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FUZZY IRRESOLVABLE SETS AND FUZZY OPEN HEREDITARILY IRRESOLVABLE SPACES

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Abstract:
In this paper, several characterizations of fuzzy irresolvable sets in fuzzy topological spaces are established. The existence of fuzzy regular closed sets and fuzzy regular open sets in fuzzy topological spaces is established by means of fuzzy irresolvable sets. By means of fuzzy resolvable and irresolvable sets, the notions of fuzzy hereditarily irresolvable spaces and fuzzy open hereditarily irresolvable spaces are introduced and studied.

Keywords: Fuzzy dense set, fuzzy somewhere dense set, fuzzy resolvable set, fuzzy hyperconnected space, fuzzy globally disconnected space, fuzzy strongly irresolvable space.

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1. INTRODUCTION

The notion of fuzzy sets as an approach to a mathematical representation of vagueness in everyday language was introduced by L.A. Zadeh [16] in his classical paper in the year 1965. The potential of fuzzy notion was realized by the researchers and has successfully been applied in all branches of Mathematics. In1968, C.L. Chang [2] introduced the concept of fuzzy topological spaces. The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. In1943, E. Hewitt [3] introduced the concepts of resolvable and irresolvable spaces in classical topology. The concept of open hereditarily irresolvable spaces was introduced by E.K. van Douwen in [15].The concept of fuzzy resolvable sets in fuzzy topological spaces was introduced and studied in [9]. In continuation of this work, the notions of fuzzy hereditarily irresolvable spaces and fuzzy open hereditarily irresolvable spaces are introduced and studied by means of fuzzy irresolvable sets. In this paper, several characterizations of fuzzy irresolvable sets in fuzzy topological spaces are established. The existence of fuzzy regular closed sets and fuzzy regular open sets in fuzzy topological spaces is established by means of fuzzy irresolvable sets. It is established that fuzzy hyperconnected spaces, are not fuzzy hereditarily irresolvable spaces and fuzzy hereditarily irresolvable spaces are not fuzzy hyperconnected spaces. The conditions, under which fuzzy topological spaces become fuzzy non-hereditarily irresolvable spaces, are also obtained in this paper.

2. PRELIMINARIES

In order to make the exposition self-contained, some basic notions and results used in the sequel, are given. In this work by (X,T) or simply by X, we will denote a fuzzy topological space due to Chang (1968). Let X be a non-empty set and I the unit interval [0,1]. A fuzzy set λ in X is a mapping from X into I. The fuzzy set 0_X is defined as $0_X(x) = 0$, for all $x \in X$ and the fuzzy set 1_X is defined as $1_X(x) = 1$, for all $x \in X$.

Definition 2.1 [2]**:** Let (X, T) be a fuzzy topological space and λ be any fuzzy set in (X, T). The interior and the closure of λ, are defined respectively as follows:

(i). $\mathrm{int}(\lambda) = \vee \{ \mu / \mu \leq \lambda, \mu \in T\}$ (ii). $\mathrm{cl}(\lambda) = \wedge \{ \mu / \lambda \leq \mu, 1-\mu \in T \}$.

Definition 2.1 [2]**:**For a family $\{\lambda_i / i \in I\}$ of fuzzy sets in (X, T), the union $\psi = \vee_i (\lambda_i)$ and intersection $\delta = \wedge_i (\lambda_i)$, are defined respectively as

(iii). $\psi(x) = sup_i\{\lambda_i(x) / x \in X\}$ (iv). $\delta(x) = inf_i\{\lambda_i(x) / x \in X\}$.

Lemma 2.1 [1]**:** For a fuzzy set λ of a fuzzy topological space X,

(i). $1-\mathrm{int}(\lambda) = \mathrm{cl}(1-\lambda)$ and (ii). $1-\mathrm{cl}(\lambda) = \mathrm{int}(1-\lambda)$.

Definition 2.2: A fuzzy set λ in a fuzzy topological space (X,T), is called a

(i). ***fuzzy dense set*** if there exists no fuzzy closed set μ in (X,T) such that $\lambda<\mu<1$.That is, $\mathrm{cl}(\lambda)= 1$, in (X,T) [5].

(ii) ***fuzzy nowhere dense set*** if there exists no non-zero fuzzy open set μ in (X,T) such that $\mu<\mathrm{cl}(\lambda)$.That is, $\mathrm{intcl}(\lambda)=0$, in (X,T) [5].

(iii). ***fuzzy simply open set*** if $\mathrm{Bd}(\lambda)$is a fuzzy nowhere dense set in(X,T). That is, λ is a fuzzy simply open set in(X, T) if $[\mathrm{cl}(\lambda) \wedge \mathrm{cl}(1-\lambda)]$, is a fuzzy nowhere dense set in (X,T) [7].

(iv). ***fuzzy simply* open set*** if $\lambda= \mu \vee \delta$, where μ is a fuzzy open set and δ is a fuzzy no where dense set in (X,T) [8].

(v). ***fuzzy somewhere dense set*** if $\mathrm{int\,cl}(\lambda) \neq 0$ in (X,T) [14].

(vi).***fuzzy resolvable set*** in (X,T) if for each fuzzy closed set μ in (X,T),$\{ \mathrm{cl}(\mu \wedge \lambda\, \mathrm{cl}(\mu \wedge [1-\lambda]) \}$ is a fuzzy nowhere dense in (X,T)[10].

(vii). ***fuzzy regular- open set*** in (X, T)if $\lambda=$ $\mathrm{int\,cl}(\lambda)$ and ***fuzzy regular – closed set*** in (X, T) if $\lambda = \mathrm{cl\,int}(\lambda)$[1].

Definition 2.3:A fuzzy topological space (X,T) is called a

(i).***fuzzy strongly irresolvable space*** if for every fuzzy dense set λ in (X,T), $\mathrm{cl\,int}(\lambda)= 1$ in (X,T)[13].

(ii). ***fuzzy hyperconnected space*** if every non - null fuzzy open subset of (X,T), is fuzzy dense in (X,T) [4].

(iii). ***fuzzy globally disconnected space*** if each fuzzy semi-open set is fuzzy open in (X,T)[11]**.**

Theorem 2.1 [12]**:** If λ is a fuzzy somewhere dense set in a fuzzy topological space(X, T), then there exist a fuzzy regular closed set η in(X, T) such that $\eta \leq \mathrm{cl}(\lambda)$.

Theorem 2.2 [9]**:** If λ is a fuzzy nowhere dense set in a fuzzy globally disconnected space (X,T), then λ is a fuzzy resolvable set in (X, T).

Theorem 2.3 [9]**:** If λ is a fuzzy dense set in a fuzzy strongly irresolvable and fuzzy globally disconnected space (X, T), then λ is a fuzzy resolvable set in (X, T).

Theorem2.4 [9]**:** If λ is a fuzzy simply* open and fuzzy dense set in a fuzzy strongly irresolvable space (X, T), then λ is a fuzzy resolvable set in (X,T).

Theorem 2.5 [10] **:** If λ is a fuzzy resolvable set in a fuzzy topological space (X,T), then for each fuzzy closed set μ in (X,T), $[\lambda \wedge (1-\lambda) \wedge \mu]$ is a fuzzy nowhere dense set in (X,T).

Theorem 2.6 [10] **:** If λ is a fuzzy open in a fuzzy hyper-connected space (X,T), then λ is a fuzzy resolvable set in (X,T).

Theorem 2.7 [1]**:** In a fuzzy topological space

(a). The closure of a fuzzy open set is a fuzzy regular closed set.

(b). The interior of a fuzzy closed set is a fuzzy regular open set.

3. FUZZY IRRESOLVABLE SETS

Definition 3.1: Let (X,T) be a fuzzy topological space. A fuzzy set λ is called a fuzzy irresolvable set if for a fuzzy closed set μ in (X,T),$\{cl(\mu\wedge \lambda) \wedge cl(\mu \wedge[1-\lambda])\}$ is a fuzzy somewhere dense in (X,T).That is, λ is a fuzzy irresolvable set in (X,T) if $intcl\{cl(\mu\wedge\lambda)\wedge cl(\mu\wedge[1-\lambda])\}\neq 0$, where $1-\mu \in T$.

Example 3.1: Let $X = \{ a, b, c\}$. Consider the fuzzy sets α, β, γ and λ defined on X as follows :

α: $X \to [0, 1]$ is defined as $\alpha(a) = 0.5$;$\alpha (b) = 0.4$; $\alpha (c) = 0.7$.

β: $X\to [0, 1]$ is defined as $\beta (a) = 0.6$; $\beta(b) = 0.5$; $\beta (c) = 0.6$.

γ: $X \to [0, 1]$ is defined as $\gamma(a) = 0.4$; $\gamma (b) = 0.6$; $\gamma (c) = 0.3$.

λ : $X\to [0, 1]$ is defined as $\lambda (a) = 0.4$; $\lambda (b) = 0.5$; $\lambda(c) = 0.5$.

Then, $T= \{ 0, \alpha, \beta, \gamma ,\alpha \vee \beta, \alpha \vee \gamma,\beta\vee \gamma,\alpha \wedge \beta,\alpha \wedge \gamma,\beta \wedge \gamma , \alpha \vee [\beta \wedge \gamma],\gamma\vee[\alpha \wedge \beta],\beta \wedge [\alpha \vee \gamma],\alpha \vee \beta\vee\gamma,1\}$ is a fuzzy topology on X. On computation, for a fuzzy closed set$1- \alpha$, $intcl(cl\{(1-\alpha)\wedge\lambda\}\wedge cl\{(1-\alpha)\wedge(1-\lambda)\}) = int(1-[\alpha \vee \beta]) = \beta \wedge \gamma \neq 0$, in (X,T). Hence λ is a fuzzy irresolvable set in (X,T).

Proposition 3.1: If λ is a fuzzy irresolvable set in a fuzzy topological space (X,T), then bd (λ) is a fuzzy somewhere dense set in (X,T).

Proof: Let λ be a fuzzy irresolvable set in(X,T). Then, for a fuzzy closed set μ in(X,T),$int\, cl\{cl(\lambda \wedge \mu) \wedge cl([1-\lambda] \wedge \mu)\} \neq 0$, in (X,T). Now $intcl\{cl(\lambda\wedge\mu)\wedge cl([1-\lambda] \wedge\mu)\}\leq intcl\{cl(\lambda) \wedge cl(\mu) \wedge cl[1-\lambda])\wedge cl(\mu)\}=intcl\{[cl(\lambda)\wedge\mu]\wedge[cl(1-\lambda)\wedge\mu]=intcl\{[cl(\lambda)\wedge cl(1-\lambda)]\wedge\mu\}=intcl\{[bd(\lambda)\wedge\mu]\}\leq intcl[bd(\lambda)]$.

Thus, $intcl\{cl (\lambda \wedge \mu) \wedge cl([1-\lambda] \wedge \mu)\} \neq 0$, implies that $intcl[bd(\lambda)]=0$, in(X,T). Thus, $bd(\lambda)$ is a fuzzy somewhere dense in (X,T).

Remark 3.1: In view of the above proposition one will have the following result:

"If λ is a fuzzy irresolvable set in a fuzzy topological space (X,T), then λ is not a fuzzy simply open set in (X,T)".

Proposition 3.2: If λ is a fuzzy irresolvable set in a fuzzy topological space(X,T), then there exists a fuzzy regular closed set η in (X,T) such that $\eta \leq bd(\lambda)$.

Proof: Let λ be a fuzzy irresolvable set in(X,T). Then, by proposition 3.1, $bd(\lambda)$ is a fuzzy somewhere dense set in (X,T) and thus $intcl[bd(\lambda)]\neq 0$, in (X,T). Then there exists a fuzzy open setδ in (X,T) such that $\delta \leq cl [bd(\lambda)]$.This implies that $cl(\delta) \leq clcl[bd(\lambda)]$ and then $cl(\delta) \leq cl [bd (\lambda)]$. Since the closure of a fuzzy open set is a fuzzy regular closed set [by theorem 2.7], cl (δ) is a fuzzy regular closed set in (X,T). Let $\eta = cl (\delta)$. Hence, there exists a fuzzy regular closed set η in (X,T) such that $\eta \leq cl [bd(\lambda)]$. Since $bd(\lambda) = cl(\lambda)\wedge cl(1 -\lambda)$, $bd(\lambda)$ is a fuzzy closed set in(X,T) and then $cl [bd(\lambda)] = bd(\lambda)$ and hence there exists a fuzzy regular closed set η in (X,T) such that $\eta \leq bd (\lambda)$.

Proposition 3.3: If λ is a fuzzy irresolvable set in a fuzzy topological space (X, T), then λ and $(1-\lambda)$ are fuzzy somewhere dense sets in (X,T).

Proof: Let λ be a fuzzy irresolvable set in(X,T). Then, for a fuzzy closed set μ in(X,T), $intcl\{ cl(\lambda \wedge \mu) \wedge cl([1-\lambda] \wedge \mu)\} \neq 0$, in (X,T). Now $intcl\{cl(\lambda\wedge\mu)\wedge cl([1-\lambda]\wedge\mu)\}=intcl\{cl(\lambda)\wedge cl(\mu)\wedge cl[1-\lambda])\wedge cl(\mu)\}=intcl\{[cl(\lambda)\wedge\mu]\wedge[cl(1-\lambda)\wedge\mu]\}=intcl\{[cl(\lambda)\wedge cl(1-\lambda)]\wedge\mu\}\leq int\{clcl(\lambda)\wedge clcl(1-\lambda)\wedge cl(\mu)\} =int\{cl(\lambda)\wedge cl(1-\lambda)]\wedge\mu\}=intcl(\lambda)\wedge intcl(1-\lambda)]\wedge int(\mu)\}$.Thus, $intcl\{cl(\lambda \wedge \mu) \wedge cl ([1-\lambda] \wedge \mu)\} \neq 0$, implies that $[intcl(\lambda)]\wedge[intcl(1-\lambda)]\wedge[int(\mu)]\}\neq 0$,in (X,T) and thus $intcl(\lambda)\neq 0$, $intcl(1-\lambda)\neq 0$ and $int(\mu)\neq 0$. Hence λ and$(1-\lambda)$ are fuzzy somewhere dense sets in(X,T).

Proposition 3.4: If a fuzzy open set λ is a fuzzy irresolvable set in a fuzzy topological space(X,T), then λ is not a fuzzy dense set in (X,T).

Proof: Let λ be a fuzzy open set in (X,T). By hypothesis, λ is a fuzzy irresolvable set in(X,T).Then, by proposition3.3, $(1-\lambda)$ is a fuzzy somewhere dense set in (X,T) and then $intcl (1- \lambda) \neq 0$, in (X,T). Now $intcl(1- \lambda) = 1- clint(\lambda) =1 -cl(\lambda)$ and then $1-cl (\lambda) \neq 0$.This implies that $cl (\lambda) \neq 1$, in (X,T). Hence λ is not a fuzzy dense set in (X,T).

Remark3.2: In view of the above proposition one will have the following result:

"The fuzzy open and fuzzy irresolvable sets in a fuzzy topological space are not fuzzy dense sets."

Proposition 3.5: If λ is a fuzzy closed set in a fuzzy topological space (X,T) in which fuzzy open sets are fuzzy irresolvable sets, then $int(\lambda) \neq 0$, in (X, T).
Proof: Let λ be a fuzzy closed set in (X, T). Then, $1-\lambda$ is a fuzzy open set in (X, T). Then, by proposition 3.4, $cl(1-\lambda) \neq 1$, and thus $1-int(\lambda) \neq 1$, in (X, T). Hence $int(\lambda) \neq 0$, in (X, T).

Proposition 3.6: If λ is a fuzzy set defined on X in a fuzzy topological space (X,T) in which fuzzy open sets are fuzzy irresolvable sets, then $1-\lambda$ is a fuzzy somewhere dense set in (X,T).
Proof: Let λ be a fuzzy set defined on X in (X,T). If $int(\lambda)$ is a non-zero fuzzy open set in (X,T), then by hypothesis, $int(\lambda)$ is a fuzzy irresolvable set in (X,T). By proposition 3.4, $cl[int(\lambda)] \neq 1$, in (X, T). This implies that $1-cl[int(\lambda)] \neq 0$ and thus $intcl(1-\lambda) \neq 0$, in (X, T). Hence $1-\lambda$ is a fuzzy somewhere dense set in (X,T).

Proposition 3.7: If λ is a fuzzy set defined on X in a fuzzy topological space (X,T) in which fuzzy open sets are fuzzy irresolvable sets, then there exists a fuzzy regular open set δ in (X, T) such that $int(\lambda) \leq \delta$.
Proof: Let λ be a fuzzy set defined on X in the fuzzy topological space (X,T). Then, by proposition 3.6, $1-\lambda$ is a fuzzy somewhere dense set in (X,T). By the Theorem2.1, there exists a fuzzy regular closed set η in (X, T) such that $\eta \leq cl(1-\lambda)$. Then, $\eta \leq 1-int(\lambda)$. This implies that $int(\lambda) \leq 1-\eta$, in (X, T). Now $1-\eta$ is a fuzzy regular open set in (X, T). Let $\delta = 1-\eta$. Thus, there exists a fuzzy regular open set δ in (X, T) such that $int(\lambda) \leq \delta$.

Proposition 3.8: If λ is a fuzzy closed set in a fuzzy topological space (X,T) in which fuzzy open sets are fuzzy irresolvable sets then λ is fuzzy somewhere dense set in (X, T).
Proof: Let λ be a fuzzy closed set in (X, T) in which fuzzy open sets are fuzzy irresolvable sets. Then, by proposition 3.5, $int(\lambda) \neq 0$, in (X, T). Now $intcl(\lambda) = int(\lambda) \neq 0$, in (X, T). Hence, λ is a fuzzy somewhere dense set in (X, T).

Proposition 3.9 : If λ is a fuzzy closed set in a fuzzy topological space (X,T) in which fuzzy open sets are fuzzy irresolvable sets, and if $\lambda \leq \mu$, then μ is a fuzzy somewhere dense set in (X, T).
Proof: Let λ be a fuzzy closed set in (X,T) in which fuzzy open sets are fuzzy irresolvable sets. Then , by proposition 3.8, λ is a fuzzy somewhere dense set in (X,T) and thus $intcl(\lambda) \neq 0$. Now $\lambda \leq \mu$ implies that $int\,cl(\lambda) \leq int\,cl(\mu)$. Then $int\,cl(\mu) \neq 0$. Hence μ is a fuzzy somewhere dense set in (X, T).

Proposition 3.10: If λ is a fuzzy somewhere dense set in a fuzzy topological space (X, T) in which fuzzy open sets are fuzzy resolvable sets, then $int[\lambda \wedge (1-\lambda)] = 0$, in (X,T).
Proof : Let λ be a fuzzy somewhere dense set in (X,T). Then , $int\,cl(\lambda) \neq 0$. By hypothesis, the fuzzy open set $intcl(\lambda)$, is a fuzzy resolvable set in (X,T). Since $intcl(\lambda) \neq 0$, there exists a fuzzy open set δ in (X, T) such that $\delta \leq cl(\lambda)$ and then $1-\delta \geq 1-cl(\lambda)$. Now $int(1-\delta) \geq int[1-cl(\lambda)]$ implies that $int(1-\delta) \geq 1-cl[cl(\lambda)]$. Thus, $int(1-\delta) \geq 1-cl(\lambda)$ in (X, T) (since $cl[cl(\lambda)] = cl(\lambda)$). Let $\gamma = 1-\delta$ .Then γ is a fuzzy closed set with $int(\gamma) \neq 0$, in (X,T). Since the non – zero fuzzy open set $int\,cl(\lambda)$ is a fuzzy resolvable set in (X, T), by theorem 2.5, for the fuzzy closed set γ, $[int\,cl(\lambda) \wedge (1-[int\,cl(\lambda)]) \wedge \gamma]$ is a fuzzy nowhere dense set in (X,T). This implies that $intcl\{[int\,cl(\lambda) \wedge (1-[int\,cl(\lambda)]) \wedge \gamma]\} = 0$, in (X,T). But $intintcl(\lambda) \wedge (1-[intcl(\lambda)]) \wedge \gamma]\} \leq intcl\{intcl(\lambda) \wedge (1-[cl(\lambda)]) \wedge \gamma)\}$, implies that $int\{intcl(\lambda) \wedge (1-[intcl(\lambda)])\gamma]\} = 0$, in (X,T). This implies that $\{intint\,cl(\lambda) \wedge int(1-[intcl(\lambda)]) \wedge int(\gamma)\} = 0$ and then $\{int\,cl(\lambda) \wedge (1-cl[int\,cl(\lambda)]) \wedge int(\gamma)\} = 0$. Now $int(\lambda) \leq int\,cl(\lambda)$, $int(\gamma) \geq 1-cl(\lambda)$, $1-cl[int\,cl(\lambda)] \geq 1-cl[\lambda]$, implies that $int(\lambda) \wedge [1-cl(\lambda)] \wedge [1-cl(\lambda)] \leq \{int\,cl(\lambda) \wedge (1-cl[int\,cl(\lambda)]) \wedge int(\gamma)\}$ and thus $int(\lambda) \wedge [1-cl(\lambda)] = 0$. This implies that $int(\lambda) \wedge int(1-\lambda) = 0$, in (X,T) and $int[\lambda \wedge (1-\lambda)] = int(\lambda) \wedge int(1-\lambda)$ implies that $int[\lambda \wedge (1-\lambda)] = 0$, in (X,T).

Proposition 3.11: If λ is a fuzzy somewhere dense set in a fuzzy topological space (X, T) in which fuzzy open sets are fuzzy resolvable sets, $cl[\lambda \vee (1-\lambda)] = 1$, in (X,T).
Proof: Let λ be a fuzzy somewhere dense set in (X,T). Since (X, T) is a fuzzy topological space in which fuzzy open sets are fuzzy resolvable sets, by proposition 3.10, $int[\lambda \wedge (1-\lambda)] = 0$, in (X,T).Then $1-int[\lambda \wedge (1-\lambda)] = 1$. This implies that $1-int[\lambda \wedge (1-\lambda)] = 1$ and then $cl(1-[\lambda \wedge (1-\lambda)]) = 1$. Now $cl(1-[\lambda \wedge (1-\lambda)]) = cl((1-\lambda) \vee [1-(1-\lambda)]) = cl[\lambda \vee (1-\lambda)]$, implies that $cl[\lambda \vee (1-\lambda)] = 1$, in (X,T).

4. FUZZY HEREDITARILY IRRESOLVABLE SPACES

Definition 4.1: A fuzzy topological space (X,T) is called a fuzzy hereditarily irresolvable space if there is no non-zero fuzzy resolvable set in (X,T).

Proposition4.1: If λ is a fuzzy set in a fuzzy topological space (X,T) in which fuzzy closed sets have zero interiors, then λ is a fuzzy resolvable set in (X,T).

Proof: Let λ be a non-zero fuzzy set defined on X in (X, T).Then, for a fuzzy closed set μ in (X, T),by hypothesis, $int(\mu)=0$,in(X,T).Now $intcl\{cl(\lambda \wedge \mu) \wedge cl([1-\lambda] \wedge \mu)\} \leq intcl\{cl(\lambda) \wedge cl(\mu) \wedge cl[1-\lambda]) \wedge cl(\mu) = intcl\{[cl(\lambda) \wedge \mu] \wedge [cl(1-\lambda) \wedge \mu]\} = intcl\{[cl(\lambda) \wedge cl(1-\lambda)] \wedge \mu\} \leq int\{clcl(\lambda) \wedge clcl(1-\lambda) \wedge cl(\mu)\} = int\{[cl(\lambda) \wedge cl(1-\lambda)] \wedge \mu\} = int\{bd(\lambda) \wedge \mu\} = int\{bd(\lambda)\} \wedge int\{\mu\} = int\{bd(\lambda)\} \wedge 0 = 0$.Thus, for a fuzzy closed set μ, $intcl\{cl(\lambda \wedge \mu) \wedge cl([1-\lambda] \wedge \mu)\}=0$ and hence λ is a fuzzy resolvable set in (X,T).

Proposition 4.2: If $\lambda \leq \mu$, for each fuzzy closed set μ with $int(\mu) = 0$ in a fuzzy topological space (X, T), then λ is a fuzzy resolvable set but not a fuzzy open set in (X,T).

Proof: Let λ be a fuzzy set defined on X and μ is a fuzzy closed set such that $int(\mu) = 0$ in (X, T). Then, by proposition 4.1, λ is a fuzzy resolvable set in (X, T). Since $\lambda \leq \mu$, $int(\lambda) \leq int(\mu)andint(\mu) = 0$, implies that $int(\lambda)=0$ and hence λ is not a fuzzy open set in (X,T).

Proposition 4.3: If λ is a fuzzy set in a fuzzy topological space (X,T) in which fuzzy open sets are fuzzy dense sets, then λ is a fuzzy resolvable set in (X,T).

Proof: Let λ be a non-zero fuzzy set in (X, T). If μ is a non-zero fuzzy closed set in (X,T),then$1-\mu$ is a fuzzy open set in (X,T) and by hypothesis, $cl(1-\mu) = 1$, in(X,T). Now$1-int(\mu) = cl(1-\mu) = 1$, implies that $int(\mu)=0$, in (X,T).Then, by proposition 4.1 λ is a fuzzy resolvable set in (X,T).

Proposition 4.4: If λ is a fuzzy set in a fuzzy topological space (X,T) in which $int\{bd(\lambda)\} = 0$, then λ is a fuzzy resolvable set in (X,T).

Proof: Let λ be a non-zero fuzzy set defined on X in (X,T). By hypothesis, $int\{bd(\lambda)\} = 0$, in(X, T). Now, for a fuzzy closed set μ in (X, T), as in the proof of proposition4.1, $intcl\{cl(\lambda \wedge \mu) \wedge cl([1-\lambda] \wedge \mu)\} \leq int\{bd(\lambda)\} \wedge int\{\mu\} = 0 \wedge int\{\mu\} = 0$ and thus, for a fuzzy closed set μ in (X,T), $intcl\{cl(\lambda \wedge \mu) \wedge cl([1-\lambda] \wedge \mu)\} = 0$, implies that λ is a fuzzy resolvable set in (X,T).

Remark 4.1: In view of the propositions 4.1, 4.3 and 4.4, one will have the following results:

(i).“The fuzzy topological spaces in which fuzzy closed sets have zero interiors, are not fuzzy hereditarily irresolvable spaces."

(ii).“The fuzzy topological spaces in which fuzzy open sets are fuzzy dense sets, are not fuzzy hereditarily irresolvable spaces."

(iii)“The fuzzy topological spaces in which boundary of fuzzy sets have zero interiors, are not fuzzy hereditarily irresolvable spaces."

Proposition 4.5: If (X, T) is a fuzzy hyperconnected space, then (X,T) is not a fuzzy hereditarily irresolvable space.

Proof: Let (X,T) be a fuzzy hyperconnected space. Then, each fuzzy open set is a fuzzy dense set in (X,T). If λ is a non-zero fuzzy set defined on X, then by proposition 4.3, λ is a fuzzy resolvable set in (X,T) and hence (X,T) is not a fuzzy hereditarily irresolvable space.

Proposition 4.6: If there exists a fuzzy nowhere dense set in a fuzzy globally disconnected space (X,T), then (X,T) is not a fuzzy hereditarily irresolvable space .

Proof: Let(X,T) be a fuzzy globally disconnected space. Suppose that λ is a fuzzy nowhere dense set in (X,T).Then, by theorem 2.2, λ is a fuzzy resolvable set in (X,T) and hence (X,T) is not a fuzzy hereditarily irresolvable space.

Proposition 4.7: If there exists a fuzzy dense set in a fuzzy strongly irresolvable and fuzzy globally disconnected space (X,T), then (X,T) is not a fuzzy hereditarily irresolvable space.

Proof: Let(X,T) be a fuzzy strongly irresolvable and fuzzy globally disconnected space. Suppose that λ is a fuzzy dense set in (X,T). Then, by theorem 2.3, λ is a fuzzy resolvable set in (X,T) and hence (X,T) is not a fuzzy hereditarily irresolvable space.

Proposition 4.8: If there exist a fuzzy simply*open and fuzzy dense set in a fuzzy strongly irresolvable space (X,T), then (X,T) is not a fuzzy hereditarily irresolvable space.

Proof: Let (X,T) be a fuzzy strongly irresolvable space. Suppose that λ is a fuzzy simply*open and fuzzy dense set in (X,T).Then, by theorem 2.4, λ is a fuzzy resolvable set in (X,T) and hence(X,T) is not a fuzzy hereditarily irresolvable space.

Remark 4.2: In view of the propositions 4.6, 4.7 and 4.8, one will have the following results:

(i) *" The existence of fuzzy nowhere dense sets in a fuzzy globally disconnected space makes it a fuzzy non-hereditarily irresolvable space ".*

(ii) *" The existence of fuzzy dense sets in a fuzzy strongly irresolvable and fuzzy globally disconnected space makes it a fuzzy non-hereditarily irresolvable space".*

(iii) *"The existence of fuzzy simply* open and fuzzy dense sets in a fuzzy strongly irresolvable space makes it a fuzzy non-hereditarily irresolvable space".*

Proposition 4.9: If (X,T) is a fuzzy hereditarily irresolvable space and if λ is a fuzzy set in (X,T), then $\lambda$is a fuzzy irresolvable set in (X,T).

Proof : Let (X,T) be a fuzzy hereditarily irresolvable space and λ be a fuzzy set defined on X in (X,T). Then, there is no non-zero fuzzy resolvable set in (X, T), implies that for a fuzzy closed set μ in (X, T), int cl$\{cl(\lambda\wedge\mu)\wedge cl\ ([1-\lambda]\wedge\mu)\} \neq 0$ and hence the fuzzy set λ is a fuzzy irresolvable set in (X,T).

Proposition 4.10: If λ is a fuzzy closed set in a fuzzy hereditarily irresolvable space (X,T), then

(i). int $(\lambda) \neq 0$;

(ii). cl int$(\lambda) \neq 1$;

(iii). For a fuzzy closed set μ in (X, T), int $(\mu) \neq 0$.

Proof: Let λ be a fuzzy closed set in (X,T). Since (X, T) is a fuzzy hereditarily irresolvable space, by proposition 4.8, λ is a fuzzy irresolvable set in (X,T).Then, for a fuzzy closed set μ in(X,T), int $cl\{cl(\lambda\wedge\mu)\wedge cl([1-\lambda]\wedge\mu)\} \neq 0$,in (X,T). Since $intcl\{cl(\lambda\wedge\mu)\wedge cl([1-\lambda]\wedge\mu)\} \leq intcl\{[cl(\lambda)\wedge cl(\mu)]\wedge cl[1-\lambda]\wedge cl(\mu) = intcl\{[\lambda\wedge\mu]\wedge[cl(1-\lambda)\wedge\mu]\} = intcl\{[\lambda\wedge cl(1-\lambda)]\wedge\mu\} \leq int\{cl(\lambda)\wedge cl\ cl(1-\lambda)\wedge cl(\mu)\} = int\{\lambda\wedge cl(1-\lambda)\wedge(\mu)\} \leq int(\lambda)\wedge int\ cl(1-\lambda)\wedge int\ (\mu)$and hence $int(\lambda)\wedge int\ cl(1-\lambda)\wedge int\ (\mu) \neq 0$.This implies that int $(\lambda) \neq 0$, int $(\mu) \neq 0$ and int cl $(1-\lambda) \neq 0$.Thus, int $(\lambda)\neq 0$, int$(\mu) \neq 0$ and $1-$cl int $(\lambda) \neq 0$.Hence if λ is a fuzzy closed set in a fuzzy hereditarily irresolvable space (X,T), then int $(\lambda) \neq 0$; cl int$(\lambda) \neq 1$ and int $(\mu) \neq 0$, for a fuzzy closed set μ in (X, T).

Proposition 4.11: If (X, T) is a fuzzy hereditarily irresolvable space, then there is no non-zero fuzzy nowhere dense set in (X,T).

Proof: Suppose that λ is a fuzzy nowhere dense set in (X,T). Then intcl $(\lambda) = 0$, in (X, T). Now, for a fuzzy closed set μ in (X, T), int cl $\{cl\ (\lambda\wedge\mu)\wedge cl\ ([1-\lambda]\wedge\mu)\} \leq$ int $cl\{cl(\lambda)\wedge cl(\mu)\wedge cl\ [1-\lambda]) \wedge cl\ (\mu)\}$ = int $cl\{[cl\ (\lambda)\wedge\mu]\wedge[cl\ (1-\lambda)\wedge\mu]\}$ = int $cl\{[cl\ (\lambda)\wedge cl\ (1-\lambda)]\wedge\mu\} \leq int\{cl\ cl(\lambda)\wedge cl\ cl\ (1-\lambda)\wedge cl(\mu)\} = int\{cl\ (\lambda)\wedge cl\ (1-\lambda)]\wedge\mu\}$ = int cl $(\lambda)\wedge$int cl $(1-\lambda)] \wedge$ int (μ)in (X,T). Now int cl$(\lambda) = 0$, implies that int cl $(\lambda)\wedge$ int cl $(1-\lambda) \wedge$ int $(\mu) = 0 \wedge$ int cl $(1-\lambda)\wedge$ int $(\mu) = 0$ and then int cl $\{cl(\lambda\wedge\mu)\wedge cl\ ([1-\lambda]\wedge\mu)\} = 0$.This will imply that λ will be a fuzzy resolvable set in (X, T), a contradiction to (X,T) being a fuzzy hereditarily irresolvable space in which non-zero fuzzy sets are fuzzy irresolvable sets in (X, T). Thus there is no non - zero fuzzy nowhere dense set in (X, T).

Proposition 4.12: If (X,T) is a fuzzy hereditarily irresolvable space, then there is no non-zero fuzzy set λ in (X, T) such that cl int $(\lambda) = 1$.

Proof: Suppose that λ is a fuzzy set in (X, T) such that cl int$(\lambda) = 1$. Then, $1 -$ cl int$(\lambda) = 0$ and this implies that int cl $(1-\lambda) = 0$, in (X,T). Now, for a fuzzy closed set μ in (X, T), int $cl\{cl\ (\lambda\wedge\mu)\wedge cl\ ([1-\lambda]\wedge\mu)\} \leq$ int cl $(\lambda)\wedge$ int cl $(1-\lambda)\wedge$ int(μ) . Now int cl $(1-\lambda) = 0$, will imply that int cl $(\lambda)\wedge$ int cl $(1-\lambda)\wedge$ int (μ) = int cl $(\lambda)\wedge 0\wedge$ int $(\mu) = 0$ and then int $cl\{cl(\lambda\wedge\mu)\wedge cl\ ([1-\lambda]\wedge\mu)\} = 0$. This will imply that λ will be a fuzzy resolvable set in (X,T), a contradiction to (X,T) being a fuzzy hereditarily irresolvable space in which non-zero fuzzy sets are fuzzy irresolvable sets in (X, T).Thus, there is no non-zero fuzzy set λ in (X, T) such that cl int$(\lambda) = 1$.

Proposition 4.13: If (X,T) is a fuzzy hereditarily irresolvable space, then there is no non - zero fuzzy simply open set in (X,T).
Proof: Let λbe a fuzzy set defined on X in (X, T). Since (X, T) is a fuzzy hereditarily irresolvable space, λ is a fuzzy irresolvable set in (X,T). Then, by proposition 3.1, int cl [bd(λ)] $\neq 0$, in (X,T).Thus λ is not a fuzzy simply open set in (X,T).
Remark 4.3: In view of the propositions 4.11 and 4.13, one will have the following result :
"In fuzzy hereditarily irresolvable spaces, there are no fuzzy simply open sets and fuzzy nowhere dense set ".

Proposition4.14: If (X,T) is a fuzzy hereditarily irresolvable space, then the fuzzy simply* open sets are fuzzy open sets in (X,T).

Proof: Let λ be a fuzzy simply*- open set in (X, T). Then, $\lambda = \mu \vee \delta$, where μ is a fuzzy open set and δ is a fuzzy nowhere dense set in (X,T). Since (X, T) is a fuzzy hereditarily irresolvable space, by proposition 4.11, then there is no non - zero fuzzy nowhere dense set in (X,T) and hence $\lambda = \mu \vee 0 = \mu$ and thus the fuzzy simply open sets in a fuzzy hereditarily irresolvable space (X, T) are fuzzy open sets in (X,T).

5. FUZZY OPEN HEREDITARILY IRRESOLVABLE SPACES

Definition 5.1: A fuzzy topological space (X, T) is called a fuzzy open hereditarily irresolvable space if each non-zero fuzzy open set is a fuzzy irresolvable set in (X,T).

Example 5.1: Let X = { a, b, c}. Consider the fuzzy sets α,β and γ defined on X as follows:

α: X $\to [0, 1]$ is defined as $\alpha(a) = 0.5$; $\alpha(b) = 0.4$; $\alpha(c) = 0.7$.

β : X$\to [0,1]$ is defined as $\beta(a) = 0.6$; $\beta(b) = 0.5$; $\beta(c) = 0.6$;

γ: X $\to [0, 1]$ is defined as $\gamma(a) = 0.4$; $\gamma(b) = 0.6$; $\gamma(c) = 0.3$.

Then, T= $\{0, \alpha, \beta, \gamma, \alpha \vee \beta, \alpha \vee \gamma, \beta \vee \gamma, \alpha \wedge \beta, \alpha \wedge \gamma, \beta \wedge \gamma, \alpha \vee [\beta \wedge \gamma], \gamma \vee [\alpha \wedge \beta], \beta \wedge [\alpha \vee \gamma], (\alpha \vee \beta \vee \gamma), 1\}$ is a fuzzy topology on X. On computation, one can easily see that for a fuzzy closed set δ in (X, T) and, for each non-zero fuzzy open set μ in (X,T), int cl (cl$\{(1-\delta)\wedge\mu\} \wedge$ cl$\{(1-\delta)\wedge(1-\mu)\}$) $\neq 0$, in (X, T). Hence the fuzzy topological space (X,T) is af uzzy open hereditarily irresolvable space.

Proposition 5.1: If λ is a fuzzy closed set in a fuzzy open hereditarily irresolvable space (X,T), then int (λ) $\neq 0$, in (X, T).
Proof : Let λ be a fuzzy closed set in (X,T). Since(X, T) is a fuzzy open hereditarily irresolvable space, fuzzy open sets in (X, T) are fuzzy irresolvable sets in(X, T)and then, by proposition 3.5, int (λ) $\neq 0$, in (X, T).

Proposition 5.2: If λ is a fuzzy closed set in a fuzzy open hereditarily irresolvable space (X,T), thenλ is a fuzzy somewhere dense set in(X, T).
Proof: Let λ be a fuzzy closed set in (X,T). Since(X, T) is a fuzzy open hereditarily irresolvable space, by proposition 5.1, int (λ) $\neq 0$, in (X, T). Now int cl (λ) = int (λ) $\neq 0$ in (X, T). Hence, λ is a fuzzy somewhere dense set in (X, T).

Proposition 5.3: If $\lambda$is a fuzzy set defined on X in a fuzzy open hereditarily irresolvable space then there exists a fuzzy regular open set δ in(X, T) such that int (λ) $\leq \delta$.
Proof: Let λ be a fuzzy set defined on X in (X, T). Since(X, T) is a fuzzy open hereditarily irresolvable space, by proposition 3.7, there exists a fuzzy regular open set δ in(X, T) such that int (λ) $\leq \delta$.

Proposition 5.4: If λ is a fuzzy set defined on X in a fuzzy open hereditarily irresolvable space (X, T), then $1 - \lambda$ is a fuzzy somewhere dense set in (X,T).
Proof: Let λ be a fuzzy set defined on X in (X, T).Then, int (λ) is a fuzzy open set in (X,T).Since (X, T) is a fuzzy open hereditarily irresolvable space, by proposition 3.4, int (λ) is not a fuzzy dense set in (X, T). That is, cl int (λ) $\neq 1$ and then 1 $-$cl int (λ) $\neq 0$.This implies that int cl ($1-\lambda$) $\neq 0$. Hence $1 - \lambda$ is a fuzzy somewhere dense set in (X,T).

Proposition 5.5: If a fuzzy topological space (X,T) is a fuzzy open hereditarily irresolvable space, then int cl $(\lambda) \neq 0$, then int $(\lambda) \neq 0$, for any non-zero fuzzy set λ in (X, T).

Proof: Let (X,T) be a fuzzy open hereditarily irresolvable space and λ be a fuzzy set defined on X such that int cl $(\lambda) \neq 0$, in (X,T). Suppose that int $(\lambda) = 0$, in (X, T).Then, cl $(1-\lambda) = 1-\text{int}(\lambda) = 1$ and then $1-\lambda$ will be a fuzzy dense set in the fuzzy open hereditarily irresolvable space(X, T), a contradiction to $1-\lambda$ being a fuzzy somewhere dense set in (X,T), [by proposition 5.4] and thus if int cl $(\lambda) \neq 0$, for a non-zero fuzzy set in a fuzzy open hereditarily irresolvable space(X, T), then int $(\lambda) \neq 0$, in (X, T).

Proposition 5.6: If (X,T) is a fuzzy hereditarily irresolvable space, then (X,T) is a fuzzy open hereditarily irresolvable space.

Proof: Let λ be a fuzzy open set in (X,T). Since (X,T) is a fuzzy hereditarily irresolvable space, the fuzzy (open) set λ is a fuzzy irresolvable set in (X,T). Hence the fuzzy open set λ is a fuzzy irresolvable set in (X,T), implies that (X,T) is a fuzzy open hereditarily irresolvable space.

Proposition 5.7: If (X,T) is a fuzzy hyperconnected space, then (X,T) is not a fuzzy open hereditarily irresolvable space.

Proof: Let λ be a fuzzy open set in the fuzzy hyperconnected space (X,T). Then, by theorem 2.6, λ is a fuzzy resolvable set in (X, T). Thus the fuzzy open set is not a fuzzy irresolvable set in (X, T), implies that (X,T) is not a fuzzy open hereditarily irresolvable space .

Remark 5.1: In view of the above propositions 4.5and 5.6, one will have the following result:
"Fuzzy hyperconnected spaces are neither fuzzy hereditarily irresolvable spaces and nor fuzzy open hereditarily irresolvable spaces."

Proposition 5.8: If λ is a fuzzy open set in a fuzzy open hereditarily irresolvable space (X,T), then λ is not a fuzzy dense set in (X, T).

Proof: Let λ be a fuzzy open set in (X, T). Since (X, T) is a fuzzy open hereditarily irresolvable space, the fuzzy open set λ is a fuzzy irresolvable set in (X,T) and then by proposition 3.4, λ is not a fuzzy dense set in (X, T).

Proposition 5.9: If (X, T) is a fuzzy open hereditarily irresolvable space, then (X,T) is not a fuzzy hyperconnected space.

Proof: Let λ be a fuzzy open set in (X, T). Since (X, T) is a fuzzy open hereditarily irresolvable space, the fuzzy open setλ is a fuzzy irresolvable set in (X,T), and then by proposition 3.4, λ is not a fuzzy dense set in (X, T). Hence (X,T) is not a fuzzy hyperconnected space.

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