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NON-ARCHIMEDEAN STABILITY AND NON-STABILITY OF QUADRATIC RECIPROCAL FUNCTIONAL EQUATION IN SEVERAL VARIABLES

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Abstract

In this paper,we prove the stability of quadratic reciprocal functional equation in several variables of the type

$$\frac{\prod_{j=2}^{k} f(x_1+x_j)}{\sum_{i=2}^{k}\prod_{j=2,j\neq i}^{k} f(x_1+x_j)} = \frac{\prod_{j=1}^{k} f(x_j)}{2\left(\sum_{i=2}^{k}\prod_{j=2,j\neq i}^{k}\sqrt{f(x_j)}\right)\prod_{j=1}^{k}\sqrt{f(x_j)}+(k-1)\prod_{j=2}^{k} f(x_j)+\sum_{i=2}^{k}\prod_{j=1,j\neq i}^{k} f(x_j)}$$

(where k is a positive integer greater than 2) in non - Archimedean fields. We also provide counter examples for singular cases for stability in non-Archimedean fields.

Keywords: Generalised Hyers-Ulam stability, quadratic reciprocal functional equation, non - Archimedean fields.

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1. INTRODUCTION

The study of stability of functional equations was encouraged by a significant question of Ulam[1], which was partially answered by Hyers[10] in Banach spaces. In the course of time, Hyers' result was generalised by T. Aoki[2], Th.M. Rassias[12] and P. Gavruta[9] under various adaptations. Since then many researchers have investigated the result for various functional equations and mappings in various spaces [3,7,11]. In 2010, Ravi and Senthil Kumar[16] obtained generalized Hyers-Ulam stability for the reciprocal functional equation

$$f(x+y) = \frac{f(x)f(y)}{f(x)+f(y)} \tag{1}$$

in the space of non-zero real numbers.The reciprocal function $f(x) = c/x$ is a solution of functional equation (1). After that various forms of the reciprocal functional equations were investigated which can be found in[13,14,17,18]. For the first time, Bodaghi and Kim [5] introduced and studied the generalised Hyers-Ulam stability for the quadratic reciprocal functional equation

$$f(2x+y)+f(x+2y)=\frac{2f(x)f(y)[4f(y)+f(x)]}{(4f(y)-f(x))^2} \tag{2}$$

in the space of non-zero real numbers. Bodaghi and Ebrahimdoost [4] generalized the above equation in non-archimedean fields. Some other forms of quadratic reciprocal functional equations can be studied found in [6,15,18].

In this paper, we investigate the generalized Hyers-Ulam stability for the reciprocal quadratic functional equation in several variables of the type

$$\frac{\prod_{j=2}^{k} f(x_1+x_j)}{\sum_{i=2}^{k}\prod_{j=2,j\neq i}^{k} f(x_1+x_j)}=\frac{\prod_{j=1}^{k} f(x_j)}{2(\sum_{i=2}^{k}\prod_{j=2,j\neq i}^{k}\sqrt{f(x_j)})\prod_{j=1}^{k}\sqrt{f(x_j)}+(k-1)\prod_{j=2}^{k} f(x_j)+\sum_{i=2}^{k}\prod_{j=1,j\neq i}^{k} f(x_j)} \tag{3}$$

where k is a positive integer greater than 2. It can be easily verified that the quadratic reciprocal function $f(x)=\frac{c}{x^2}$ is a solution of the functional equation (3).

2. PRELIMINARIES

A *Non-Archimedean field* is a field K equipped with a function $|\blacklozenge|: K\to R$ such that for any $r,s\in K$ we have

- $|r|\geq 0$ and the equality holds if and only if $r=0$,
- $|rs|=|r||s|$,
- $|r+s|\leq max\{|r|,|s|\}$.

The third condition is called the strict triangle inequality. By second, we have $|1|=|-1|=1$. Thus, by induction, it follows from the third condition that $|n|\leq 1$ for each integer n. We always assume in addition that $|\blacklozenge|$ is non trivial, i.e. there is an $r_o\in K$ such that $|r_o|\notin\{0,1\}$.

2.1 Definition

Let X be a vector space over a field K with a non-Archimedean valuation $|*|$. A function $||*||: X\to[0,\infty)$ is called a *non-Archimedean norm* if the following conditions hold:

- $||x||=0$ iff $x=0$ for all $x\in X$;
- $||rx||=|r|||x||$ for all $r\in K$ and $x\in X$;
- (strong triangle inequality) $||x+y||\leq max\{||x||,||y||\}$for all $x,y\in X$.

Then $(X,||*||)$is called a *non-Archimedean normed space.*

2.2 Definition

Let $\{x_n\}$ be a sequence in a non-Archimedean normed space X.

1. A sequence $\{x_n\}_{n=1}^{\infty}$ in a non-Archimedean space is a *Cauchy sequence* if and only if, the sequence $\{x_{n+1}-x_n\}_{n=1}^{\infty}$ converges to zero.
2. The sequence $\{x_n\}$ is said to be convergent if, for any $\varepsilon>0$, there are a positive integer N and $x\in X$ such that $||x_n-x||\leq\varepsilon$ for all $n\geq N$. Then, the point $x\in X$ is called the limit of the sequence $\{x_n\}$, which is denoted by $lim_{n\to\infty}x_n=x$.
3. If every Cauchy sequence in X converges, then the non-Archimedean normed space X is called a *non-Archimedean Banach space.*

3. STABILITY OF FUNCTIONAL EQUATION (1) IN NON- ARCHIMEDEAN FIELD

In this section we will prove the generalized Hyers-Ulam stability of cubic functional equation (3) in non-Archimedean fields. Throughout this paper, suppose that X and Y are non-Archimedean field and complete non-Archimedean field, respectively. For convenience, define the difference operator $D_kf: X^k\to Y$ such that

$$D_kf(x_1,x_2,\ldots,x_k)=\frac{\prod_{j=2}^{k} f(x_1+x_j)}{\sum_{i=2}^{k}\prod_{j=2,j\neq i}^{k} f(x_1+x_j)}-\frac{\prod_{j=1}^{k} f(x_j)}{2(\sum_{i=2}^{k}\prod_{j=2,j\neq i}^{k}\sqrt{f(x_j)})\prod_{j=1}^{k}\sqrt{f(x_j)}+(k-1)\prod_{j=2}^{k} f(x_j)+\sum_{i=2}^{k}\prod_{j=1,j\neq i}^{k} f(x_j)}$$

Also let us assume that

$$\sum_{i=2}^{k}\prod_{j=2,j\neq i}^{k} f(x_1+x_j)\neq 0,$$

$$2(\sum_{i=2}^{k}\prod_{j=2,j\neq i}^{k}\sqrt{f(x_j)})\prod_{j=1}^{k}\sqrt{f(x_j)}+(k-1)\prod_{j=2}^{k} f(x_j)+\sum_{i=2}^{k}\prod_{j=1,j\neq i}^{k} f(x_j)\neq 0$$

for all $x_i\in X; i=1,2,3,\ldots,k; k\geq 3$ and $x_1\neq x_i$ for all $i; 2\leq i\leq k; k\geq 3$; and set $\frac{0^{k-1}}{0^{k-2}}=0$.

3.1 Theorem

Let $\Phi: X^k \to Y$ be a function satisfying

$$lim_{n\to\infty}\frac{1}{2^{2n}}\Phi(\frac{x}{2^n},\frac{x}{2^n},\ldots\ldots,\frac{x}{2^n}) = 0 \quad (4)$$

for all $x_i \in X$. Suppose that $f: X \to Y$ satisfies the following inequality

$$|D_k f(x_1, x_2, \ldots\ldots, x_k)| \le \Phi(x_1, x_2, \ldots\ldots, x_k) \quad (5)$$

for all $x_i \in X$, then there exists a quadratic reciprocal mapping $Q: X \to Y$ satisfying (3) and the inequality

$$|Q(x) - f(x)| \le max\{\frac{|k-1|}{2^{2j}}\Phi(\frac{x}{2^{j+1}},\frac{x}{2^{j+1}},\ldots\ldots,\frac{x}{2^{j+1}}); j \in N \cup \{0\}\} \quad (6)$$

Moreover if,

$$lim_{m\to\infty} lim_{n\to\infty} max\{\frac{|k-1|}{2^{2j}}\Phi(\frac{x}{2^{j+1}},\frac{x}{2^{j+1}},\ldots\ldots,\frac{x}{2^{j+1}}); m \le j \le n+m\} = 0 \quad (7)$$

then Q is a unique quadratic reciprocal mapping satisfying (6).

Proof: Existence- Replacing x_i by $\frac{x}{2}$ for i = 1,2,...,k in (5) and multiplying by $(k-1)$, we get

$$|f(x) - \frac{1}{2^2}f(\frac{x}{2})| \le |k-1|\Phi(\frac{x}{2},\frac{x}{2},\ldots\ldots,\frac{x}{2}) \quad (8)$$

for all $x \in X$. Replacing x by $\frac{x}{2^{n-1}}$ in (8) and dividing both sides by $2^{2(n-1)}$,we obtain

$$|\frac{1}{2^{2(n-1)}}f(\frac{x}{2^{n-1}}) - \frac{1}{2^{2n}}f(\frac{x}{2^n})| \le \frac{|k-1|}{2^{2(n-1)}}\Phi(\frac{x}{2^n},\frac{x}{2^n},\ldots\ldots,\frac{x}{2^n}) \quad (9)$$

for all $x \in X$. It can be easily seen from (4) and (8) that the sequence $\{\frac{1}{2^{2n}}f(\frac{x}{2^n})\}_{n\ge1}$ is a cauchy sequence.Since Y is complete,therefore $\{\frac{1}{2^{2n}}f(\frac{x}{2^n})\}_{n\ge1}$ is convergent. Hence we can define

$$Q(x) = lim_{n\to\infty}\{\frac{1}{2^{2n}}f(\frac{x}{2^n})\} \quad (10)$$

for all $x \in X$. With the help of induction ,we can easily show that

$$|f(x) - \frac{1}{2^{2n}}f(\frac{x}{2^n})| \le |\sum_{j=0}^{n-1}\frac{1}{2^{2j}}f(\frac{x}{2^j}) - \frac{1}{2^{2(j+1)}}f(\frac{x}{2^{(j+1)}})|$$

$$\le max\{|\frac{1}{2^{2j}}f(\frac{x}{2^j}) - \frac{1}{2^{2(j+1)}}f(\frac{x}{2^{(j+1)}})|: 0 \le j < n\}$$

$$\le max\{\frac{|k-1|}{|2^{2j}|}\Phi(\frac{x}{2^{j+1}},\frac{x}{2^{j+1}},\ldots\ldots,\frac{x}{2^{j+1}}); 0 \le j < n\} \quad (11)$$

for all $n \in N$ and for all $x \in X$. By taking limit $n \to \infty$ in (11) and using (10), we get that inequality (6) is true. By (4), (5) and (10), we get

$$|D_k Q(x_1, x_2, \ldots, x_k)| = lim_{n\to\infty}|\frac{1}{2^{2n}}D_k f(\frac{x}{2^n},\frac{x}{2^n},\ldots,\frac{x}{2^n})| \le lim_{n\to\infty}|\frac{1}{2^{2n}}\Phi(\frac{x}{2^n},\frac{x}{2^n},\ldots,\frac{x}{2^n})|$$

for all $x \in X$.Therefore, by letting $n \to \infty$ we can say that the function Q satisfies (3) and hence Q(x) is a reciprocal quadratic reciprocal function.

Uniqueness-To prove the uniqueness of Q, let $Q^*: X \to Y$ be another function which satisfies (6). Then

$$|Q(x) - Q^*(x)| = lim_{j\to\infty}\frac{1}{|2|^{2j}}|Q(\frac{x}{2^j}) - Q^*(\frac{x}{2^j})|$$

$$\le lim_{j\to\infty}\frac{1}{|2|^{2j}}max\{|Q(\frac{x}{2^j}) - f(\frac{x}{2^j})|, |f(\frac{x}{2^j}) - Q^*(\frac{x}{2^j})|\}$$

$$\le lim_{j\to\infty} lim_{n\to\infty} max\{\frac{1}{|2|^{2(j+i)}}\Phi(\frac{x}{2^{i+1}},\frac{x}{2^{i+1}},\ldots\ldots,\frac{x}{2^{i+1}}); j \le i < n+j\}$$

$$= 0 \text{ for all } x \in X.$$

Therefore $Q = Q^*$. Hence the proof.

3.2 Theorem

Let $\Phi: X^k \to Y$ be a function satisfying

$$lim_{n\to\infty} 2^{2n}\Phi(2^n x, 2^n x, \ldots\ldots, 2^n x) = 0 \quad (12)$$

for all $x_i \in X$.Suppose that $f: X \to Y$ satisfies the inequality (5) for all $x_i \in X$, then there exists a quadratic reciprocal mapping $Q: X \to Y$ satisfying (3) and the inequality

$$|Q(x) - f(x)| \le max\{|k-1|2^{2j+2}\Phi(2^j x, 2^j x, \ldots\ldots, 2^j x); j \in N \cup \{0\}\} \quad (13)$$

Moreover if,

$$lim_{m\to\infty} lim_{n\to\infty} max\{|k-1|2^{2j+2}\Phi(2^j x, 2^j x, \ldots\ldots, 2^j x); m \le j \le n+m\} = 0 \quad (14)$$

then Q is a unique quadratic reciprocal mapping satisfying (13).

Proof: Replacing x_i by x for i = 1,2,..,k in (5) and applying similar arguments as in previous theorem we can get the desired result.

3.3 Corollary

Let p be any real number and θ is non-negative real number. If $f: X \to Y$ satisfies

$$|D_k f(x_1, x_2, \ldots\ldots x_k)| = \begin{cases} \theta(\sum_{i=1}^{k} |x_i|^p) & p < -2 \quad \text{or} \quad p > -2 \\ \theta(\prod_{i=1}^{k} |x_i|^{p/k}) & p < -2 \quad \text{or} \quad p > -2, \\ \theta((\sum_{i=1}^{k} |x_i|^{k\alpha} + \prod_{i=1}^{k} |x_i|^{\alpha}) & \alpha < \frac{-1}{k} \quad \text{or} \quad \alpha > \frac{-1}{k} \end{cases}$$

for all $x_i \in X$, then there exists a unique quadratic reciprocal mapping $Q: X \to Y$ satisfying (3) and the inequality for all $x_i \in X$

$$|Q(x) - f(x)| = \begin{cases} \frac{\theta k(k-1)|x|^p}{2^p} & p < -2 \\ 2^2\theta k(k-1)|x|^p & p > -2 \\ \frac{\theta(k-1)|x|^p}{2^p} & p < -2 \\ 2^2\theta(k-1)|x|^p & p > -2 \\ \frac{\theta(k^2-1)|x|^{k\alpha}}{2^{k\alpha}} & \alpha < \frac{-1}{k} \\ 2^2\theta(k^2-1)|x|^{k\alpha} & \alpha > \frac{-1}{k} \end{cases}$$

Proof: Choosing appropriate $\Phi(x_1, x_2, \ldots, x_k)$ in above theorems we can get the desired results.

4. COUNTER-EXAMPLES

In this section we will provide examples to show the non-stability of functional equation (3) for $p = 2$ and $\alpha = \frac{-1}{k}$ in **R** with usual metric $|.|$ in Corollary (3.3) using well-known counter example provided by Z. Gajda [8]. Consider the function $\Phi: \mathbf{R}^* \to \mathbf{R}$ defined as

$$\Phi(x) = \begin{cases} \frac{\vartheta}{x^2} & \text{for } x \in (1, \infty) \\ \vartheta & \text{otherwise} \end{cases}$$

where $\vartheta > 0$ is a constant, and let for all $x \in \mathbf{R}^*$ the function $f: \mathbf{R}^* \to \mathbf{R}$ be defined as

$$f(x) = \sum_{n=0}^{\infty} \frac{\Phi(2^{-n}x)}{2^{2n}} \tag{15}$$

4.1 Theorem

If $f: \mathbf{R}^* \to \mathbf{R}$ as defined in (15) satisfies the functional inequality

$$|D_k f(x_1, x_2, \ldots, x_k)| \le \frac{20\vartheta}{3(k-1)} (\sum_{j=1}^{k} |x_j|^{-2}) \tag{16}$$

for all $x_j \in \mathbf{R}^*$. Therefore there does not exist a quadratic reciprocal mapping $Q: \mathbf{R}^* \to \mathbf{R}$ and a constant $\rho > 0$ such that

$$|f(x) - Q(x)| \le \rho |x|^{-2} \tag{17}$$

for all $x \in \mathbf{R}^*$.

Proof: $|f(x)| = |\sum_{n=0}^{\infty} \frac{\Phi(2^{-n}x)}{2^{2n}}| \le \sum_{n=0}^{\infty} \frac{|\Phi(2^{-n}x)|}{|2^{2n}|} \le \sum_{n=0}^{\infty} \frac{\vartheta}{2^{2n}} = \frac{4\vartheta}{3}$.

Hence the function is bounded by $4\vartheta/3$. If $(\sum_{n=1}^{k} |x_j|^{-2}) \ge 1$ then L.H.S. of (16) is less than $\frac{20\vartheta}{3(k-1)}$. Suppose that $0 < (\sum_{n=1}^{k} |x_j|^{-2}) < 1$. Then there exists a positive number m such that

$$\frac{1}{2^{2(m+1)}} \le (\sum_{n=1}^{k} |x_j|^{-2}) \le \frac{1}{2^{2m}} \tag{18}$$

Hence we can say that $2^{2m}(\sum_{n=1}^{k} |x_j|^{-2}) < 1$

Or $$\frac{x_j}{2^m} > 1 \quad \text{for} \quad j = 1,2, \ldots, k$$

Or $$\frac{x_j}{2^{(m-1)}} > 2 > 1 \quad \text{for} \quad j = 1,2, \ldots, k$$

and consequently $\frac{x_j}{2^{(m-1)}}, \frac{x_j + x_1}{2^{(m-1)}} > 1$ for j=1,2,....,k. Therefore, for each n= 0,1,2,....,m-1,we have $\frac{x_j}{2^n}, \frac{x_j + x_1}{2^n} > 1$ for j=1,2,....,k.

and $D_k\Phi(2^{-n}x_1, 2^{-n}x_2, \ldots, 2^{-n}x_k) = 0$ for n= 0,1,2,....,m-1.Using (15) and definition of function , we can easily calculate that

$$|D_k f(x_1, x_2, \ldots, x_k)| = \left| \frac{\prod_{j=2}^{k} f(x_1+x_j)}{\sum_{i=2}^{k} \prod_{j=2, j\neq i}^{k} f(x_1+x_j)} - \frac{\prod_{j=1}^{k} f(x_j)}{2(\sum_{i=2}^{k} \prod_{j=2,j\neq i}^{k} \sqrt{f(x_j)}) \prod_{j=1}^{k} \sqrt{f(x_j)} + (k-1)\prod_{j=2}^{k} f(x_j) + \sum_{i=2}^{k} \prod_{j=1,j\neq i}^{k} f(x_j)} \right|$$

$$\leq \frac{\prod_{j=2}^{k} \sum_{n=m}^{\infty} (\vartheta/2^{2n})}{(k-1)\prod_{j=2}^{k-1} \sum_{n=m}^{\infty} (\vartheta/2^{2n})} + \frac{\prod_{j=1}^{k} \sum_{n=m}^{\infty} (\vartheta/2^{2n})}{4(k-1)\prod_{j=2}^{k} \sum_{n=m}^{\infty} (\vartheta/2^{2n})}$$

$$\leq \frac{5}{4(k-1)} \frac{\vartheta}{2^{2m}} (1 - \frac{1}{4})^4 \leq \frac{20\vartheta}{3(k-1)} (\sum_{j=1}^{k} |x|^{-2})$$

for all $x_j \in \mathbf{R}^*$. Hence (16) is proved. Next we claim that the quadratic reciprocal functional equation is not stable for p= -2 in corollary (3.3). Assume that there exists a quadratic reciprocal functional equation $Q: \mathbf{R}^* \to \mathbf{R}$ satisfying (17). Therefore ,we have

$$|f(x)| \leq (1+\rho)|x|^{-2} \tag{19}$$

Next, we can choose a positive integer r with $r\vartheta > \rho + 1$. If $x \in (1, 2^{r-1})$ then $2^{-n}x \in (1, \infty)$ for all n=0,1,2,....r-1 and therefore

$$|f(x)| = |\sum_{n=0}^{\infty} \frac{\Phi(2^{-n}x)}{2^{2n}}| \geq \sum_{n=0}^{r-1} \frac{2^{2n}\vartheta/x^2}{2^{2n}} = \frac{r\vartheta}{x^2} > (\rho+1)x^{-2}$$

which is a contradiction to (19), hence the proof.

4.2 Theorem

If $f: \mathbf{R}^* \to \mathbf{R}$ as defined in (15) satisfies the functional inequality

$$|D_k f(x_1, x_2, \ldots, x_k)| \leq \frac{20\vartheta}{3(k-1)} (\sum_{n=1}^{k} |x|^{-2}) \tag{20}$$

for all $x_j \in \mathbf{R}^*$. Therefore there does not exist a quadratic reciprocal mapping $Q: \mathbf{R}^* \to \mathbf{R}$ and a constant $\rho > 0$ such that

$$|f(x) - Q(x)| \leq \rho(\sum_{i=1}^{k} |x_i|^{-1} + \prod_{i=1}^{k} |x_i|^{\frac{-1}{k}}) \tag{21}$$

for all $x \in \mathbf{R}^*$.

Proof: Using arguments of previous theorem we can easily get the desired result.

REFERENCES

[1] S. M. Ulam, Problems in Modern Mathematics, *Science Editions*, JohnWiley and Sons, New York, NY, USA, 1964.

[2] T. Aoki, On the stability of the linear transformation in Banach spaces. *J. Math. Soc. Jpn.*, Vol. 2(1950), 64-66.

[3] H. Azadi Kenary, On the stability of a cubic functional equation in random normed spaces, *J. Math. Ext.*, Vol. 4, No. 1(2009), 1 -11.

[4] A Bodaghi,Y Ebrahimdoost, On the stability of quadratic reciprocal functional equation in non-Archimedean fields, *Asian-Eur. J. Math.*, Vol. 9, No. 1(2016), Article ID 1650002.

[5] A Bodaghi, S.O. Kim, Approximation on the quadratic reciprocal functional equation. *J. Funct. Spaces Appl.* 2014, Article ID 532463.

[6] Bodaghi, J.M. Rassias, C. Park, Fundamental stabilities of an alternative quadratic reciprocal functional equation in non-Archimedean fields. *Proc. Jangjeon Math. Soc.*, Vol. 18, No.3(2015), 313-320.

[7] M. Eshaghi Gordji, M. B. Savadkouhi, Stability of mixed type cubic and quartic functional equations in random normed spaces, *J. Inequal. Appl.*, 2009(2009), Article ID 527462, 9 pp.

[8] Z. Gajda, On stability of additive mappings, *Int. J. Math. Math. Sci.*, Vol. 14 (1991), 431-434.

[9] P. Gavruta, A generalization of the Hyers-Ulam-Rassias stability of approximately additive mappings, *J. Math. Anal. Appl.*, Vol. 184, No. 3(1994), 431-436.

[10] D. H. Hyers, On the stability of the linear functional equation, *Proc. Natl. Acad. Sci. USA*, Vol. 27, No. 4(1941), 222-224.

[11] D. H. Hyers, G. Isac, Th. M. Rassias, Stability of Functional Equations in Several Variables, Birkhauser, Basel, Switzerland, 1998.

[12] Th. M. Rassias, On the stability of the linear mapping in Banach spaces, *Proc. Amer. Math. Soc.*, Vol. 72, No. 2(1978), 297-300.

[13] K. Ravi, J. M. Rassias, B. V. Senthil Kumar, Ulam stability of generalized reciprocal functional equation in several variables, *Int. J. Appl. Math. Stat.* 19(D10) (2010) 119.

[14] K. Ravi, J. M. Rassias, B. V. Senthil Kumar, Ulam stability of reciprocal difference and adjoint funtional equations, *Australian J. Math. Anal.Appl.* 8(1) (2011), Article ID 13, 118.

[15] K. Ravi, J.M. Rassias, B.V. Senthil Kumar, A. Bodaghi, Intuitionistic fuzzy stability of a

reciprocal-quadratic functional equation, *Int. J. Appl. Sci. Math.* Vol. 1, No. 1(2014), 9-14.

[16] K. Ravi, B.V. Senthil Kumar, Ulam-Gavruta-Rassias stability of Rassias reciprocal functional equation. *Glob. J. Appl. Math. Math. Sci.*,Vol. 3, No. 1-2(2010), 57-79.

[17] K. Ravi, E. Thandapani, B.V. Senthil Kumar, Stability of reciprocal type functional equations. *Panam. Math. J.* Vol. 21, No. 1(2011), 59-70.

[18] K. Ravi, E. Thandapani, B.V. Senthil Kumar, Solution and stability of a reciprocal type functional equation in several variables, *J. Nonlinear Sci. Appl.*, Vol. 7 (2014), 18-27.