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ACCELERATION MOTION OF A SINGLE VERTICALLY FALLING NON-SPHERICAL PARTICLE IN INCOMPRESSIBLE NON-NEWTONIAN FLUID BY DIFFERENT METHODS

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Abstract

An analytical investigation is applied for acceleration motion of a vertically falling non-spherical particle in a Shear-thinning ($n< 1$) and Shear-thickening ($n> 1$) power low fluid. The acceleration motion of a vertically falling non-spherical particle in non-Newtonian fluid can be described by the force balance equation (Basset-Boussinesq-Ossen equation).The main difficulty in the solution of this equation lies in the nonlinear term due to the nonlinearity nature of the drag coefficient. Varinational Iterations Method (VIM) and Numerical Method (Runge- Kutta 4th order method) are used to solve the present problem. The results were compared with those obtained from VIM by R-K 4thorder method. We find that VIM which was used to solve such non-linear differential equation with fractional power is simpler and more accurate than other methods. Analytical results also indicate that the velocity in a falling procedure is significantly increased with approaching flow behavior to $n \rightarrow 2$ that validated the results obtained by the numerical method. Acceleration motion of a vertically falling single non-spherical particle decreases as the behavior index increases and the particle falling in high behavior index fluid attains its terminal velocity earlier as compared to its motion in a low behavior index fluid. To obtain the results for all different methods, the software MATLAB is used.

Keywords: Acceleration motion, non-spherical particle, Shear-thickening power low fluid, Shear-thinning power low fluid, Varinational Iteration method (VIM).

1. INTRODUCTION

The problem of acceleration motion of vertically falling spherical and non-spherical particles in a Shear-thinning ($n< 1$) and Shear-thickening ($n> 1$) power low Non-Newtonian fluids is relevant to many situations of practical interest. In many processes it is often essential to obtain the path of particles that accelerate in the fluid region for designing or improving the process. Gorji et al. [1] find the solution of unsteady motion of vertically falling spherical particles in non-Newtonian fluid by Collocation Method and achieve good results. Recently Kaur & Garg [2, 3] investigated the acceleration motion of a single vertically falling non-spherical particle in incompressible Newtonian fluid and Radiation effect on velocity of a vertically falling non-spherical particle in incompressible Newtonian fluid by Diagonal Pade' [3/3] approximants. Also several works have been done to study the unsteady motion of particles in Newtonian fluid [4-5]. Bagchi and Chhabra reported the distance traveled by accelerating spherical particles in downward vertical motion of particles in power law fluid [6]. From literature review, majority of the studies have described the motion of solid particles in Newtonian suspension only, however many slurries and concentrated suspensions, which are used in materials processing industry behave as non-Newtonian liquids [7]. Along with the same proposition, many researchers realized the physical significance of some analytical methods such as the Homotopy Perturbation Method (HPH), Homotopy Analysis Method (HAM), Varinational Iteration Method (VIM)[8]. To solve the present problem Varinational Iteration Method (VIM) is used and validated with Numerical Method (R-K 4^{th} order method).

Nomenclature		**Greek symbols**	
Acc	acceleration, m/s^2	α, β, γ	Constants
C_D	Drag coefficient	φ	Sphericity
D	Particle diameter, m	μ	Dynamic viscosity,kg/ms
g	acc. due to gravity,m/s^2	ρ	Fluid density,kg/m^3
m	particle mass,kg	ρ_s	Spherical partical density, kg/m^3
Re	Reynolds number		
t	time,s		
u	Velocity,m/s		

2. PROBLEM STATEMENT

The consideration of one-dimensional acceleration motion of a rigid body, non-spherical particle with equivalent volume diameter D, mass m and density ρ_s which is vertically falling in an infinite extent of incompressible non-Newtonian fluid of density ρ and viscosity μ is considered here, where, u represents the velocity of the non-spherical particle at any instant time t, and g is the acceleration due to gravity [Cf. 9]. Thus, the Basset –Boussinesq-Ossen (BBO) equation for the unsteady motion of particle in a fluid is given by [see, 10]

$$m\frac{du}{dt} = mg\left(1-\frac{\rho}{\rho_s}\right) - \frac{\pi D^2 \rho C_D}{8}u^n - \frac{\pi\rho D^3}{12}\frac{du}{dt} \quad (1)$$

$$C_D = f(Re, n) \quad (1a)$$

The drag coefficient could be obtained from Stokes law in the following form:

$$C_D = \frac{24}{Re}X(n), \text{ where } Re = \frac{\rho u^2}{K u^n D^{-n}} \text{ is the Reynolds number and} \quad (1b)$$

$X(n) = 6^{\frac{n-1}{2}}\left(\frac{3}{n^2+n+1}\right)^{n+1}$ is the deviation factor.

Here C_D is the drag coefficient. In right hand side of the Eq.(1), the 1st term represents the buoyancy effect, the 2nd term corresponds to drag resistance and the 3rd term is associated with the added mass effect which is due to the acceleration of fluid around the particle. The complexity of the above equation arises due to the non-linear nature of drag coefficient. So by rewriting force balance Eq. (1) of motion of the particle as,

$$\alpha\frac{du}{dt} + \beta(n)u^n - \gamma = 0,\ u(0) = 0 \quad (1c)$$

in which $\alpha = m + \frac{1}{12}\pi D^3\rho$, $\beta(n) = 3\pi K X(n) D^{2-n}$, $\gamma = mg\left(1-\frac{\rho}{\rho_s}\right)$

For $\alpha = \beta = \gamma = 1$, Eq. (1c) can be written as follows-

$$\frac{du}{dt} + u^n - 1 = 0,\ u(0) = 0 \quad (2)$$

3. VARINATIONAL ITERATION METHOD (VIM)

In 1997, Jihuan He introduced the Varinational Iteration Method (VIM) [11] to solve such nonlinear ordinary and partial differential equations. He's Varinational iteration method (VIM) has been extensively applied as a powerful tool for solving various kinds of problems [12, 13, 14, 15]. Liu and Gurram have solved the problems of free vibration involving an Euler-Bernoulli beam by VIM and obtained accurate results [16]. Slota obtained results for the Heat equation by VIM which were same as the exact solution [17]. To clarify the VIM, we consider the following differential

$$Lu(t) + Nu(t) = g(t) \quad (3)$$

where L is a linear operator, N is a nonlinear operator and $g(t)$ is a non-homogeneous term. By using the Varinational iteration method, a correction functional can be constructed as

$$u_{n+1}(t) = u_n(t) + \int_0^t \lambda\{Lu_n(\zeta) + N\tilde{u}_n(\zeta) - g(\zeta)\}d\zeta \quad (3a)$$

where λ is a general Lagrange multiplier, which can be determined by the help of Varinational theory, the subscript $_n$ means the nth approximation; u_n is restricted variation and $\delta\tilde{u}_n = 0$ [18, 19]. According to VIM, firstly we will find Lagrange multiplier and then trial function u_0 to get the successive iterations $u_{n+1}, n \geq 0$ which converge to the exact solution. The solution is $u = \lim_{n\to\infty} u_n$

To solve eq. (1c) using VIM, the correction functional can be constructed as follows:

$u_{n+1}(t)=u_n(t)+\int_0^t \lambda\{\alpha\frac{du_n(s)}{ds}+\beta u^n(s)-\gamma\}ds$ (3b)

For $\alpha=\beta=\gamma=1$, Eq.(3b) becomes

$u_{n+1}(t)=u_n(t)+\int_0^t \lambda\{\frac{du_n(s)}{ds}+u^n(s)-1\}ds$ (4b)

The stationary condition can be obtained as follows:

$1+\lambda(t)_{s=t}=0$ (4c)

Subsequently, the Lagrangian multiplier is obtained as:

$\lambda=-1$ (4d)

$u_{n+1}(t)=u_n(t)-\int_0^t\{\frac{du_n(s)}{ds}+u^n(s)-1\}ds$ with condition $u_0(t)=0,$ (4e)

4. RUNGE-KUTTA 4th ORDER METHOD

In numerical analysis, the Runge-Kutta methods are a family of implicit and explicit iterative methods. These methods were developed around 1900 by German mathematicians C. Runge and M. W. Kutta. The current problem is initial value problem (IVP) of 1st order. So for obtaining a numerical solution, we can apply numerical method (R-K 4th order method) (also called the mid-point method [see, 20])

$u'(t)=1-u^n$, with initial condition $u(0)=0$ (5)

and f is a function of time and velocity

i.e. $f(t,u)=1-u^n, u(0)=0$ (5a)

5. RESULTS AND DISCUSSION

Table.1.The velocity results of single vertically falling non-spherical particle in incompressible non-Newtonian Fluid for $\alpha=\beta=\gamma=1$

time	Shear-thinning (n< 1) Non-Newtonian media						Shear-thickening(n> 1) Non-Newtonian media					
	n=0.5		n=0.7		n=0.9		n =1.2		n =1.5		n =2.0	
in Sec	VIM	R-K method	VIM	R-K method	VIM	R-K method	VIM	R-K method	VIM	R-K method	VIM	R-K method
0.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.1	0.0789	0.0813	0.0883	0.0891	0.0934	0.0937	0.0971	0.0972	0.0987	0.0987	0.0997	0.0997
0.2	0.1404	0.1474	0.1619	0.1655	0.1753	0.1771	0.1868	0.1875	0.1928	0.1931	0.1973	0.1974
0.3	0.1905	0.2054	0.2240	0.2331	0.2466	0.2520	0.2678	0.2703	0.2803	0.2814	0.2910	0.2913
0.4	0.2313	0.2573	0.2761	0.2937	0.3077	0.3195	0.3395	0.3459	0.3595	0.3630	0.3787	0.3799
0.5	0.2643	0.3043	0.3189	0.3486	0.3590	0.3806	0.4011	0.4145	0.4293	0.4376	0.4583	0.4621
0.6	0.2902	0.3472	0.3532	0.3983	0.4006	0.4359	0.4523	0.4766	0.4885	0.5052	0.5280	0.5370
0.7	0.3096	0.3866	0.3792	0.4437	0.4327	0.4861	0.4926	0.5325	0.5360	0.5659	0.5857	0.6044
0.8	0.3230	0.4230	0.3975	0.4853	0.4556	0.5316	0.5218	0.5828	0.5710	0.6202	0.6293	0.6640
0.9	0.3308	0.4566	0.4082	0.5233	0.4692	0.5730	0.5395	0.6280	0.5926	0.6684	0.6570	0.7163
1.0	0.3333	0.4879	0.4118	0.5583	0.4737	0.6106	0.5455	0.6685	0.6000	0.7110	0.6667	0.7616

Table1 shows the velocity versus time display of non-spherical particle in non-Newtonian fluid by VIM and R-K 4th order method. Figs. 1, 2 & 3 show velocity profiles of vertically falling single non-spherical particle in Shear-thinning power law fluid & Shear-thickening power law fluid. In this, u(vertically) denotes the velocity results of particle w.r.t. time t(horizontally) in seconds in non-Newtonian fluid. Solution is obtain by R-K 4thorder.These figures show that the particle velocity is increasing as the behavior index is increasing and approaching to $n\rightarrow 2$.

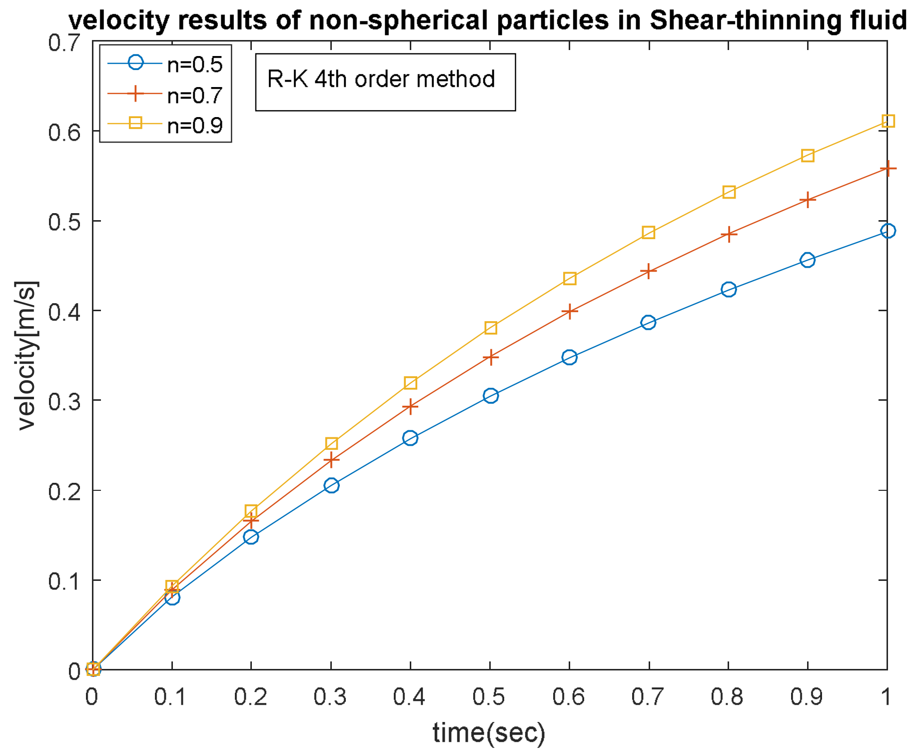

Figure 1: Velocity results of vertically falling non spherical particle in Shear-thinning (n< 1)Non-Newtonian media

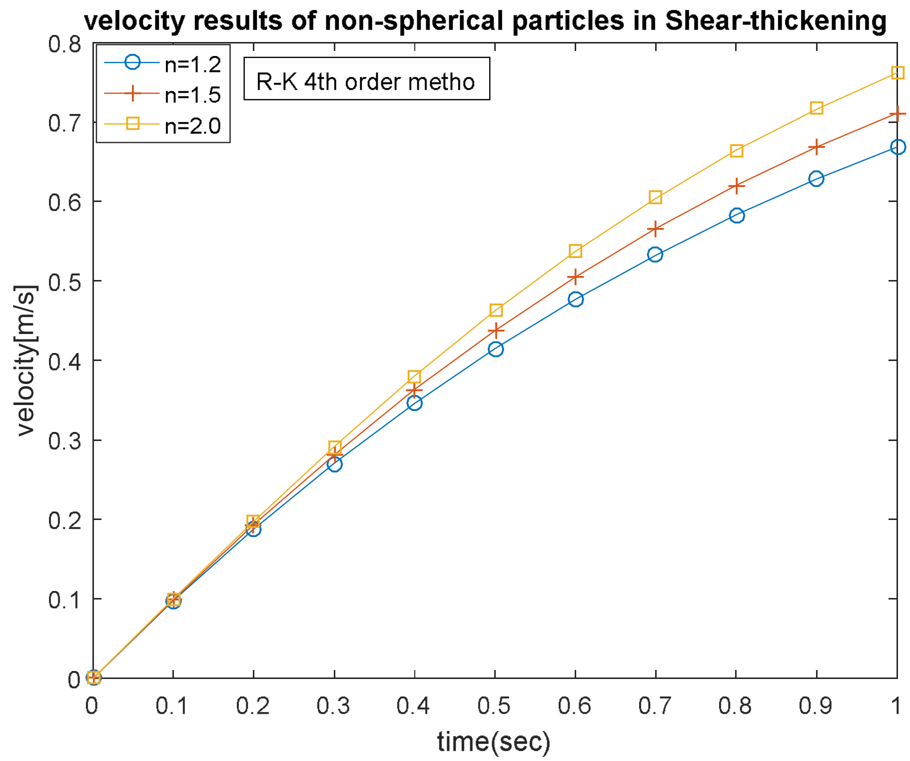

Figure 2: Velocity results of vertically falling non spherical particle in Shear-thickening (n> 1) Non-Newtonian media

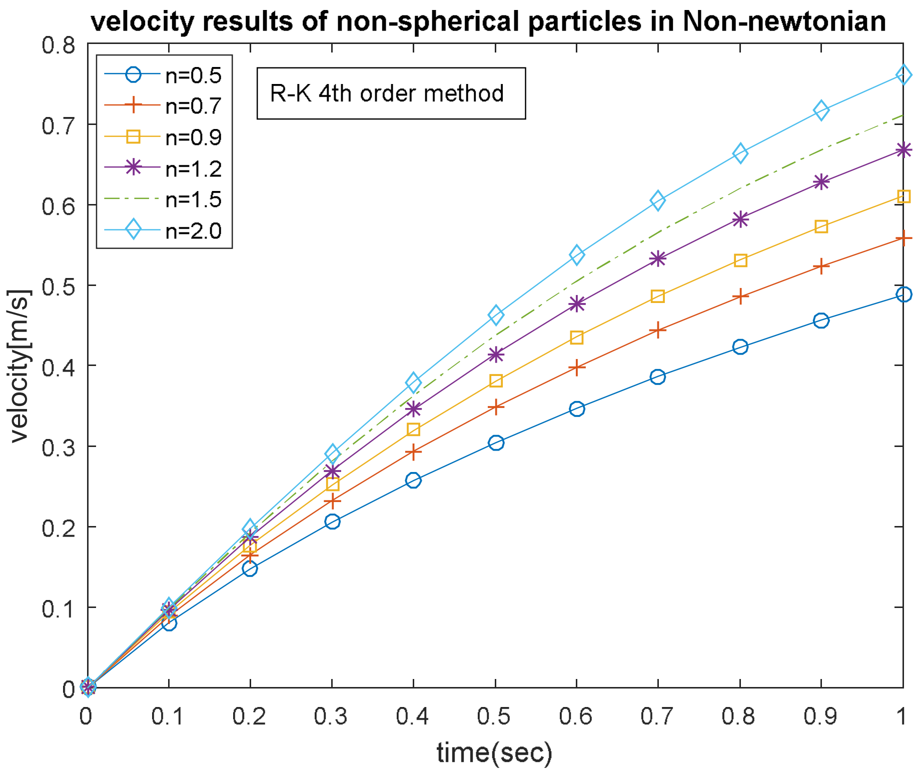

Figure 3: Velocity results of vertically falling non spherical particle in Non-Newtonian media

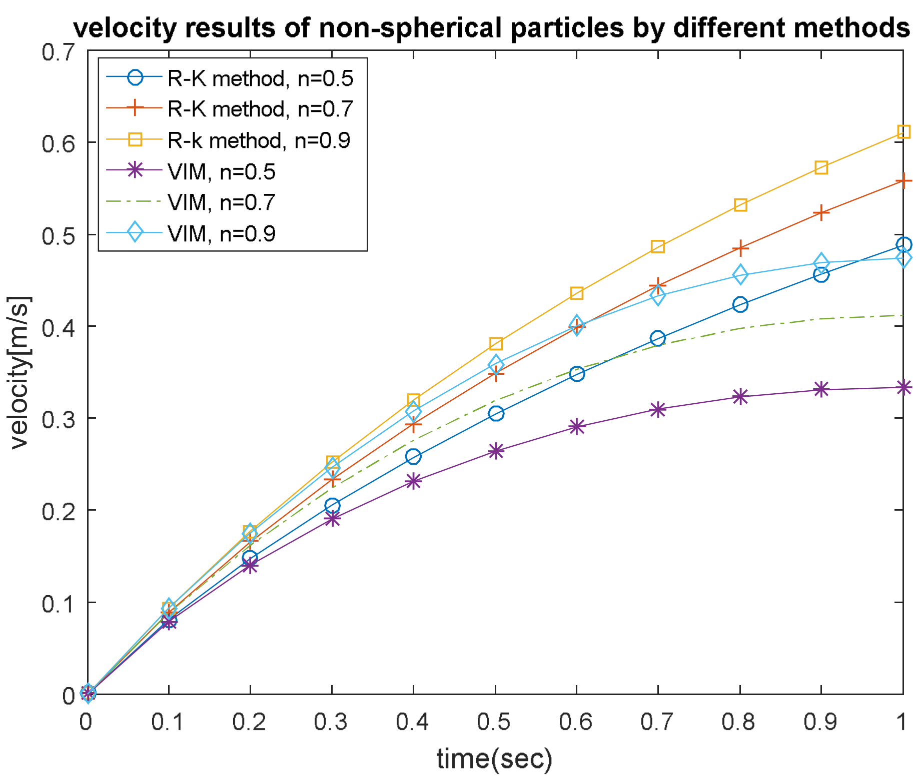

Figure 4: Comparison between VIM and R-K 4th order method (Shear-thinning (n< 1)) Non-Newtonian media

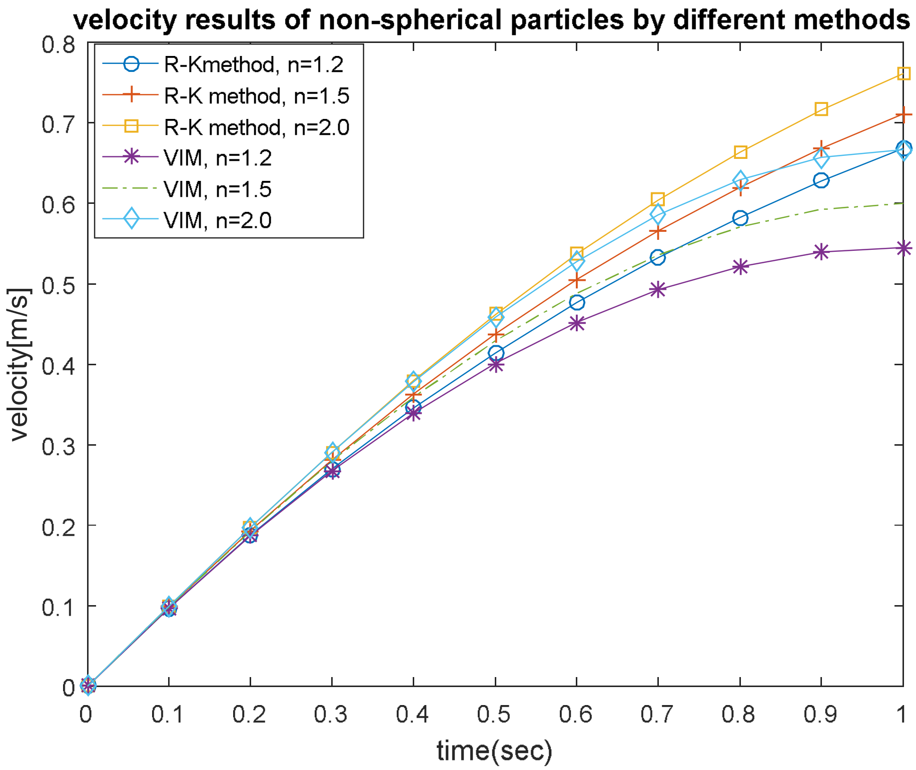

Figure 5: Comparison between VIM and R-K 4th order method (Shear-thickening (n> 1) Non-Newtonian media

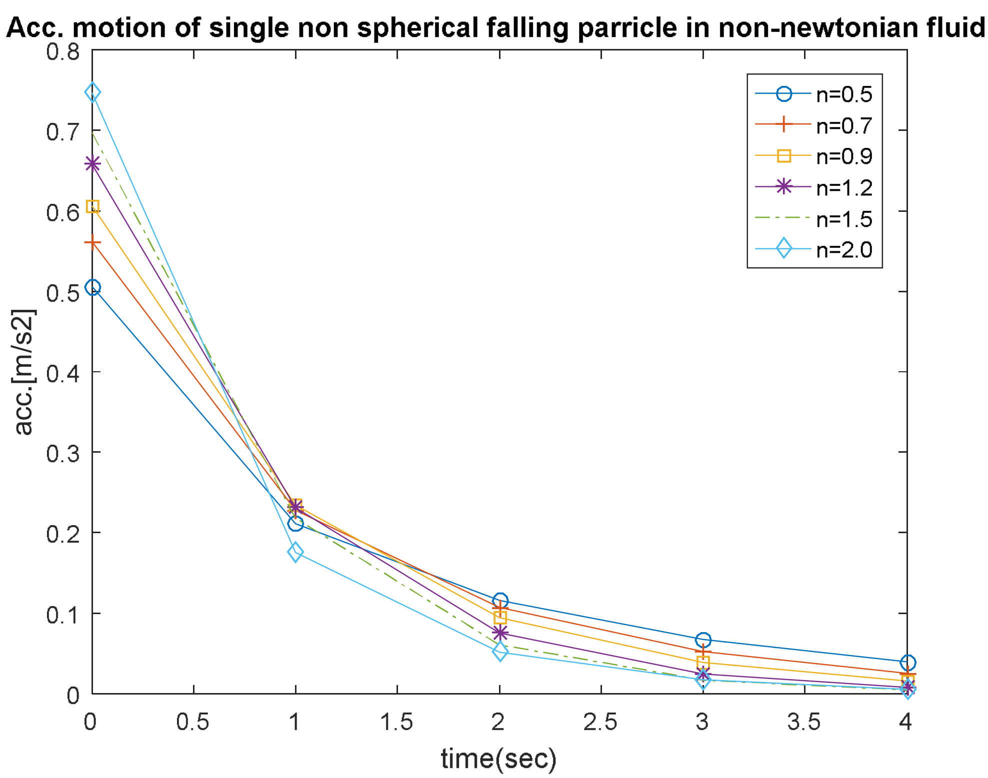

Figure 6: Acceleration results of vertically falling non spherical particle in Non-Newtonian media

Figs. 4 & 5 shows velocity results of vertically falling single non-spherical particle in Shear-thinning power law fluid & Shear-thickening power law fluid by Variation Iteration Method(VIM) and compare the results with

R-K 4^{th} order method. Above figures indicate that the current method gives considerably good results. Fig. 6. shows the acceleration results of the present problem. It depicts that as the behavior index (n) increases, the particle attains its terminal velocity in a shorter period. i.e. the acceleration of single particle which is falling in Shear-thickening power law non-Newtonian fluid reaches early at zero (i.e., particle is not accelerating). As n(behavior index) decreases, the particle takes more time to reach at terminal velocity.

6. CONCLUSION

The achievement of this work is to apply the current method VIM in order to study the nonlinear differential equation of 1^{st} order with initial condition that govern the acceleration motion of a vertically falling non-spherical particle in incompressible non-Newtonian fluid. The current method is applied without using any linearization, discretization, restrictions or transformations. From above discussion, it is clear that the VIM is in good agreement with numerical method and provides highly reliable results. Also, the current method can be used to develop the valid solution of other nonlinear differential equations. In addition, the above discussion shows that the particle velocity is increasing as the behavior index increases and the particle attains its terminal velocity in short period. It also depicts that the acceleration of a single particle which is falling in Shear-thickening power law non-Newtonian fluid reaches early at zero, while, as n(behavior index) decreases, the particle takes more time to attain its terminal velocity.

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