, 12(1), 159 -164.

[2]. Kadilar, C. and Cingi, H. (2003). A study on the chain ratio-type estimator, *Hacett. Jour. Math. Statist.*, 32, 105-108.

[3]. Singh, H.P. and Rathour, A.(2010). A study on the class of chain ratio-type estimator, *Jour. Statist. Res. Iran,* 7(2), 187-199.

[4]. Singh H.P. and Tailor, R. (2003). Use of known correlation coefficient in estimating the finite population mean, Statist. *Tran.*, 6, 555-560.

[5]. Singh, H.P. and Pal, S.K. (2015). A new chain ratio-ratio-type exponential estimator using auxiliary information in sample surveys, *Int. Jour. Math Appli.*, 3(4-B), 37-46.

[6]. Singh, H.P., Lashkari, P., and Pal, S.K. (2016): New product type and ratio type exponential estimators of the population mean using auxiliary information in sample surveys. *Jour. Statist.*, 23, 67-85.

[7]. Singh, H.P., Pal, S.K., and Solanki, R. S. (2017): A new class of estimators of finite population mean in sample surveys. *Commun. Statist. Theo. Meth.*, 46(6), 2630-2637.

[8]. Solanki, R.S., Singh, H.P. and Pal, S.K. (2013): Improved estimation of finite population mean in sample survey. *Jour. Adv. Comp.*, 1(2), 70-78.

[9]. Singh, R., Chauhan, P., Swain, N. and Smarandache, F. (2008). Ratio estimators in simple random sampling using information on attribute. *Pak. Jour. Statist. Opera. Res.*, 4(1), 47-53.

[10]. Sisodia, B.V.S. and Dwivedi, V.K.(1981). A modified ratio estimator using coefficient of variation of auxiliary variable, *Jour Ind. Soc. Agri. Statist Res.*,7(2), 187-199.

[11]. Swain, A.K.P.C. (2014). On an improved ratio type estimator of finite population mean in sample surveys, *Rev. Invest. Opera.*, 35(1), 49-57.

[12]. Upadhyaya, L.N. and Singh, H.P. (1999). Use of transformed auxiliary variable in estimating the finite population mean, *Biometrical Jour.*, 41, 627- 636.