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ON PRIME AND SEMIPRIME INTUITIONISTIC FUZZY k-IDEALS OF HEMIRINGS

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Abstract: Hemirings are one of the Universal algebras generalizing an associative ring sharing the same properties as a ring except that the second binary operation is assumed to be a semigroup rather than a group. Many researchers made a detailed study on different aspects of hemirings. This paper introduces the notions of prime and semiprime intuitionistic fuzzy left k-ideals of hemirings. We also investigate some of their properties.

Keywords: Hemirings, Fuzzy left k-ideals, Intuitionistic fuzzy left k-ideals, prime intuitionistic fuzzy left k-ideals, semiprime intuitionistic fuzzy left k-ideals , Regular hemiring.

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1. INTRODUCTION

After the introduction of fuzzy sets by Zadeh (1965) in his classic paper [1], its generalization called intuitionistic fuzzy set has been provided by Atanassov [2-3].The concept of extensions of interval- valued intuitionistic fuzzy ideals in semirings was studied and some of the properties were discussed by Balasubramanian and Raja [4], and Thirumagaland and Murugadas [5] presented the three different types of intuitionistic fuzzy prime ideals of semirings. Abdullah et al. [6] developed the concept of (α, β) −intuitionistic fuzzy subhemiring and ideals of hemirings and they investigated some related properties. The notions of intuitionistic fuzzy left k-ideals of semirings and intuitionistic fuzzy left h-ideals of hemirings was introduced by Akram and Dudek [7] and by Dudek in [8] respectively. They described the characteristic, normal and completely normal intuitionistic fuzzy left k-ideals of semirings and intuitionistic fuzzy left h-ideals of hemirings respectively. Dheena and Mohanraaj in [9] introduced intuitionistic fuzzy right k-ideals of semirings. Balasubramanian and Raja [10] introduced the concept of the product of intuitionistic fuzzy k-ideals of semirings and proved some important properties. In this paper, we try to answer one of the question posed in the conclusion of Kumbhojkar [11]. In [11] the author concludes his paper by the comment , " In our opinion the future study of intuitionistic fuzzy ideals of semirings and hemirings can be connected with : (1) investigating semiprime and prime intuitionistics fuzzy k-ideals …". Kumbhojkar [11] redefined prime and semiprime fuzzy h-ideals of hemirings just to resolve the drawbacks of the definition of prime fuzzy h-ideals of hemirings given by Zhan and Dudek [12]. For further information about this we refer the reader to see the papers [11-12]. In this paper, prime and semiprime intuitionistic fuzzy k-ideals of hemirings are defined and some of their properties are proved in similar lines as done by the author of [11] with some modifications that the paper concerned with intuitionistic fuzzy left k-ideals. This paper also assumes L to be a complete Heyting algebra.

2. PRELIMINARIES

Definition 2.1: A system (R,+ $\cdot$) where R is a non empty set and $+$ and $\cdot$ are binary operations on R is called a hemiring if :

(i). (R , +) is commutative semigroup with zero element 0.

(ii). (R ,$\cdot$) is a semigoup.

(iii). a (b + c) = ab + ac and (a+ b)c = ac + bc and 0$\cdot$x = x .0 for all a,b,c, and x ∈R.

A hemiring (R,+ $\cdot$) is said to be commutative if $a \cdot b = b \cdot a\ \forall a, b \in R$. The identity element of

(R,+ $\cdot$) is denoted by 1 and throughout this paper R denotes the hemiring (R,+ $\cdot$).

Definition 2.2: A non - empty subset I of R, closed under addition of R and is such that for all $x \in R$ and $a \in I$ we have $xa \in I$, then I is called left ideal of R .

Definition 2.3: A left ideal I of R satisfying the property : if $y, z \in I$ and $x \in R$, $x + y = z$ implies $x \in I$ is called a left k- ideal.

Analogous definitions can be given for the right cases.

Definition 2.4: An L-fuzzy set μ of non – empty set X is a mapping μ : X ⟶L. For any fuzzy set μ of X and t ∈(0,1] , the upper and lower t-level cuts of μ can be defined as follows respectively.

U($\mu : t) = \{x \in X : \mu(x) \geq t\}$ and L($\mu : t) = \{x \in X : \mu(x) \leq t\}$. $\bar{\mu}$ = 1- $\mu(x)$ is called the complement of μ .

The intersection and union of two fuzzy sets can be defined as follows :

(i) $\mu \cap v(x) = \min\{\mu(x), v(x)\}$.
(ii) $\mu \cup v(x) = \max\{\mu(x), v(x)\}$

Definition 2.5: A fuzzy set $\mu :$ X ⟶L satisfying the conditions:

(i) $\mu(x + y) \geq \mu(x) \wedge \mu(y)$
(ii) $\mu(xy) > \mu(y)$ for x and y in R is called a fuzzy left ideal.

Definition 2.6: A fuzzy left ideal μ satisfying the condition: if for all x, y and z are in R, $x + y = z \Rightarrow \mu(x) \geq$ min $\{\mu(y), \mu(z)\}$ is called a fuzzy left k-ideal.

Similarly, the right cases can be defined.

A fuzzy k-ideal is one which is both a fuzzy right and left k-ideal .

Definition: 2.7: (see, [8]) A left k-ideal P of a hemiring R is called prime, if P ≠ R and for any left k-ideals I and J of R , we have $IJ \subseteq P$ implies $I \subseteq P$ $J \subseteq P$.

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Definition 2.8: A left k-ideal I of a hemiring R is called semiprime, if I ≠ R , and for any left k-ideal J of R, we have $JJ \subseteq I$ implies $J \subseteq I$.

Proposition 2.9: (see,[11]) A proper left k-ideal P of a hemiring R is prime iff for all $x, y \in R, xRy \subseteq P \Rightarrow x \in P$ or $y \in P$.

Proposition 2.10: (see,[11]) If R is a commutative hemiring with unity, then a proper k-ideal P of a hemiring R is prime iff for all $x, y \in R, xy \in R \Rightarrow x \in R$ or $y \in R$.

Definition 2.11: (see,[11]) A fuzzy left k-ideal P : R → L is called prime if it is non-constant and, for all x, y ∈ R and $\alpha \in L$, the following condition is satisfied :

$$P(xry) \geq \alpha \Rightarrow P(x) \geq \alpha \text{ or } P(y) \geq \alpha \; \forall r \in R.$$

Definition 2.12: (see,[11]) A fuzzy left k-ideal J : R → L is called semiprime if J is non-constant and $\forall x \in R$ *and* $\alpha \in L$, the following condition is satisfied:

$$J(xrx) \geq \alpha \Rightarrow J(x) \geq \alpha \;\; \forall r \in R.$$

Definition 2.13: An object of the form $\left\{\left(x, \mu_A(x), \lambda_A(x)\right) : x \in R\right\}$ where, $\mu_A : X \to L$ *and* $\lambda_A : X \to L$ define the degree of membership and non-membership of the element x of R, respectively and for each $x \in R$, satisfying $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$ is called an intuitionistic fuzzy set (IFS) A in R.

For the sake of simplicity, the symbol A =(μ_A, λ_A) is used to denote the IFS A in R .
Clearly, for every fuzzy set μ, we can have an IFS: $A = \{(x, \mu(x), 1 - \mu(x)) : x \in X\}$.
For two intuitionistic fuzzy sets, A =(μ_A, λ_A) and B =(μ_B, λ_B) we have the following definitions:

(a) $A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x)$ *and* $\lambda_A(x) \geq \lambda_B(x) \; \forall \in X$.
(b) $A = B \Leftrightarrow A \subseteq B$ *and* $B \subseteq A$.
(c) $A \cap B = (\mu_A \cap \mu_B, \lambda_A \cap \lambda_B)$
(d) $A \cup B = (\mu_A \cup \mu_B, \lambda_A \cup \lambda_B)$

3. INTUTIONISTIC FUZZY IDEALS OF HEMIRINGS

Definition 3.1: (see, [7-9,13]) An intuitionistic fuzzy set A =(μ_A, λ_A) is said to be an intuitionistic fuzzy left ideal of R if :

(i) $\mu_A(x + y) \geq \mu_A(x) \wedge \mu_A(y)$
(ii) $\lambda_A(x + y) \leq \lambda_A(x) \vee \lambda_A(y)$
(iii) $\mu_A(x\,y) \geq \mu_A(y)$
(iv) $\lambda_A(xy) \leq \lambda_A(y) \; \forall x, y \in R$.

Similar definitions can be given for the right cases.
An intuitionistic fuzzy ideal of R is one which is both intuitionistic fuzzy right and left ideal.

Definition 3.2: An intuitionistic fuzzy left ideal A = (μ_A, λ_A) is said to be an intuitionistic fuzzy left k-ideal if it satisfies :

(i) $x + y = z \Rightarrow \mu_A(x) \geq \mu_A(y) \wedge \mu_A(z)$
(ii) $x + y = z \Rightarrow \lambda_A(x) \leq \lambda_A(y) \vee \lambda_A(z), \forall x, y, z \in R$.

Analogous definitions can be given for the right cases.

Definition 3.3: (see, [7-9]) Let A =(μ_A, λ_A)be an IFS of a set R and let $s, t \in [0,1]$ .Then the set

$$R_A^{(s,t)} = \{ x \in R : \mu_A(x) \geq s \, , \lambda_A(x) \leq t \}$$

is called an (s,t) – level subset of A =(μ_A, λ_A).
The set $(s,t) \in Im\,\mu_A \times Im\,\lambda_A : s + t \leq 1$ is called the image of $A = (\mu_A, \lambda_A)$.

Observation:

$R_A^{(s,t)} = \left\{ x \in R : \mu_A(x) \geq s \,,\, \lambda_A(x) \leq t \right\} = \{ x \in R : \mu_A(x) \geq s \} \cap \left\{ x \in R : \lambda_A(x) \leq t \right\} = \cup(\mu_A\,;\, s) \cap L\left(\lambda_A ; t\right)$.

4. BASIC RESULTS ON INTUTIONISTIC FUZZY IDEALS (k-IDEALS)

Proposition 4.1: (see, [7]) Every intuitionistic fuzzy left k-ideal is an intuitionistic fuzzy left ideal.

Theorem 4.2: (see, [7,9]) An intuitionistic fuzzy set $A = (\mu_{A\,,} \lambda_A)$ in R is an intuitionistic fuzzy right (left) ideal in R iff $\cup(\mu_A\,;\, t)$ is a right(left) ideal in a hemiring R and $L\left(\lambda_A ; t\right)$ is a right (left) ideal in a hemiring R for all t ∈[0, 1], whenever nonempty.

Theorem 4.3: (see, [7,9]) An intuitionistic fuzzy set A = ($\mu_{A\,,} \lambda_A$) in R is an intuitionistic fuzzy right (left) ideal in R iff $R_A^{(s,t)}$ is a right (left) ideal for all s, t ∈ [0, 1] whenever nonempty.

Theorem 4.4: (see, [7,9]) An intuitionistic fuzzy set A = ($\mu_{A\,,} \lambda_A$) is an intuitionistic fuzzy right (left) k-ideal in R iff $\cup(\mu_A\,;\, t)$ is a right (left) k-ideal in a hemiring R and $\left(\lambda_A ; t\right)$ is a right (left) k-ideal in a hemiring R for all t ∈ R whenever nonempty.

Theorem 4.5: (see, [7,9]) An intuitionistic fuzzy set A =($\mu_{A\,,} \lambda_A$) is an intuitioistic fuzzy right(left) k-ideal in R iff $R_A^{(s,t)}$ is a right (left) k-ideal for all s and t ∈[0, 1] whenever nonempty.

4.1. Prime intuitionistic fuzzy k-ideals of hemirings.

Definition 4.1.1: An intuitionistic fuzzy left k-ideal A =($\mu_{A\,,} \lambda_A$) is called prime if the following conditions are satisfied : $\forall x, y \in R \text{ and } \alpha \in L$,

(i) $\mu_A(xry) \geq \alpha \Rightarrow \mu_A(x) \geq \alpha$ or $\mu_A(y) \geq \alpha \;\; \forall\, r \in R$

(ii) $\lambda_A(xry) \leq \alpha \Rightarrow \lambda_A(x) \leq \alpha \text{ or } \lambda_A(y) \leq \alpha \;\; \forall\, r \in R$

Proposition 4.1.2: An intuitionistic fuzzy left k-ideal A =($\mu_{A\,,} \lambda_A$) is prime iff $\cup(\mu_A\,;\, t)$ and $L\left(\lambda_A ; t\right)$ are prime ideals of R.

Proof: Let an intuitionistic fuzzy left k-ideal $A = (\mu_{A\,,} \lambda_A)$ be prime. Clearly by the theorem IV.2 of [9] and corollary 3.12 of [7],

$\cup(\mu_A\,;\, t)$ and $L\left(\lambda_A ; t\right)$ are left k-ideals of R. Let R≠$\cup(\mu_A\,;\, t) \neq \emptyset$ and xRy$\subseteq \cup(\mu_A\,;\, t)$

$\Rightarrow \mu_A(xry) \geq t \;\forall\, r \in R$

$\Rightarrow \mu_A(x) \geq t$ or $\mu_A(y) \geq t$ (since, A = $(\mu_{A\,,} \lambda_A)$ is prime)

$\Rightarrow x \in \cup(\mu_A\,;\, t) \; or, y \in (\mu_A\,;\, t)$.

Thus $\cup(\mu_A\,;\, t)$ is a prime ideal of R .

Similarly, let xRy$\subseteq L\left(\lambda_A \,;\, t\right)$

$\Rightarrow \lambda_A(xry) \leq t \;\forall\, r \in R$

$\Rightarrow \lambda_A(x) \leq t \; or, \lambda_A(y) \leq t$ (since, A = $(\mu_{A\,,} \lambda_A)$ is prime)

$\Rightarrow x \in L\left(\lambda_A ; t\right) \; or, \; y \in L\left(\lambda_A ; t\right)$

Hence $L\left(\lambda_A ; t\right)$ is prime ideal of R. The converse will be proved by contradiction.

Let $\cup(\mu_A\,;\, t) \; and \; L\left(\lambda_A ; t\right)$ be prime ideals of R for t ∈ L.

Suppose A = $(\mu_{A\,,} \lambda_A)$ be an intuitionistic fuzzy ideal which is not prime.

$\Rightarrow \exists\, \alpha \in L$ and $x, y \in R$ such that

$\mu_A(xry) \geq \alpha \;\; \forall\, r \in R$, but $\alpha > \mu_A(x)$, and $\alpha > \mu_A(y)$. Further,

$\lambda_A(xry) \le \alpha \ \forall\, r \in R$, but $\alpha < \lambda_A(x)$ and $\alpha < \lambda_A(y)$
$\Rightarrow xRy \subseteq \cup(\mu_A\,;\,\alpha)$ but $x \notin \cup(\mu_A\,;\,\alpha), y \notin \cup(\mu_A\,;\,\alpha)$ and $xRy \subseteq L\left(\lambda_A;\alpha\right)$ but $x \notin L\left(\lambda_A;\alpha\right)$ and $y \notin L\left(\lambda_A;\alpha\right) \Rightarrow \cup(\mu_A\,;\,\alpha)$and $L\left(\lambda_A;\alpha\right)$ are not prime ideals of R.
This is a contradiction to the hypothesis.

Theorem 4.1.3: If R is a commutative hemiring with unity, and L is totally ordered then the intuitionistic fuzzy k-ideal A = (μ_A, λ_A) is prime iff $\mu_A(xy) = \max\{\mu_A(x), \mu_A(y)\}$ and

$$\lambda_A(xy) = \min\{\lambda_A(x), \lambda_A)(y)\}$$

Proof: Suppose that an intuitionistic fuzzy left k-ideal A = (μ_A, λ_A) is prime.
Let $\mu_A(xy) = \alpha\,,\ \alpha \in L$

$$\Rightarrow \mu_A(xry) = \mu_A(rxy) \ge \mu_A(r) \vee \mu_A(xy) \ge \alpha \ \forall\, r \in R \Rightarrow \mu_A(rxy) \ge \alpha$$

$\Rightarrow \mu_A(x) \ge \alpha \vee \mu_A(y) \ge \alpha$ (as A = (μ_A, λ_A) is prime).

If $\mu_A(x) \ge \alpha$, then $\mu_A(xy) \ge \mu_A(x) \ge \alpha = \mu_A(xy) \Rightarrow \mu_A(xy) = \mu_A(x)$
If $\mu_A(y) \ge \alpha$, then $\mu_A(xy) \ge \mu_A(y) \ge \alpha = \mu_A(xy) \Rightarrow \mu_A(xy) = \mu_A(y)$
Thus, both the cases give that : $\mu_A(xy) = \max\{\mu_A(x), \mu_A(y)\}$.
Similarly, let $\lambda_A(xy) = \alpha\,,\ \alpha \in L$

$$\Rightarrow \lambda_A(xry) = \lambda_A(rxy) \le \lambda_A(r) \vee \lambda_A(xy) \le \alpha$$

$$\Rightarrow \lambda_A(xry) \le \alpha$$

$\Rightarrow \lambda_A(x) \le \alpha$ *or* $\lambda_A(y) \le \alpha$ (as A = (μ_A, λ_A) is prime).
Now, if $\lambda_A(x) \le \alpha$, then $\lambda_A(xy) \le \lambda_A(x) \le \alpha = \lambda_A(xy)$

$$\Rightarrow \lambda_A(xy) = \lambda_A(x)$$

if $\lambda_A(y) \le \alpha$, then $\lambda_A(xy) \le \lambda_A(y) \le \alpha = \lambda_A(xy)$
Thus , both the cases yield that : $\lambda_A(xy) = \lambda_A(x) \vee \lambda_A)(y)$.
To prove the converse, suppose that $\mu_A(xy) = \max\{\mu_A(x), \mu_A(y)\}$ and

$$\lambda_A(xy) = \min\{\lambda_A)(x), \lambda_A)(y)\}$$

Let $\mu_A(xry) \ge \alpha \ \forall\, r \in R \ \&\ \lambda_A(xry) \le \alpha \ \forall r \in R$.
Now, putting $r = 1$, which gives that $\mu_A(xy) \ge \alpha \ \&\ \lambda_A(xy) \le \alpha$

$$\Rightarrow \max\{\mu_A(x), \mu_A(y)\} \ge \alpha$$

and $\min\left\{\lambda_A(x), \lambda_A(y)\right\} \le \alpha$, (by the hypothesis)
$\Rightarrow \mu_A(x) \ge \alpha$ *or* $\mu_A(y) \ge \alpha \ \&\ \lambda_A(x) \le \alpha$ *or* $\lambda_A(y) \le \alpha$
Hence A = (μ_A, λ_A) is a prime intuitionistic fuzzy k-ideal.

4.2 Semiprime intuitionistic fuzzy k-ideals of hemirings

Definition 4.2.1: An intuitionistic fuzzy left k-ideal A = (μ_A, λ_A) of R is called semiprime if , for all x, y ∈ R and $\alpha \in L$, the following conditions are satisfied:

(i) $\mu_A(xrx) \ge \alpha \Rightarrow \mu_A(x) \ge \alpha \ \forall\, r \in R$
(ii) $\lambda_A(xrx) \le \alpha \Rightarrow \lambda_A(x) \le \alpha \quad \forall\, r \in R$.

Proposition 4.2.2: An intuitionistic fuzzy left k-ideal A = (μ_A, λ_A) of R is semiprime iff $\cup(\mu_A\,;\,t)$ *and* $L\left(\lambda_A;t\right)$ are semiprime ideals of R .

Proof: Let an intuitionistic fuzzy left k-ideal A = (μ_A, λ_A) be semiprime. Clearly by the theorem IV.2 of [9] and corollary 3.12 of [7],

$\cup\,(\mu_A\,;\,t)$ *and* $L\left(\lambda_A; t\right)$ are left k-ideals of R .

Let R$\neq\cup\,(\mu_A\,;\,t)\;\neq\emptyset$ and xRx$\subseteq\cup\,(\mu_A\,;\,t)$.

$\Rightarrow \mu_A(xrx) \geq t\;\forall\, r \in R$

$\Rightarrow \mu_A(x) \geq t$ (as A = (μ_A, λ_A) is semiprime)

$\Rightarrow x \in \cup\,(\mu_A\,;\,t)$.

Thus $\cup\,(\mu_A\,;\,t)$ is a semiprime ideal of R .

Similarly, let xRx$\subseteq L\left(\lambda_A\,;\,t\right)$

$\Rightarrow \lambda_A(xrx) \leq t\;\forall\, r \in R$

$\Rightarrow \lambda_A(x) \leq t$ (as A = (μ_A, λ_A) is semiprime)

$\Rightarrow x \in\; L\left(\lambda_A; t\right)$

hence $L\left(\lambda_A; t\right)$ is a semiprime ideal of R. We prove the converse by contradiction.

Let $\cup\,(\mu_A\,;\,t)$ *and* $L\left(\lambda_A; t\right)$ be semiprime ideals of R for t ∈ L.

Suppose A $=$ (μ_A, λ_A) be an intuitionistic fuzzy ideal which is not semiprime.

$\Rightarrow \exists\;\alpha \in L$ and $x \in R$ such that

$\mu_A(xrx) \geq \alpha\;\;\forall\, r \in R$, but $\alpha > \mu_A(x)$ and

$\lambda_A(xrx) \leq \alpha\;\;\forall\, r \in R$, but $\alpha < \lambda_A(x)$

$\Rightarrow$ xRx$\subseteq\cup\,(\mu_A\,;\,\alpha)$, but $x \notin \cup\,(\mu_A\,;\,\alpha)$ and xRx$\subseteq L\left(\lambda_A; \alpha\right)$, but $y \notin L\left(\lambda_A; \alpha\right)$

$\Rightarrow \cup\,(\mu_A\,;\,\alpha)$ and $L\left(\lambda_A; \alpha\right)$ are not prime ideals of R.

This is a contradiction to the hypothesis.

Corollary 4.2.3: The intersection of semiprime intuitionistic fuzzy left k-ideals of R is a semiprime intuitionistic fuzzy left k-ideal.

Proof : Let A =(μ_A, λ_A), and B =(μ_B, λ_B) be two semiprime intuitionistic fuzzy left k-ideals of R.

Let $\mu_{A\cap B}(xrx) \geq \alpha$ and $\lambda_{A\cap B}(xrx) \leq \alpha$.

$\Rightarrow \min\{\mu_A(xrx),\,\mu_B(xrx)\} \geq \alpha\;\&\max\{\lambda_A(xrx),\,\lambda_B(xrx)\} \leq \alpha.$

$\Rightarrow \mu_A(xrx) \geq \alpha$, $\mu_B(xrx) \geq \alpha$ and $\lambda_A(xrx) \leq \alpha$, $\lambda_B(xrx) \leq \alpha$

$\Rightarrow \mu_A(x) \geq \alpha\;\;\&\,\mu_B(x) \geq \alpha$, and, $\lambda_A(x) \leq \alpha\;\;\&\,\lambda_B(y) \leq \alpha$ (since A & B are semiprimes)

$\Rightarrow \min\{\mu_A(x), \mu_B(x)\} \geq \alpha$, *and* $\max\left\{\lambda_A(x),\,\lambda_B(x)\right\} \leq \alpha.$

$\Rightarrow \mu_{A\cap B}(x) \geq \alpha$ and $\lambda_{A\cap B}(x) \leq \alpha.$

Thus, $A \cap B = (\mu_A \cap \mu_{B,}\,\lambda_A \cup \lambda_B)$ is semiprime.

Theorem 4.2.4: If R is a commutative hemiring with unity, and L is totally ordered, then the intuitionistic fuzzy left k-ideal A = (μ_A, λ_A) is semiprime iff $\mu_A(x^2) = \max\{\mu_A(x), \mu_A(x)\}$and $\lambda_A(x^2) = \min\left\{\lambda_A(x),\,\lambda_A(x)\right\}$.

Proof: By putting y = x in the proof of the Theorem 4.1.3, this result follows.

Theorem 4.2.5: An intuitionistic fuzzy left k-ideal A =(μ_A, λ_A) of a commutative hemiring R with unity is semiprime iff $\mu_A(x^n) = \mu_A(x)$ and $\lambda_A(x^n) = \lambda_A(x)\;\forall x\,, \forall n \geq 1.$

Proof: The forward direction of this theorem will be proved by induction. The result holds for n = 2 (by the Corollary 4.1.6 above). Let k ≥ 2, and assume $\mu_A(x^n) = \mu_A(x)$ and $\lambda_A(x^n) = \lambda_A(x)\;\forall n, 1 \leq n \leq k.$

$\mu_A(x^{k+1}) = \mu_A(x)$ and $\lambda_A(x^{k+1}) = \lambda_A(x)$.

(i) If k is odd , then k = 2m + 1, m < k.

This implies, $\mu_A(x^{k+1}) = \mu_A(x^{2m+2}) = \mu_A((x^{m+1})^2) = \mu_A(x)$

and $\lambda_A(x^{k+1}) = \lambda_A(x^{2m+2}) = \lambda_A\big(((x^{m+1})^2)\big) = \lambda_A(x)$

$\Rightarrow \mu_A(x^{k+1}) = \mu_A(x)$ and $\lambda_A(x^{k+1}) = \lambda_A(x)$

(ii). If k is even , then k = 2m, m < k

$$\Rightarrow \mu_A(x) \leq \mu_A(x^{k+1}) = \mu_A(x^{2m+1}) \leq \mu_A(x^{2m+2}) = \mu_A((x^{m+1})^2) = \mu_A(x^{m+1}) = \mu_A(x)$$

and $\lambda_A(x) \geq \lambda_A(x^{k+1}) = \lambda_A(x^{2m+1}) \geq \lambda_A(x^{2m+2}) = \lambda_A((x^{m+1})^2) = \lambda_A(x)$

$\Rightarrow \mu_A(x^{k+1}) = \mu_A(x)$ and $\lambda_A(x^{k+1}) = \lambda_A(x)$

Hence, $\mu_A(x^n) = \mu_A(x)$ and $\lambda_A(x^n) = \lambda_A(x)$

The converse of the theorem follows from the Corollary 4.1.6 by putting n = 2.

Definition 4.2.6: (see, [7,9]) A hemiring R is said to be regular if for all a ∈ R, there exists x ∈ R such that $a = axa$.

Theorem 4.2.7: If R is a commutative regular hemiring with unity, then every intuitionistic fuzzy left k-ideal A = (μ_A, λ_A) of R is semiprime.

Proof : Let x ∈ R. Since R is regular, there exits a ∈ R such that $x = xax$. If this result is applied repeatedly , it yields $x = x^n a^{n-1}\ \forall n \geq 2$. Now $\mu_A(x) = \mu_A(x^n a^{n-1}) \geq \mu_A(x^n) \geq \mu_A(x)$ and $\lambda_A(x) = \lambda_A(x^n a^{n-1}) \leq \lambda_A(x^n) \leq \lambda_A(x)$

$\Rightarrow \mu_A(x^n) = \mu_A(x)$ and $\lambda_A(x^n) = \lambda_A(x)$.

Hence, by the Theorem 4.2.4 above it follows that A =(μ_A, λ_A) is semiprime.

5. CONCLUSION

In this paper, we prove the necessary and sufficient conditions for an intuitionistic fuzzy left k-ideal $A = (\mu_A, \lambda_A)$ to be prime and semiprime and also discuss some related results. The author of this paper suggests that the future study of prime and semiprime intuitionistic fuzzy k-ideals of hemirings can be focused on constructing the spectrum of prime and semiprime intuitionistic fuzzy k-ideals of hemirings.

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