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VARIANCE BALANCED INCOMPLETE BLOCK DESIGNS

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Abstract

This paper provides two new series for the construction for General Efficiency Balanced Incomplete Block Design using the combinatorial arrangement of Efficiency Balanced Incomplete Block Designs.

Keywords: General Efficiency Balanced Incomplete Block Design, Efficiency Balanced Incomplete Block Designs, Variance Balanced Incomplete Block Designs.

1. INTRODUCTION

The arrangement of 'v' treatments in 'b' blocks of sizes $k_1, k_2, \ldots, k_b$ (when none of the block contains all treatments) and the treatments are appearing exactly in $r_1, r_2, \ldots, r_v$ blocks such that n_1 pairs of treatments occur in λ_1 blocks, n_2 pairs occur in λ_2 blocks and so on then the resulting design is said to be a General Incomplete Block Design. The C matrix for an incomplete block design is in the form $C = R–NK^{-1}N'$ where N is the incidence matrix of the incomplete block design and N′ is its transpose, $R = diag(r_1, r_2, \ldots r_v)$ and $K= diag(k_1, k_2, \ldots k_b)$.

Definition 1.1: An incomplete block design is said to be 'Balanced' if all the non-zero characteristic roots of C matrix are equal. When $r_i = r$ and $k_j = k$, then $C = rI_v – k^{-1}[(r-\lambda)I_v + \lambda J_{v,v}] \Rightarrow C= \left[\frac{\lambda v}{k}\right]I - \left[\frac{\lambda}{k}\right]J = \theta\, [\, I_v – v^{-1}J_{vv}]$ where $\theta = \lambda v/k$ is the characteristic root of C matrix.

Definition 1.2: An incomplete block design is said to be 'Efficiency Balanced' if all the v-1 treatment contrasts are estimated with the same efficiency μ where $\mu= v/k$ or $1- (\lambda v/rk)$ is non zero Eigen value of $M = \mu I + (1-\mu) Jr'$ depends on whether the design is orthogonal or non orthogonal.

Definition 1.3: An incomplete block design is said to be 'Variance Balanced' if all the elementary contrasts are estimated with the same precision, i.e the estimation of all treatment contrasts have the same variance.
Several authors provided methods for the construction of Balanced Incomplete Block Designs with nested rows and columns. Some of them are Ehrenfeld [6], Agrawal and Prasad [1,2], Das and Ghosh [4], Cheng [3], David and Uddin[5], Sreenath [11,12,13], Uddin and Morgan [17,18,19], Uddin [14,15,16], Prasad, Gupta and Voss [8], Prasad [9]. Hishida and Jimbo [7], etc.

Singh and Dey [10] first introduced a balanced incomplete block design with nested rows and columns and they have given several methods of construction and examples. Agrawal and Prasad [1,2] provided methods for the construction of Balanced Incomplete Block Design with nested rows and columns using the concepts of method of differences. Further, Uddin and Morgan [17, 18, 19] and Sreenath [11,12] also worked on Balanced Incomplete Block Design with nested row and column design. Uddin [14] provided four new infinite series of designs, in which each of the row, column and block component designs is a Balanced Incomplete Block Design, while Udddin [15] presented a recursive method for the construction of Balanced Incomplete Block Designs with nested rows and columns. Uddin and Morgan [19] constructed two infinite series, which are universally optimum for the analysis with recovery of row and column information, using the known series. Prasad, Gupta and Voss [8] presented a paper on optimal nested row and column design. One of these series achieves orthogonality with just v-1 replicates of v treatments, fewer than required by the latin square. Hishida and Jimbo [7] presented a paper in which the given constructions generate Balanced Incomplete Block Design Row-Columns having the same parameters as several series due to Uddin and Morgan [17] as their special cases. Uddin and Morgan [17, 18, 19] developed a method of construction of class of universally optimal structurally incomplete row column design.

This paper provides two new series for the construction of Variance Balanced Incomplete Block Design using the combinatorial arrangement of Efficiency Balanced Incomplete Block Designs.

2. COMBINATORIAL CONSTRUCTION OF NEW SERIES OF GEBIBD'S

Method 2.1: Let N be the incidence matrix of an Efficiency Balanced Incomplete Block Designs with parameters v, b, r, k. Arrange the incidence matrix N in the form

$$N' = \begin{bmatrix} N & N \\ J & J \end{bmatrix} \tag{2.1}$$

The resulting design is the incidence matrix of a variance balanced design with parameters $v'=v+1$, $b'= 2b$, $r' = (2b, 2r)$, $k_j'= (2b, k+1)$, where r = (2b, 2r) stands for 'v' treatments each replicated '2r' times and the remaining one treatment replicated '2b' times.
The method is illustrated in the example 2.1.

Example 2.1: Let us consider N be the incidence matrix of an Efficiency Balanced Incomplete Block Design with parameters v = 6, b = 11, r = 4 or 5, k = 2, 4. The resulting design is the incidence matrix of a variance balanced design with parameters $v'=7$, $b'= 22$, $r' = 8$ or 22, $k'= 3$ or 5.

Table 2.1: Construction of Variance Balanced Design

N	VBIBD
$\begin{bmatrix} 1&1&1&1&0&0&0&0&0&0&0 \\ 1&1&0&0&1&1&0&0&0&0&0 \\ 1&1&0&0&0&0&1&1&0&0&0 \\ 1&1&0&0&0&0&0&0&1&1&0 \\ 0&0&1&0&1&0&1&0&1&0&1 \\ 0&0&0&1&0&1&0&1&0&1&1 \end{bmatrix}$	$\left[\begin{array}{ccccccccccc\|cccccccccc} 1&1&1&1&0&0&0&0&0&0&0&1&1&1&1&0&0&0&0&0&0 \\ 1&1&0&0&1&1&0&0&0&0&0&1&1&0&0&1&1&0&0&0&0 \\ 1&1&0&0&0&0&1&1&0&0&0&1&1&0&0&0&0&1&1&0&0 \\ 1&1&0&0&0&0&0&0&1&1&0&1&1&0&0&0&0&0&0&1&1 \\ 0&0&1&0&1&0&1&0&1&0&1&0&0&1&0&1&0&1&0&1&0 \\ 0&0&0&1&0&1&0&1&0&1&1&0&0&0&1&0&1&0&1&0&1 \\ \hline 1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1 \end{array}\right.$

Method 2.2: The combinatorial construction of Variance Balanced Incomplete Block Design using Efficiency Balanced Incomplete Block Design is presented below.

Let N_1 and N_2 be the incidence matrices of two Efficiency Balanced Incomplete Block Designs with parameters v, b_1, r_1, k_1, and v, b_2, r_2, k_2 . Arrange the incidence matrices N_1 and N_2 in the form

$$N' = \begin{bmatrix} N_1 & N_2 \\ J & J \end{bmatrix} \tag{2.2}$$

The resulting design is the incidence matrix of a variance balanced design with parameters $v'=v+1$, $b'= b_1 + b_2$, $r_1' = b_1+ b_2$, $r_i' = r_1+ r_2$, $i = 1, 2, \dots v+1$, $k_j'= (k_1+1)$ for $j = 1, 2, \dots, b_1$; $k_j' = (k_2+1)$ $j = b_1+1, \dots , b_1+b_2$.
The method is illustrated in the example 2.2.

Example 2.2: Let N_1 be the incidence matrix of an Efficiency Balanced Incomplete Block Design with parameters $v = 6$, $b_1 = 11$, $r_1 = 4$ or 5, $k_1 = 2, 4$ and N_2 be the incidence matrix of an Efficiency Balanced Incomplete Block Design with parameters $v = 6$, $b_2 = 21$, $r_2 = 15$, or 30, $k_2 = 5$. The resulting design is the incidence matrix of a variance balanced design with parameters $v'=7$, $b'= 32$, $k' = 3, 5$ or 6, $r'= 19, 32$ or 35.

Efficiency Balanced Designs Incidence Matrices	Variance Balanced Design
$N_1 = \begin{bmatrix} 1&1&1&1&0&0&0&0&0&0&0 \\ 1&1&0&0&1&1&0&0&0&0&0 \\ 1&1&0&0&0&0&1&1&0&0&0 \\ 1&1&0&0&0&0&0&0&1&1&0 \\ 0&0&1&0&1&0&1&0&1&0&1 \\ 0&0&0&1&0&1&0&1&0&1&1 \end{bmatrix}$ $N_2 = \begin{bmatrix} 1&1&1&1&1&0&1&1&1&1&0&1&1&1&1&1&1&0&0&0&0 \\ 1&1&1&1&0&1&1&1&1&0&1&1&1&1&0&0&0&1&1&1&0 \\ 1&1&1&0&1&1&1&1&0&1&1&1&0&0&1&1&0&1&1&0&1 \\ 1&1&0&1&1&1&1&0&1&1&1&0&1&0&1&0&1&1&0&1&1 \\ 1&0&1&1&1&1&0&1&1&1&1&0&0&1&0&1&1&0&1&1&1 \\ 0&1&1&1&1&1&1&1&1&1&1&2&2&2&2&2&2&2&2&2&2 \end{bmatrix}$	$N' = \begin{bmatrix} N_1 & N_2 \\ J & J \end{bmatrix}$

Note: A variance balanced design with unequal replications is not an efficiency balanced design but a Balanced Incomplete Block Design with unequal replications becomes a variance balanced design.

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REFERENCES

[1] Agrawal, H. L. and Prasad, J. (1982). Some methods of construction of Balanced Incomplete Block Design with nested row and columns, Biometrika, 69, 481-483.
[2] Agrawal, H. L. and Prasad. J (1983). On construction of Balanced Incomplete Block Design with nested rows and columns, Sankhya-B, Vol. 45(3), 345-350.
[3] Cheng, C.S. (1986). A method for constructing Balanced Incomplete Block Design with nested rows and columns, Biometrika, Vol. 73(3), 695-700.
[4] Das, M.N. and Ghosh, D.K. (1985). Balancing Incomplete Block Designs, Sankhya-B, Vol. 47, 67-77.
[5] David, C. and Uddin, N. (1999). On optimal row-column designs for two treatments, Journal of Statistical Research, Vol. 33, 27-34.
[6] Ehrenfeld S. (1953). On the Efficiency of Experimental Designs, Annals of Mathematical Statistics, Vol. 26, 247-255.
[7] Hishida T. and Jimbo M. (2002). Construction of Balanced Incomplete Block Design with nested rows and columns, Journal of Statistical Planning and Inference, Vol. 106, 47-56.
[8] Prasad, R., Gupta, V.K. and Voss, D. (2001). Optimal nested row - column designs, Journal of Indian Society for Agricultural Statistics, Vol. 54(2), 244-257.
[9] Prasad, R. (2001). Block design with nested rows and columns, IASRI, Library Avenue, New Delhi.
[10] Singh. M and Dey, A. (1979). Block design with nested rows and columns, Biometrika, Vol. 66, 321-326.
[11] Sreenath, P.R. (1989). Miscellanea construction of some Balanced Incomplete Block Designs with nested rows and columns, Biometrika, Vol. 76(2), 399-402.

[12] Sreenath, P.R. (1991). Construction of Balanced Incomplete Block Design with nested rows and columns through the method of differences, Sankhya-B, Vol. 53(3), 352-358.

[13] Sreenath, P.R. (1999). Balanced Incomplete Block Design with nested rows and columns, Technical Report, IASRI, New Delhi.

[14] Uddin, N. (1992). Construction for some Balanced Incomplete Block Designs with nested rows and columns, Journal of Statistical Planning and Inference, 31, 253-261.

[15] Uddin, N. (1995). On recursive construction for Balanced Incomplete Block Designs with nested rows and columns, Metrika, 42, 341-345.

[16] Uddin, N. (2002). A method of construction of class of universally optimal structurally incomplete row-column designs, Statistics and Probability Letters, 59, 93- 98.

[17] Uddin N. and Morgan, J. P. (1990). Some constructions for Balanced Incomplete Block Design with nested rows and column, Biometrika, 77 (1), 193-202.

[18] Uddin, N. and Morgan, J. P. (1991). Two construction of Balanced Incomplete Block Design with nested rows and columns, Statistic Sinica, 1, 229-232.

[19] Uddin, N. and Morgan, J. P. (1997). Orthogonal sets of Balanced Incomplete Block Design with nested rows and columns, Journal of Statistical Planning and Inference, 59, 369-378.