Original Article

Bulletin of Pure and Applied Sciences.
Vol. 37E (Math & Stat.), No.2, 2018.P.260-266
Print version ISSN 0970 6577
Online version ISSN 2320 3226
DOI: 10.5958/2320-3226.2018.00026.7

EFFECT OF VARYING THERMAL CONDUCTIVITY AND VISCOSITY IN UNSTEADY FREE STREAM FLOW OVER STRETCHING SHEET

Vikas Poply*

Author Affiliation:
Faculty of Mathematics, KLP College, Rewari, Haryana 123401, India

***Corresponding Author:**
Vikas Poply, Faculty of Mathematics, KLP College, Rewari, Haryana 123401, India

E-mail: vikaspoply@gmail.com

Received on 24.02.2018, Accepted on 18.07.2018

Abstract

The aim of this paper is to investigate two dimensional unsteady flow of a viscous incompressible fluid about stagnation point on stretching sheet in the presence of time dependent free stream velocity. It is assumed that fluid viscosity and thermal diffusivity are assumed to vary as linear function of temperature. Fluid is considered in the presence of radiation effect. Using time-dependent stream function, partial differential equations corresponding to the momentum and energy equations are converted into non-linear differential equations. Numerical solutions of these equations are obtained by using Runge-Kutta Fehlberg method with the help of shooting technique. In the present work the effect of unsteadiness parameter, stretching sheet parameter, temperature-dependent fluid viscosity parameter and thermal diffusivity parameter on flow and heat transfer characteristics are discussed. Skin-friction and Nusselt number at the sheet are computed and discussed.

Keywords: unsteadiness, stretching sheet, stagnation point flow, variable physical property.

1. INTRODUCTION

Crane [1] initially investigated the flow caused by the stretching of a sheet in its own plane with a velocity varying linearly with the distance from a fixed point. The stagnation-point flow towards a stretching surface in the presence of free stream velocity has been analyzed by Gupta and Mahapatra [2] and they reported that as the stretching velocity exceeds the free stream velocity, an inverted boundary layer is formed. Singh et al. [3] and [4] reported the effect of orthogonal flow and oblique flow over stretching surfaces respectively. Pop and Na [5] analyzed the unsteady flow over stretching surface and reported that the unsteady flow would approach the steady flow situation after long passage of time. Elbashbeshy and Bazid [6] studied the heat transfer of an unsteady boundary layer flow over stretching sheet. They reported that thermal boundary layer thickness and momentum boundary layer thickness decrease with unsteady parameter. Ishak et al. [7] investigated boundary layer flow over a continuous stretching permeable surface. They found that the heat transfer rate at the surface increase with unsteadiness parameter.

Takhar et al. [8] investigated the unsteady magneto hydrodynamic flow due to the impulsive motion of a stretching sheet. They reported that the surface heat transfer increases up to a certain portion of time, beyond that it decreases. Free convection heat transfer with radiation effect over the isothermal stretching sheet and a flat sheet near the stagnation point have been investigated respectively by Ghaly and Elbarbary [9], and Pop et al. [10]. They found that the boundary layer thickness increases with radiation. El-Aziz [11] studied the thermal radiation effects over an unsteady stretching sheet. Mukhopadhyay [12] studied the unsteady boundary layer flow and heat transfer past a porous stretching sheet in the presence of variable viscosity and thermal diffusivity.

In this paper our concern is to investigate two dimensional unsteady flow of a viscous incompressible fluid about stagnation on stretching sheet in the presence of time dependent free stream velocity, time dependent varying fluid viscosity and thermal diffusivity. Fluid is considered in the influence of thermal radiation. The effect of different parameters (viz. unsteadiness, temperature-dependent fluid viscosity, variable thermal diffusivity) on velocity and temperature fields are investigated and analyzed with the help of their graphical representation.

2. FORMULATION OF PROBLEM

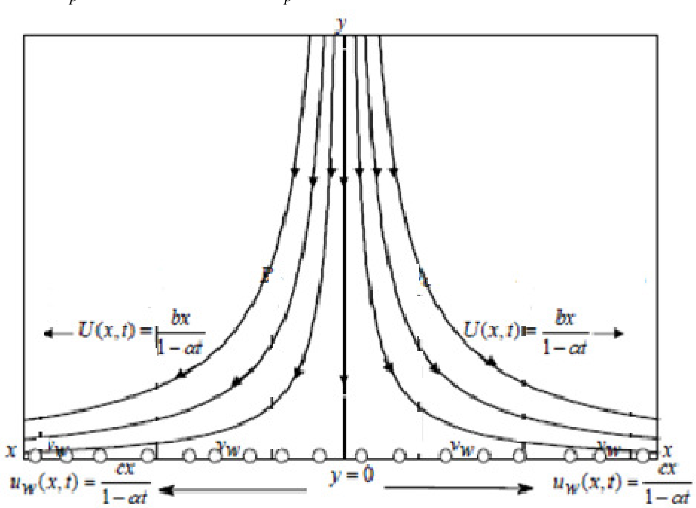
Consider unsteady two-dimensional forced convection flow of a viscous incompressible electrically conducting fluid past a heated stretching sheet. Fluid is considered in the influence of thermal radiation effect. The fluid occupies the upper half plane i.e. $y > 0$. The sheet has uniform temperature T_∞ and moving with non-uniform velocity $u_w(x,t) = \frac{cx}{1-\alpha t}$ where c is the initial stretching sheet rate and $\frac{c}{1-\alpha t}$ is the effective stretching rate which is increasing with time, c and α are positive constants with dimension (time)$^{-1}$. The temperature of the sheet is different from that of ambient medium. The fluid viscosity is assumed to vary with temperature while the other fluid properties are assumed constants. The governing equations of continuity, momentum and energy under the influence of radiation in the boundary layer are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(1)

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + \frac{1}{\rho}\frac{\partial}{\partial y}(\mu\frac{\partial u}{\partial y}) \quad (2)$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\rho C_p}\frac{\partial}{\partial y}(K\frac{\partial T}{\partial y}) - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \quad (3)$$



Physical Model of the Problem

where u and v are the velocity components along x and y axes respectively, $u_w(x,t) = \frac{cx}{1-\alpha t}$ is the free stream velocity of fluid, b is a positive constant with dimension (time)$^{-1}$. σ is electrical conductivity, T is the temperature, ρ is density of fluid, K is the coefficient of thermal conductivity (dependent on temperature) and C_p is specific heat at constant pressure, μ is the fluid density,

Here q_r is approximated by Rosseland approximation, which gives: $q_r = -\frac{4\sigma_s}{3k}\frac{\partial T^4}{\partial y}$ (4)

where k is mean absorption coefficient, σ_s is Stefan-Boltzmann constant. It is assumed that the temperature difference within the flow is so small that T^4 can be expressed as a linear function of T_∞. This can be obtained by expanding T^4 in a Taylor series about T_∞ and neglecting the higher order terms. Thus, we get $T^4 \cong 4T_\infty^3 T - 3T_\infty^4$

Therefore, using above equation in (4), change in radiative flux with respect to y has been obtained as

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma_s T_\infty^3}{3k}\frac{\partial^2 T}{\partial y^2} \tag{5}$$

Boundary conditions for the given model are:

$$u = u_w(x,t) = \frac{cx}{1-\alpha t},\ v = 0 \text{ and } T = T_w(x,t) \text{ at } y = 0$$

$$u = U(x,t) = \frac{bx}{1-\alpha t},\ \ T = T_\infty \text{ at } y = \infty \tag{6}$$

$T_w(x,t) = T_\infty + \frac{T_0 \operatorname{Re}_x (1-\alpha t)^{1/2}}{2}$ is the wall temperature, where $\operatorname{Re}_x = \frac{u_w x}{v}$ is the local Reynold number based on the stretching velocity u_w, T_0 is the reference temperature such that $0 \le T_0 \le T_\infty$.

3. METHOD OF SOLUTION

Now we introduce the stream function $\psi(x, y)$ which is defined by

$$u = \frac{\partial \psi}{\partial y},\ v = -\frac{\partial \psi}{\partial x} \text{ and } \theta = \frac{T - T_\infty}{T_w - T_\infty}. \tag{7}$$

The temperature-dependent fluid viscosity is given by $\mu = \mu^*[a + d(T_w - T)]$, where μ^* is the constant value of the coefficient of viscosity far away from the sheet and a, d are constants and $d > 0$. The variation of thermal diffusivity with the dimensionless temperature is written as $K = K_0(1+\beta\theta)$, where β is a parameter which depends on the nature of the fluid, K_0 is the value of thermal conductivity at the temperature T_w and the similarity variable $\eta = \sqrt{\frac{c}{(1-\alpha t)v}}\, y$, $\psi(x, y) = \sqrt{\frac{cx}{1-\alpha t}}\, x f(\eta)$ and $T = T_\infty + T_0\left[\frac{cx^2}{2v}\right](1-\alpha t)^{-3/2}\theta(\eta)$

With the help of the above relations, the governing equations finally reduce to:

$$(a + A - A\theta)f''' + ff'' - (f')^2 - h(\frac{\eta f''}{2} + f') + \lambda^2 + h\lambda - A\theta' f'' = 0 \tag{8}$$

$$(3R\beta\theta + 3R + 4)\theta'' - 3R\Pr\left[\frac{h}{2}(3\theta + \eta\theta') + 2f'\theta - f\theta'\right] + 3R\beta(\theta')^2 = 0 \tag{9}$$

where the corresponding boundary conditions reduce to

$$f(0) = 0,\ f'(0) = 1,\ \theta(0) = 1 \text{ and } f'(\infty) = \lambda,\ \theta(\infty) = 0 \tag{10}$$

and $h = \frac{\alpha}{c}$ is the unsteadiness parameter, , $R = \frac{kK}{4\sigma_s T_\infty^3}$ is radiation parameter, $\Pr = \frac{\mu c_p}{K}$ is the Prandtl number, $\lambda = \frac{b}{c}$ is the ratio of free stream velocity parameter to stretching parameter, $A = d(T_w - T_\infty)$ is the temperature dependent fluid viscosity parameter and β is the variable thermal diffusivity.

The physical quantity of interest is the skin friction and Nusselt number. The skin-friction coefficient of the sheet is given by

$$C_f = 2\,\mathrm{Re}^{-1/2} f''(0) \quad (11)$$

The rate of heat transfer in terms of the Nusselt number at the sheets is given by

$$N_u = -\mathrm{Re}^{-1/2}\,\theta'(0) \quad (12)$$

4. RESULTS AND DISCUSSION

In the absence of an analytical solution of a problem, a numerical solution is indeed an obvious and natural choice. Thus, the governing boundary layer and thermal boundary layer equations (8) and (9) with boundary conditions (10) are solved using Runge-Kutta Fehlberg method with shooting technique. Different values of unsteadiness parameter h, temperature-dependent fluid viscosity A, variable diffusivity β, and the ratio of free stream velocity parameter to stretching velocity parameter λ are taken.

It is observed from Table 1 that skin friction decreases as unsteadiness parameter increase for $\lambda < 1$. Here, the negative value of $f''(0)$ means the solid surface exerts a drag force on the fluid. This is due to the development of the velocity boundary layer which is caused solely on the stretching plate.

Table 1: Values of $f''(0)$ and $-\theta'(0)$ for different values of h when $\lambda = 0.5$, $\mathrm{Pr} = 0.5$, $R = 1$, $\beta = -1$, $A = 1$

h	$f''(0)$	$-\theta'(0)$
0	-0.740547	0.954213
0.6	-0.828231	1.122422
1	-0.883327	1.222675

Fig. 1 represents variation of velocity profile of the fluid with unsteadiness parameter for $\lambda < 1$. It is observed that with increase in unsteadiness parameter h, the fluid velocity decreases initially and then increases after crossing the transition point. Temperature is found to be decreasing with increasing unsteadiness parameter as shown in Fig. 2. We also notice that the impact of unsteadiness parameter on temperature profile is more pronounced than on the velocity profile.

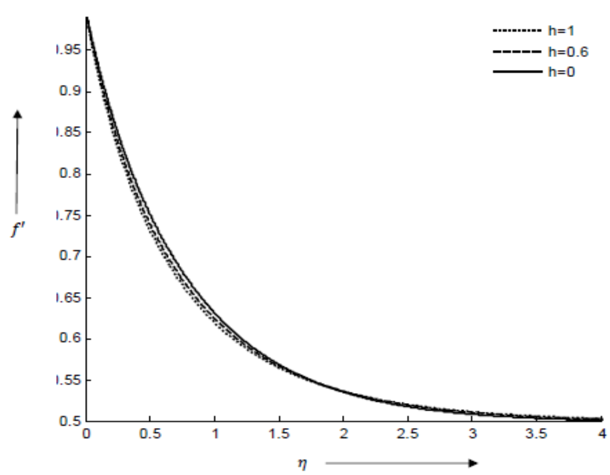


Fig. 1: Velocity profile $f'(\eta)$ for different values of h when $\lambda = 0.5$, $\mathrm{Pr} = 0.5$, $R = 1$, $\beta = -1$, $A = 1$

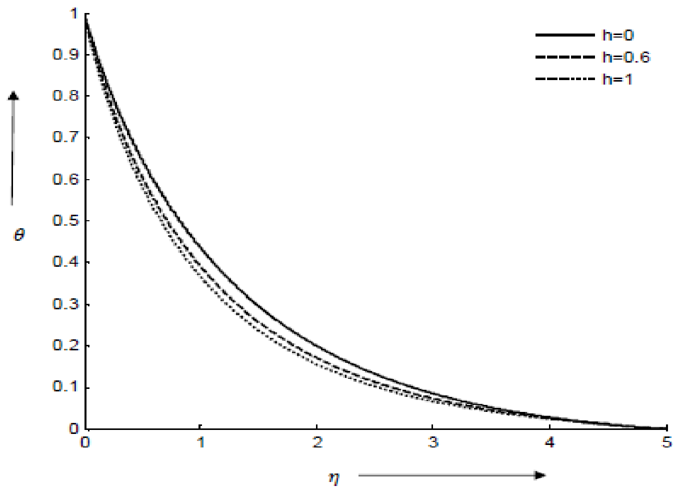


Fig. 2: Temperature profile $\theta(\eta)$ for different values of h when $\lambda = 0.5$, $\Pr = 0.5$, $R = 1$, $\beta = -1$, $A = 1$

As the parameter A increases, the fluid viscosity decreases which results in an increment of boundary layer thickness. In Fig. 3 variations of the fluid velocity with η for several values of A are shown. This exhibits that the fluid velocity decreases with increasing value of A. With increasing A, the thermal boundary thickness decreases, which in turns causes to decrease the velocity profile. Table 2 gives the values of $f''(0)$ and $-\theta'(0)$ for different values of A. It is clear from the Table 2 that fluid viscosity parameter A only affects the velocity of the fluid.

Table 2: Values of $f''(0)$ and $-\theta'(0)$ for different values of A when $h = 0.8$, $\lambda = 0.5$, $\Pr = 0.5$, $R = 1$, $\beta = 0$

A	$f''(0)$	$-\theta'(0)$
1	-0.83812508	0.7868209
2	-0.90479159	0.7878757
3	-0.96807764	0.7887263

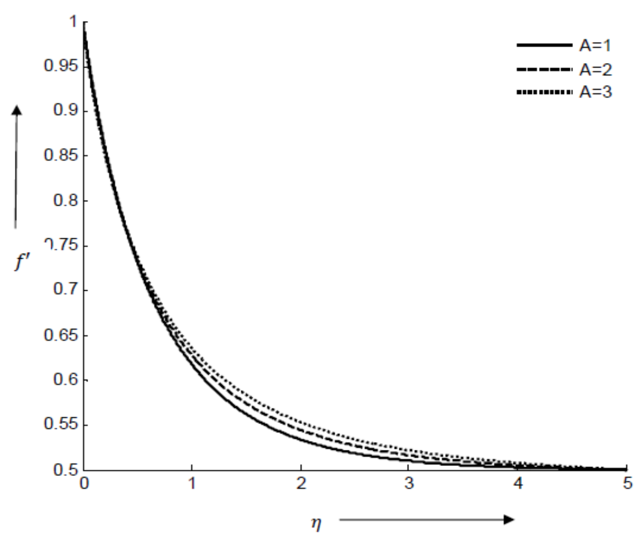


Fig. 3: Velocity profile $f'(\eta)$ for different values of A when $h = 0.8$, $\lambda = 0.5$, $\Pr = 0.5$, $R = 1$, $\beta = 0$.

The effect of thermal diffusivity parameter on velocity and temperature are given in Fig. 4. It represents the fluid velocity profile for several values of β. The temperature at a particular point of the sheet increases with increasing value of β. This is due to thickening of the thermal boundary layer as a result of increasing thermal diffusivity so, the fluid velocity decreases. In this case $\beta = 0$ gives the result in case of uniform thermal diffusivity.

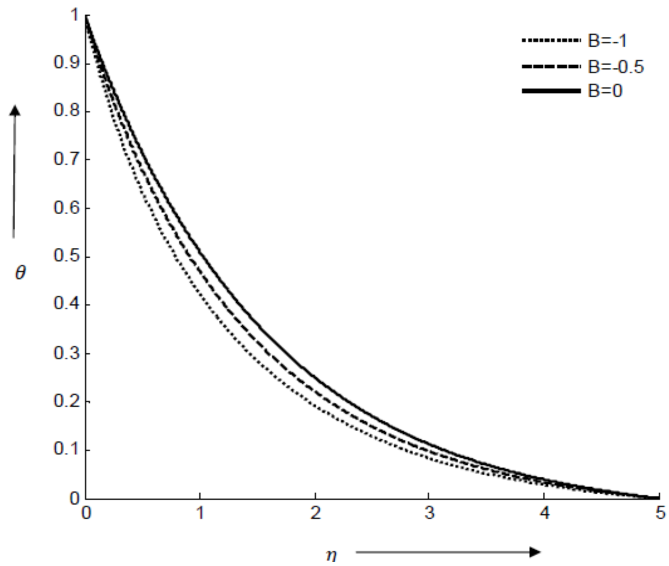


Fig. 4: Temperature profile $\theta(\eta)$ for different values of β when $h = 0.2$, $\lambda = 0.5$, $\Pr = 0.5$, $R = 1$, $A = 0$.

Table 3 below gives the values of $f''(0)$ and $-\theta'(0)$ for different values of β from where it is clear that the fluid viscosity parameter β only affects the temperature of the fluid.

Table 3: Values of $f''(0)$ and $-\theta'(0)$ for different values of β when $h = 0.2$, $\lambda = 0.5$, $\Pr = 0.5$, $R = 1$, $A = 0$.

β	$f''(0)$	$-\theta'(0)$
0	-0.692910	0.674234
-0.5	-0.692910	0.798978
-1	-0.692906	1.008758

5. CONCLUSIONS

The two dimensional viscous, incompressible, unsteady flow and heat transfer of an electrically conducting fluid on stretching sheet with variable fluid viscosity and thermal diffusivity in the presence to time dependent free stream velocity is investigated above. Fluid is considered under the influence of radiation effect. Numerical solution for the governing equations is obtained which allows the computation of the flow and heat transfer characteristics for various values of unsteadiness parameter, variable thermal diffusivity and temperature dependent fluid viscosity. The main results of the paper can be summarized as follows:-

- In the presence of free stream parameter λ a transition point occurs in the velocity profile.
- With increase in unsteadiness parameter the fluid velocity decreases initially and then increases after crossing the transition point.
- Temperature is found to decrease with increasing unsteadiness parameter.
- Fluid velocity increases with increasing fluid viscosity parameter.
- Temperature profile increases with increasing thermal diffusivity.

REFERENCES

[1] Crane, I.J., Flow past a stretching plate, J. Appl. Math. Phys. (ZAMP) 21 (1970) 645–647.

[2] Gupta, A.S., Mahapatra, T.R., Stagnation-point flow towards a stretching surface, The Canadian Journal of Chemical Engineering 81 (2003) 258-263.

[3] Singh, P., Tomer, N.S., Kumar, S., Sinha, D., Effect of Radiation and Porosity Parameter on Magnetohydrodynamic Flow due to Stretching Sheet in Porous Media, Thermal Sciences 15 (2) (2011) 517-525.

[4] Singh, P., Tomer, N.S., Kumar, S., Sinha, D., MHD oblique stagnation-point flow towards a stretching sheet with heat transfer, International Journal of Applied Mathematics and Mechanics 6 (13) (2010) 94-111.

[5] Pop, I., Na, T., Unsteady flow past a stretching sheet, Mechanics Research Communications 23 (4) (1996) 413-422.

[6] Elbashbeshy, E.M.A., Bazid, M.A.., Heat transfer over an unsteady stretching surface, Heat and Mass Transfer 41 (2004) 1-4.

[7] Ishak, A., Nazar, R., Pop, I., Heat transfer over an unsteady stretching permeable surface with prescribed wall temperature, Nonlinear Analysis: Real World Applications 10 (2009) 2909-2913.

[8] Takhar, H.S., Chamkha, A.J., Nath, G., Unsteady three-dimensional MHD boundary layer due to the impulsive motion of a stretching surface, Acta Mechanica 146 (2001) 59-71.

[9] Ghaly, A.Y., Elbarbary, E.M.E., Radiation effect on MHD free –convection flow of a gas at a stretching surface with a uniform free stream, Journal of Applied Mathematics 2 (2) (2002) 93–103.

[10] Pop, I., Pop, S.R., Grosan, T., Radiation effects on the flow near the stagnation point, Technische Mechanik 25 (2) (2004) 100-106.

[11] El-Aziz, M.A., Radiation effect on the flow and heat transfer over an unsteady stretching sheet, International Communications in Heat and Mass Transfer 36 (2009) 521-524.

[12] Mukhopadhyay, S., Unsteady boundary layer flow and heat transfer past a porous stretching sheet in presence of variable viscosity and thermal diffusivity, International Journal of Heat and Mass Transfer (2009), 5213-5217.