Original Article

Bulletin of Pure and Applied Sciences.
Vol. 38E (Math & Stat.), No.1, 2019. P.25-30

Available online at
www.bpasjournals.com

Print version ISSN 0970 6577
Online version ISSN 2320 3226
DOI: 10.5958/2320-3226.2019.00003.1

ECONOMIC ORDER QUANTITY MODEL WITH PRODUCTION PLAN FOR SEQUENTIALLY CONVERTIBLE ITEM

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Received on 01.08.2018, Revised on 29.11.2018, Accepted on 24.12.2018

Abstract: An economic quantity and production plan is developed for sequentially convertible items for deteriorating items. The initial form of an item sequentially converts into another form and different nature of items. In this process conversion cost and times need to convert the item. We have considered that demand of an item is different for different converted items, also the deterioration rates are different. For example, milk is converted into curd, curd into butter and butter into ghee after different durations of time. Managerial insights are provided for convertible items. Conversion of items needs conversion cost and conversion time.

Keywords: Inventory, Sequentially Convertible item, Conversion cost, Conversion time.

2010 AMS Mathematics Subject Classification: 90B05, 90B50.

1. INTRODUCTION

The item convertibility is a need of hour in the market for inventory managers due to few major reasons, like preventive duration of item may be limited otherwise item may get destroyed, the demand of converted item may be higher than the original one and the profit earned from last converted form may be the highest. This paper suggests an inventory model for items convertible in nature as per the need and demand. In the past, Harris [1] and many other researchers have suggested inventory models with variety of demand functions. There are many products that follow a logarithmic demand pattern and still there is a need to develop some new inventory models for such type of products. Also a business could be started with shortage like advance booking of LPG gas, electricity supply and pre public offer of equity shares before the proper functioning of a company. We incorporate two features: one is the logarithmic demand and other is start of business with shortage in the proposed model. Few items available in the market are of high demand for people like sugar, wheat, oil whose shortage can break the customer's faith and arrival pattern. This motivates retailers to order for excess units of item for inventory in spite of being deteriorated. Moreover, deterioration is manageable for many items by virtue of modern advanced storage technologies. Inventory model presents a real life problem (situation) which helps to run the business smoothly. Our aim is to solve the problem of the business which start with shortage and in which the demand of the products follow the logarithmic demand.

A constrained optimization model was suggested by Silver and Moon [8] which describes the problem of how best to allocate convertible units among the end items when there are a number of units available that can be converted into any one of the end items. The authors there assumed that each of these end items could be purchased at unit cost from the market which is higher than the unit conversion costs. Moon et al. [14] revisited the same problem with the application of inflation and time value of the money. Life-cycle inventory analysis of waste incineration in Switzerland is a useful contribution discussed by Hellweg et al. [6] concluding that the choice of landfill model has a significant influence on the results of life-cycle assessment of waste incineration.

Teng and Chang [18] suggested an economic production quantity model for deteriorating items when the demand rate depends not only on-display stock, but also on the selling price per unit of item which may be influenced by economic policy, political scenario or agriculture productivity or both get affected. Qi *et al.* [19] analyzed the supply chain-coordination under demand disruption in deterministic scenario. The supply shortages for managerial purpose were investigated by Yang *et al.* [20] and they obtained the solution by greedy method. A number of structural properties of the inventory system were analytically presented by Samanta and Roy [21] by the determination of production cycle time and backlog for deteriorating item. Chandel and Khedlekar [22] solved the inventory problem by using two and three warehouses setups at different locations and compared them and suggested the central economical replenishment policy for different locations. Kumar and Sharma [23] formulated the replacement policy for perishable item by using queuing theory approach and presented an optimal policy.

Shukla and Khedlekar [15] introduced a three-component demand rate for newly launched deteriorating items based on two different marketing policies with constant and time dependent demand. Chena et al. [11] suggested a replenishment policy allowing shortages for a product life-cycle. Federgrum and Heching [5] analyzed the price in an incapacitated inventory system with stochastic demand for single item having time dependent parameters. Further useful contributions, in this stream of literature, are due to Matsuyama [7], Hsueh [16], Shukla et al. [3], Chu [9], Khedlekar and Agrawal [13] and You [10], Shukla and Khedlekar [2]. A thought from literature survey appears as rare research contributions exist in the area of modeling of convertible items, specially when the state-wise conversion is a focus. This has motivated us to consider the problem in the present mathematical structure.

This paper suggests an inventory model for items convertible in nature as per need and demand of the market as shown in fig. 1.

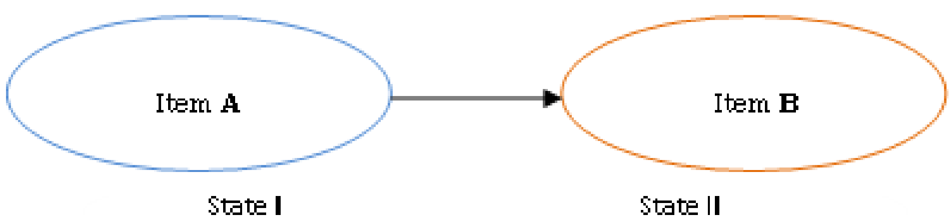

Figure 1: Conversion of items

2. ASSUMPTIONS AND NOTATIONS

Assume that the inventory of a convertible product which is maintained which is in state I is avaialable. Suppose q quantity of product in state I is available to sell for time t_1. The demand of the initial product in state I is assumed to be exponentially decreasing and the product in this state cannot preserved a longer duration of time so, it needs to be converted into another state II with conversion cost. The inventory is depleted due to the demand and deterioration both and at the end of time T the inventory reduces to zero. Suppose replenishment is instantaneous and the lead time is zero between conversions and inventory depletion as shown in fig. 2.

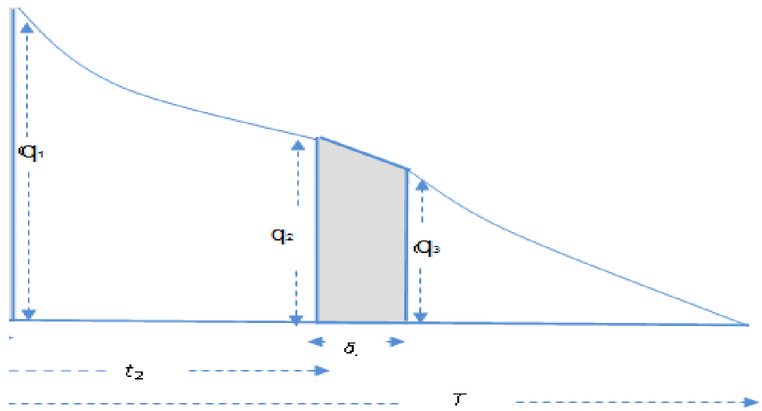

Figure 2: (Inventory depletion of convertible items)

The proposed model is developed under the following assumptions:

q the initial quantity of convertible product available in state I.

$\lambda e^{\mu t}$ the demand of the initial product in state I, where λ is the initial demand and μ is parameter governing the decreasing trend of the product.

a the demand of product the in state II, where a is positive and b is any real value.

d the demand of product in state III, d is a positive value.

C_0 unit purchasing cost of product in state I.

t_1 time period when the product converts from state I to state II,

q_1 the unsold quantity of the product in state I $(q_1 < q)$ at time t_1.

q_2 the quantity of product in state II (after first conversion) at time $t_1 + \delta_1$.

q_3 the sold quantity product in state II $(q_3 < q_2)$.

δ_1 the time required for conversion of product from state I to state II

C_1 the conversion cost to convert from state I to state II,

α_1 the conversion ratio from state I to state II.

3. PROPOSED MODEL

Suppose a quantity q of convertible product in state I is purchased at rate C_0 by an inventory management as initial stock and it is resold till time t_1. A quantity q_1 rremains after sale till time t_1, and there after at time t_2, inventory holder converts it into another product in state II with conversion cost C_1 and conversion ratio α_1 that is amount of converted product in state II is $q_2 = \alpha_1 q_1$. Differential equations for products in state I and state II and the amount of on hand inventories $I_1(t)$ and $I_2(t)$ are shown below in equations (1) and (2) respectively.

Rate of decay of inventory in state I is $\left\{-\frac{d}{dt} I_1(t)\right\}$ which is the rate of demand $\left\{\lambda\, e^{-\mu t}\right\}$, so the differential equation follows equation (1). As per fig. 2, the boundary conditions are on hand inventory initially $I(0)$ is q and at time t_1 is $I(t_1)$ equals to q_1.

$$\frac{d}{dt} I_1(t) = -\lambda e^{-\mu t},\ 0 \le t \le t_1 \text{ with boundary condition } I_1(0) = q,\ I_1(t_1) = q_1 \text{ and } q_1\ \alpha_1 = q_2 \tag{1}$$

$$\frac{d}{dt} I_1(t) + \vartheta I_1(t) = a + bt, \qquad t_1 + \delta_1 \le t \le T \tag{2}$$

with the boundary condition $I_2(t_1 + \delta_1) = q_2$, $I_2(t_2) = q_3$ and $q_3 \alpha_2 = q_4$

On solving equation (1), we get

$$q=\frac{q_2}{\alpha_1}+\frac{\lambda}{\mu}\left(1-e^{-\mu t_1}\right) \quad \text{and} \quad I_1(t)=\frac{q_2}{\alpha_1}+\frac{\lambda}{\mu}\left(e^{-\mu t}-e^{-\mu t_1}\right) \tag{3}$$

Holding cost H_1 for product in state I over time interval $[0, t_1]$ is

$$H_1(t_1)=\int_0^{t_1} I_1(t)dt,$$

$$=\frac{h_1 t_1 q_2}{\alpha_1}-\frac{h_1\lambda}{\mu^2}\left(e^{-\mu t_1}+\mu t_1 e^{-\mu t_1}-1\right) \tag{4}$$

where q_2 remains constant after integration.

Solving equation (2), we get

$$I_2(t)e^{\theta t}=-\int_{t_1+\delta_1}^{t}(a+bt)e^{\theta t}dt+E \quad \text{with conditions } I_2(t_1+\delta_1)=q_2,\ I_2(t_2)=q_3,\ q_3\alpha_2=q_4 \tag{5}$$

$$E=q_3e^{\theta t_2}+\int_{t_1+\delta_1}^{t_2}e^{\theta t}(a+bt)\,dt \quad \text{and } E=q_2e^{\theta(t_1+\delta_1)}$$

$$q_2=\frac{q_4}{\alpha_2}(1+\theta t_2-\theta t_1-\theta\delta_1)+a(t_2-t_1-\delta_1)(1-\theta t_1-\theta\delta_1)+\frac{1}{2}(a\theta+b)\left(t_2^2-(t_1+\delta_1)^2\right)-\frac{b\theta}{2}(t_1+\delta_1)\left(t_2^2-(t_1+\delta_1)^2\right)+\frac{b\theta}{3}\left(t_2^3-(t_1+\delta_1)^3\right)$$

Holding cost for product in state II over time interval $[t_1+\delta_1, t_2]$ is

$$H_2(t)=\int_{t_1+\delta_1}^{t_2}h_2 I_2(t)\,dt$$

$$H_2(t)=\frac{h_2q_4}{\alpha_2}\left\{(1+\theta t_2)(t_2-t_1-\delta_1)-\frac{1}{2}\left(t_2^2-(t_1+\delta_1)^2\right)\right\}+ah_2(t_2-t_1-\delta_1)^2+\frac{h_2}{2}(a\theta+b)\left(t_2^2-(t_1+\delta_1)^2\right)(t_2-t_1-\delta_1)$$

$$-\frac{b\theta h_2}{4}\left(t_2^2-(t_1+\delta_1)^2\right)^2+\frac{b\theta h_2}{3}\left(t_2^3-(t_1+\delta_1)^3\right)(t_2-t_1-\delta_1)-\frac{a\theta h_2}{2}\left(t_2^2-(t_1+\delta_1)^2\right)(t_2-t_1-\delta_1) \tag{6}$$

$$q_2=\frac{d}{\alpha_2}(T-t_2-\delta_2)(1+\theta t_2-\phi t_2-\phi\delta_2-\theta t_1-\theta\delta_1)+\frac{d\phi}{2\alpha_2}\left(T^2-(t_2+\delta_2)^2\right)+\frac{b\theta}{3}\left(t_2^3-(t_1+\delta_1)^3\right) \tag{7}$$

Total cost (TC) in inventory system over $[0, T]$ is the sum of the purchasing cost (qC_0), conversion costs from state I to state II (q_1C_1), conversion cost from state II to state III (q_3C_2), holding costs in different states $(H_1+H_2+H_3)$ and setup cost C_3.

$$TC=qc_0+q_1c_1+q_2c_2$$

$$TC=\frac{q_2}{\alpha_1}(C_0+C_1+h_1t_1)+\frac{q_4}{\alpha_2}\left\{C_2+h_2(1+\theta t_2)(t_2-t_1-\delta_1)-\frac{\theta h_2}{2}\left(t_2-(t_1+\delta_1)^2\right)\right\}+\frac{\lambda C_0}{\mu}\left(1-e^{-\mu t_1}\right)+ah_2(t_2-t_1-\delta_1)^2$$

$$-\frac{\lambda h_1}{\mu^2}\left(\mu t_1e^{-\mu t_1}+e^{-\mu t_1}-1\right)+\frac{h_2}{2}(a\theta+b)\left(t_2^2-(t_1+\delta_1)^2\right)(t_2-t_1-\delta_1)+\frac{b\theta h_2}{4}\left(t_2^2-(t_1+\delta_1)^2\right)$$

$$-\frac{h_2a\theta}{2}(t_2-t_1-\delta_1)\left(t_2^2-(t_1+\delta_1)^2\right)+\frac{b\theta h_2}{3}\left(t_2^3-(t_1+\delta_1)^3\right)(t_2-t_1-\delta_1)+h_3dT\left(1+\frac{\phi T}{2}\right)(T-t_2-\delta_2)$$

$$-\frac{h_3d}{2}(1+\phi T)\left(T^2-(t_2+\delta_2)^2\right)+\frac{h_3d\phi}{6}\left(T^3-(t_2+\delta_2)^3\right)+C_3 \tag{8}$$

To find the optimum condition for the values of t_1, t_2 and T subject to the condition that when cost function is minimum, we differentiate above equation w. r. t. t_1, and T and equate to zero.

$$\frac{\partial TC}{\partial t_1}=\frac{\partial TC}{\partial t_2}=0$$

$$\frac{\partial TC}{\partial t_1}=\frac{h_1 q_2}{\alpha_1}+\frac{1}{\alpha_1}\left(C_0+C_1+h_1 t_1\right)\frac{\partial q_2}{\partial t_1}+C_0\lambda e^{-\mu t_1}+h_1\lambda\ t_1 e^{-\mu t_1}-\frac{q_4 h_2}{\alpha_2}\left(1+\theta t_2-\theta t_1-\theta\delta_1\right)+\frac{a\theta h_2}{2}(t_2-t_1-\delta_1)(t_1+\delta_1)$$

$$+\frac{1}{\alpha_2}\left\{C_2+h_2(1+\theta t_2)(t_2-t_1-\delta_1)-\frac{h_2\theta}{2}(t_2^2-(t_1+\delta_1)^2)\right\}\frac{\partial q_4}{\partial t_1}-2ah_2(t_2-t_1-\delta_1)-h_2(a\theta+b)(t_1+\delta_1)(t_2-t_1-\delta_1)$$

$$-\frac{b\theta h_2}{3}\left(t_2^3-(t_1+\delta_1)^3\right)$$

$$+h_2\left(t_2^2-(t_1+\delta_1)^2\right)\left\{b\theta(t_1+\delta_1)-\frac{b}{2}\right\}-b\theta h_3(t_1+\delta_1)^2(t_2-t_1-\delta_1)+\frac{a\theta h_2}{2}(t_2-t_1-\delta_1)(t_1+\delta_1)=0$$

$$\frac{\partial TC}{\partial t_2}=\frac{1}{\alpha_1}\left(C_0+C_1+h_1 t_1\right)\frac{\partial q_2}{\partial t_2}+\frac{q_4 h_2}{\alpha_2}(1+\theta t_2-\theta t_1-\theta\delta_1)+2ah_2(t_2-t_1-\delta_1)+\frac{1}{\alpha_2}\left\{C_2+h_2(1+\theta t_2)(t_2-t_1-\delta)-\frac{h_2\theta}{2}\left(t_2^2-(t_1+\delta_1)^2\right)\right\}\frac{\partial q_4}{\partial t_2}$$

$$h_2(a\theta+b)t_2(t_2-t_1-\delta_1)+bh_2\left(t_2^2-(t_1+\delta_1)^2\right)\left(\frac{1}{2}-\theta t_2\right)+\frac{b\theta h_2}{3}\left(t_2^3-(t_1+\delta_1)^3\right)+b\theta h_2(t_2-t_1-\delta_1)t_2^2-h_3 dT\left(1+\frac{\phi T}{2}\right)$$

$$+h_3 d(t_2+\delta_2)\left\{1+\phi T-\frac{\phi t_2+\phi\delta_2}{2}\right\}-a\theta h_2(t_2-t_1-\delta_1)t_2=0$$

Let the positive value of t_1, and t_2 obtained by solution the above equations be t_1^*, and t_2^*. To show that the total cost function TC minimum at t_1^*, and t_2^* values we write the Hessian matrix H_i as

$$H_1=\frac{\partial^2 TC}{\partial t_1^{\ 2}},$$

$$H_2=\begin{bmatrix}\dfrac{\partial^2 TC}{\partial t_1^{\ 2}} & \dfrac{\partial^2 TC}{\partial t_1\,\partial t_2}\\[2mm] \dfrac{\partial^2 TC}{\partial t_1\,\partial t_2} & \dfrac{\partial^2 TC}{\partial t_2^{\ 2}}\end{bmatrix},$$

If H_1, H_2 are positive on substituting values of the second order derivatives in the above determinant then TC will be optimum.

4. CONCLUSION

An economic quantity model for sequentially convertible items is developed in this paper. The deterioration of converted items is incorporated and the total cost function calculated is shown to be an optimum solution. The conversion is in a sequential manner so that one item converts into other state-wise. It is found that the conversion model construction is possible and the relevant mathematical expressions are derivable. The suggested model performs realistic outputs for given input data set obtained through the real world. One can apply the optimization techniques on the convertible item inventory models. For model builders and researchers, an open problem is to develop better realistic models for sequentially convertible items.

ACKNOWLEDGEMENT

The authors are thankful to referee for their fruitful comments which helped to improve the quality of the manuscript.

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