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Effects of chemical reaction and heat absorption on hydromagnetic flow past a moving plate through porous medium with ramped wall temperature in the presence of Hall current and thermal radiation

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Abstract

This paper deals with the study of effects of chemical reaction and heat absorption on an unsteady free convective flow of a viscous, incompressible, electrically conducting and optically thin fluid past an infinite vertical moving plate through porous medium with ramped wall temperature in the presence of Hall current and thermal radiation. The governing equations are converted into non-dimensional form and then solved by using the Laplace transform technique. The expressions are obtained for fluid velocity, fluid temperature, species concentration, skin friction, Nusselt number and Sherwood numbers. Effects of pertinent flow parameters are presented graphically and discussed quantitatively. It is found that the increase in chemical reaction and heat absorption parameters contribute in thinning the momentum and thermal boundary layers.

Keywords: Free convection, Chemical reaction, Hall current, Thermal radiation, Heat absorption.

Mathematics Subject Classification (2010) 76D05, 76D10, 76S05.

1. INTRODUCTION

MHD convection flow problems are very significant in science and technology. The study of free convection flow is important in geothermal systems, thermal insulations, oil extraction, iron blast furnaces and ground water hydrology. Keeping in view such facts Bejan and Khair [4], Takhar et al. [19], Chaudhary and Jain [6], Singh and Kumar [18], Ahmed et al. [1], Das et al. [8], Kundu et al. [13], Chand and Thakur [5] and Das et al. [9] studied hydromagnetic natural convection flow past a vertical plate considering different aspects of the problem.

The phenomenon of heat generation / absorption is observed in convection in Earth's mantle, fire and combustion modeling and fluids undergoing exothermic / endothermic chemical reaction. Kinyanjui et al. [12] considered the MHD free convection heat and mass transfer flow of a heat generating fluid past an impulsively started infinite vertical porous plate with Hall current and radiation absorption effect. Shankar et al. [17] examined radiation and mass transfer effects on MHD free convection fluid flow embedded in a porous medium with heat generation / absorption. The effect of Hall current on unsteady MHD natural convection flow of an electrically conducting, viscous, incompressible and temperature dependent heat absorbing fluid past an accelerated moving infinite vertical plate with ramped temperature through a porous medium in the presence of thermal diffusion has been analyzed by Seth et al. [16]. MHD free convection heat and mass transfer of an electrically conducting, viscous, incompressible and heat absorbing fluid flow past a vertical infinite flat plate embedded in non-Darcy porous medium has been investigated by Pandit et al.[14].

In chemical industry, chemical reaction plays an important role in tubular reactors, oxidation of solid materials and synthesis of ceramic materials. A chemical reaction may be of zero or first or second order. The first order reaction is the simplest chemical reaction in which the rate of reaction is directly proportional to the species concentration. The effect of chemical reaction on free convective heat and mass transfer problems has been discussed by several researchers in various situations. A numerical analysis to study the unsteady magneto hydrodynamic convective flow of a viscous, incompressible, electrically conducting Newtonian fluid along a vertical permeable plate in the presence of a homogeneous first order chemical reaction and taking into account thermal radiation effects has been carried out by Balla and Naikoti [3]. Ahmed and Kalita [2] analyzed non-linear MHD flow with heat and mass transfer characteristics of an incompressible, viscous, electrically conducting and Newtonian fluid over a vertical oscillating porous plate embedded in a porous medium in the presence of homogeneous chemical reaction of first order and thermal radiation effects. A numerical study based on finite difference scheme to investigate the effect of variable viscosity and thermal conductivity with chemical reaction on a transient MHD free convective mass transfer flow of an incompressible viscous electrically conducting, Newtonian fluid past a suddenly started infinite vertical plate with ramped wall temperature and concentration in presence of appreciable radiation heat transfer with viscous dissipation and Joulian heat and uniform transverse magnetic field has been presented by Hazarika and Doley [10]. Seth et al. [15] investigated Soret and Hall effects on unsteady MHD free convection heat and mass transfer flow of a viscous, incompressible, electrically conducting and optically thick radiating fluid past an impulsively moving infinite vertical plate with ramped temperature through a uniform porous medium in a rotating system in the presence of first order chemical reaction. Hussain et al. [11] analyzed MHD free convective heat and mass transfer flow over an accelerated moving vertical plate in the presence of heat absorption and chemical reaction with ramped temperature and ramped surface concentration through a porous medium in a rotating system taking Hall effects into account.

The main objective of this work is to extend the work of Das et al. [8] by studying the effects of chemical reaction and heat absorption on an unsteady, hydromagnetic, free convective flow of a viscous, incompressible, electrically conducting and optically thin fluid past an infinite vertical plate through porous medium with variable temperature in the presence of thermal radiation and Hall current. The Laplace transform technique is used to find the exact solution.

2. FORMULATION OF THE PROBLEM

Consider an unsteady free convective flow of a viscous, incompressible, electrically conducting and optically thin fluid past an infinite vertical plate through porous medium. The coordinate system is chosen in such a way that x^*-axis is taken along the plate in the upward direction, y^*-axis normal to it and z^*-axis perpendicular to $x^* y^*$-plane. A uniform magnetic field of strength B_0 is applied perpendicular to the plate. First order chemical reaction, heat absorption, radiation and Hall current effects are also taken into consideration.

Initially, at time $t^* \leq 0$ both the fluid and the plate are at rest and assumed to be at the same temperature T_∞^*. At time $t^* > 0$, the plate at $z^* = 0$ starts moving in its plane with uniform velocity U_0 and is heated with temperature $T_\infty^* + \left(T_w^* - T_\infty^*\right)\frac{t^*}{t_0^*}$. Since the plate is infinitely long in x^* and y^* directions, therefore all the physical quantities except pressure depend upon z^* and t^* only.
Under the usual Boussinesq approximation, equations governing the flow are given by

$$\frac{\partial w^*}{\partial z^*} = 0 \Rightarrow w^* = 0 \tag{1}$$

$$\frac{\partial u^*}{\partial t^*} = \upsilon \frac{\partial^2 u^*}{\partial z^{*2}} + g\beta\left(T^* - T_\infty^*\right) + g\beta_c\left(C^* - C_\infty^*\right) - \frac{\sigma B_0^2}{\rho\left(1+m^2\right)}\left(u^* - mv^*\right) - \frac{\upsilon u^*}{K^*} \tag{2}$$

$$\frac{\partial v^*}{\partial t^*} = \upsilon \frac{\partial^2 v^*}{\partial z^{*2}} - \frac{\sigma B_0^2}{\rho\left(1+m^2\right)}\left(v^* + mu^*\right) - \frac{\upsilon v^*}{K^*} \tag{3}$$

$$\rho C_p \frac{\partial T^*}{\partial t^*} = \kappa \frac{\partial^2 T^*}{\partial z^{*2}} - \frac{\partial q_r^*}{\partial z^*} - Q^*\left(T^* - T_\infty^*\right) \tag{4}$$

$$\frac{\partial C^*}{\partial t^*} = D\frac{\partial^2 C^*}{\partial z^{*2}} - Kr^*\left(C^* - C_\infty^*\right) \tag{5}$$

In above equations u^*, v^*, w^* – denote the components of velocity in the boundary layer in x^*, y^* and z^* directions respectively; T^* – the temperature of fluid in the boundary; T_∞^* – the temperature of the free stream; T_w^* – the temperature of the plate; C^* – species concentration in the fluid in the boundary layer region; C_∞^* – species concentration in the fluid far away from the plate; t^* – the time; t_0^* – the characteristics time; σ – the electrical conductivity; β – the volumetric coefficient of thermal expansion; β_c – volumetric coefficient of expansion with species concentration ρ – the density of fluid; g – the acceleration due to gravity; υ – the kinematic viscosity; K^* – the permeability of the medium; C_p – the heat capacity of fluid at constant pressure; B_0 – the magnetic field strength; κ – the thermal conductivity of the fluid; q_r^* – the radiative heat flux, Q^*-coefficient of heat absorption, D-molecular diffusivity and Kr^*-chemical molecular diffusivity.

The initial and boundary conditions for velocity and temperature profile are:

$$\left.\begin{aligned} &u^* = 0, v^* = 0, T^* = T_\infty^*, C^* = C_\infty^* \text{ for all } z^* \text{ and } t^* \le 0 \\ &u^* = U_0, v^* = 0, T^* = T_\infty^* + \left(T_w^* - T_\infty^*\right)\frac{t^*}{t_0^*}, C^* = C_w^* \text{ at } z^* = 0 \text{ for } t^* > 0 \\ &u^* \to 0, v^* \to 0, T^* \to T_\infty^*, C^* \to C_\infty^* \text{ as } z^* \to \infty \text{ for } t^* > 0 \end{aligned}\right\} \quad (6)$$

Following Cogley et al. [7], it is assumed that fluid is optically thin with a relatively low density and radiative heat flux is given by

$$\frac{\partial q_r^*}{\partial z^*} = 4\left(T^* - T_\infty^*\right) I \quad (7)$$

where $I = \int_0^\infty K_{\lambda_w} \left(\frac{\partial e_{\lambda p}}{\partial T^*}\right)_w d\lambda$ (8)

In equations (7) K_λ is the absorption coefficient, λ is the wavelength, $e_{\lambda p}$ is the Plank's function and the subscript 'w' points out that all quantities have been evaluated at the temperature T_∞^* which is the temperature of the wall at time $t^* \le 0$.

On the use of the equation (7), the equation (4) becomes

$$\rho C_p \frac{\partial T^*}{\partial t^*} = \kappa \frac{\partial^2 T^*}{\partial z^{*2}} - 4\left(T^* - T_\infty^*\right) I \quad (9)$$

To solve above equations, introducing following non-dimensional variables and parameters:

$$\left.\begin{aligned} &u = \frac{u^*}{U_0},\ v = \frac{v^*}{U_0}, z = \frac{z^* U_0}{\upsilon},\ t = \frac{t^*}{t_0},\ \theta = \frac{\left(T^* - T_\infty^*\right)}{\left(T_w^* - T_\infty^*\right)}, C = \frac{\left(C^* - C_\infty^*\right)}{\left(C_w^* - C_\infty^*\right)}, \Pr = \frac{\mu C_p}{\kappa}, \\ &M^2 = \frac{\sigma B_0^2 \upsilon}{\rho U_0^2}, K = \frac{K^* U_0^2}{\upsilon^2}, R = \frac{4 I \upsilon^2}{U_0^2 \kappa}, Gr = \frac{g\beta\upsilon\left(T_w^* - T_\infty^*\right)}{U_0^3}, Gm = \frac{g\beta_c\upsilon\left(C_w^* - C_\infty^*\right)}{U_0^3} \\ &Kr = \frac{Kr^*\upsilon}{U_0^2}, Q = \frac{Q^*\upsilon^2}{\kappa U_0^2}, Sc = \frac{\upsilon}{D} \end{aligned}\right\} \quad (10)$$

Using these dimensionless quantities, equations (2), (3), (5) and (9) transform to

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial z^2} + Gr\theta + GmC - \frac{M^2\left(u - mv\right)}{\left(1 + m^2\right)} - \frac{u}{K} \quad (11)$$

$$\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial z^2} - \frac{M^2 (v + mu)}{\left(1 + m^2\right)} - \frac{v}{K} \quad (12)$$

$$\Pr\frac{\partial\theta}{\partial t}=\frac{\partial^2\theta}{\partial z^2}-\left(R+Q\right)\theta \tag{13}$$

$$Sc\frac{\partial C}{\partial t}=\frac{\partial^2 C}{\partial z^2}-KrScC \tag{14}$$

where $M^2, K, Gr, Gm, \Pr, R, Q, Kr$ and Sc represents the magnetic parameter, permeability parameter, Grashof number, modified Grashof number, Prandtl number, radiation parameter, heat absorption parameter, chemical reaction parameter and Schmidt number respectively.

In the above non-dimensionalisation process, the characteristics time t_0^* can be defined as $t_0^*=\dfrac{\upsilon}{U_0^2}$

The corresponding initial and boundary conditions are

$$\left.\begin{array}{l} u=0, v=0, \theta=0, C=0 \text{ for all } z \text{ and } t\le 0 \\ u=1, v=0, \theta=t, C=1 \text{ at } z=0 \text{ for } t>0 \\ u\to 0, v\to 0, \theta\to 0, C\to 0 \quad z\to\infty \text{ for } t>0 \end{array}\right\} \tag{15}$$

3. METHOD OF SOLUTION

To solve the system of equations (11) and (12), we combine these equations as follows and get

$$\frac{\partial F}{\partial t}=\frac{\partial^2 F}{\partial z^2}-a_1F+Gr\theta+GrC \tag{16}$$

where $F=u+\iota v,\ a_1=\dfrac{M^2\left(1+\iota m\right)}{\left(1+m^2\right)}+\dfrac{1}{K}$ and $\iota=\sqrt{-1}$

The corresponding initial and boundary conditions are

$$\left.\begin{array}{l} F=0, \theta=0, C=0 \text{ for all } z \text{ and } t\le 0 \\ F=1, \theta=t, C=1 \text{ at } z=0 \text{ for } t>0 \\ F\to 0, \theta\to 0, C\to 0 \text{ as } z\to\infty \text{ for } t>0 \end{array}\right\} \tag{17}$$

Applying Laplace transformation on equations (13), (14) and (17) and on using initial conditions, we get

$$\frac{d^2\overline{F}}{dz^2}-(s+a_1)\overline{F}=-Gr\overline{\theta}-Gm\overline{C} \tag{18}$$

$$\frac{d^2\overline{\theta}}{dz^2}-\left(R+Q+s\Pr\right)+\overline{\theta}=0 \tag{19}$$

$$\frac{d^2\overline{C}}{dz^2}-\left(Kr+s\right)Sc\overline{C}=0 \tag{20}$$

where $\overline{F}(z,s)=\int_0^\infty F(z,t)e^{-st}dt, \overline{\theta}(z,s)=\int_0^\infty \theta(z,t)e^{-st}dt$ and $\overline{C}(z,s)=\int_0^\infty C(z,t)e^{-st}dt$

The corresponding boundary conditions are

$$\left.\begin{aligned}&\overline{F}(0,s)=\frac{1}{s},\ \overline{\theta}(0,s)=\frac{1}{s^2},\overline{C}(0,s)=\frac{1}{s}\\&\overline{F}\to 0,\ \overline{\theta}\to 0,\overline{C}\to 0 \text{ as } z\to\infty\end{aligned}\right\} \tag{21}$$

The solution of equations (18), (19) and (20) subject to boundary conditions (21) are given by

The solution of equations (18), (19) and (20) subject to boundary conditions (21) are given by

$$\overline{F}(z,s)=\begin{cases}\left(\frac{1}{s}+\frac{b_1}{s^2(s-a_2)}+\frac{b_2}{s(s-a_3)}\right)e^{-z\sqrt{(s+a_1)}}-\frac{b_1}{s^2(s-a_2)}e^{-z\sqrt{(R+Q+s\Pr)}}-\frac{b_2}{s(s-a_3)}e^{-z\sqrt{(Kr+s)Sc}} & \text{when } Sc\neq 1, \Pr\neq 1\\ \left(\frac{1}{s}+\frac{b_3}{s^2}+\frac{b_4}{s}\right)e^{-z\sqrt{(s+a_1)}}-\frac{b_3}{s^2}e^{-z\sqrt{(R+Q+s)}}-\frac{b_4}{s}e^{-z\sqrt{(Kr+s)}} & \text{when } Sc=1, \Pr=1\end{cases} \tag{22}$$

$$\overline{\theta}(z,s)=\begin{cases}\frac{1}{s^2}e^{-z\sqrt{(R+Q+s\Pr)}} & \text{when } \Pr\neq 1\\ \frac{1}{s^2}e^{-z\sqrt{(R+Q+s)}} & \text{when } \Pr=1\end{cases} \tag{23}$$

$$\overline{C}(z,s)=\begin{cases}\frac{1}{s}e^{-z\sqrt{(Kr+s)Sc}} & \text{when } Sc\neq 1\\ \frac{1}{s}e^{-z\sqrt{(Kr+s)}} & \text{when } Sc=1\end{cases} \tag{24}$$

Taking inverse Laplace transform of equations (22), (23) and (24), we get the following expressions for velocity, temperature and concentration profile:

$$F(z,t)=\begin{cases}\frac{1}{2}\left[\left(1-\frac{b_1}{a_2^2}-\frac{b_2}{a_3}-\frac{b_1 t}{a_2}-\frac{b_1 z}{2a_2\sqrt{a_1}}\right)e^{z\sqrt{a_1}}erfc\left(\frac{z}{2\sqrt{t}}+\sqrt{a_1 t}\right)+\left(1-\frac{b_1}{a_2^2}-\frac{b_2}{a_3}-\frac{b_1 t}{a_2}+\frac{b_1 z}{2a_2\sqrt{a_1}}\right)e^{-z\sqrt{a_1}}erfc\left(\frac{z}{2\sqrt{t}}-\sqrt{a_1 t}\right)\right] \\ +\frac{b_1 e^{a_2 t}}{2a_2^2}\left[e^{z\sqrt{a_1+a_2}}erfc\left(\frac{z}{2\sqrt{t}}+\sqrt{(a_1+a_2)t}\right)+e^{-z\sqrt{a_1+a_2}}erfc\left(\frac{z}{2\sqrt{t}}-\sqrt{(a_1+a_2)t}\right)\right] \\ +\frac{b_2 e^{a_3 t}}{2a_3}\left[e^{z\sqrt{a_3+a_1}}erfc\left(\frac{z}{2\sqrt{t}}+\sqrt{(a_3+a_1)t}\right)+e^{-z\sqrt{a_3+a_1}}erfc\left(\frac{z}{2\sqrt{t}}-\sqrt{(a_3+a_1)t}\right)\right] \\ +\frac{1}{2}\left[\left(\frac{b_1}{a_2^2}+\frac{b_1 t}{a_2}+\frac{b_1 z\sqrt{\Pr}}{2a_2\sqrt{a_4}}\right)e^{z\sqrt{a_4 \Pr}}erfc\left(\frac{z\sqrt{\Pr}}{2\sqrt{t}}+\sqrt{a_4 t}\right)+\left(\frac{b_1}{a_2^2}+\frac{b_1 t}{a_2}-\frac{b_1 z\sqrt{\Pr}}{2a_2\sqrt{a_4}}\right)e^{-z\sqrt{a_4 \Pr}}erfc\left(\frac{z\sqrt{\Pr}}{2\sqrt{t}}-\sqrt{a_4 t}\right)\right] \\ -\frac{b_1 e^{a_2 t}}{2a_2^2}\left[e^{z\sqrt{\Pr(a_2+a_4)}}erfc\left(\frac{z\sqrt{\Pr}}{2\sqrt{t}}+\sqrt{(a_2+a_4)t}\right)+e^{-z\sqrt{\Pr(a_2+a_4)}}erfc\left(\frac{z\sqrt{\Pr}}{2\sqrt{t}}-\sqrt{(a_2+a_4)t}\right)\right] \\ +\frac{b_2}{2a_3}\left[e^{z\sqrt{KrSc}}erfc\left(\frac{z\sqrt{Sc}}{2\sqrt{t}}+\sqrt{Krt}\right)+e^{-z\sqrt{KrSc}}erfc\left(\frac{z\sqrt{Sc}}{2\sqrt{t}}-\sqrt{Krt}\right)\right] \\ -\frac{b_2}{2a_3}e^{a_3 t}\left[e^{z\sqrt{(Kr+a_3)Sc}}erfc\left(\frac{z\sqrt{Sc}}{2\sqrt{t}}+\sqrt{(Kr+a_3)t}\right)+e^{-z\sqrt{(Kr+a_3)Sc}}erfc\left(\frac{z\sqrt{Sc}}{2\sqrt{t}}-\sqrt{(Kr+a_3)t}\right)\right] \\ \qquad\qquad\qquad\text{when } Sc\neq 1, \Pr\neq 1 \\ \frac{1}{2}\left[\left(1+b_4+b_3 t+\frac{b_3 z}{2\sqrt{a_1}}\right)e^{z\sqrt{a_1}}erfc\left(\frac{z}{2\sqrt{t}}+\sqrt{a_1 t}\right)+\left(1+b_4+b_3 t-\frac{b_3 z}{2\sqrt{a_1}}\right)e^{-z\sqrt{a_1}}erfc\left(\frac{z}{2\sqrt{t}}-\sqrt{a_1 t}\right)\right] \\ -\frac{b_3}{2}\left[\left(t+\frac{z}{2\sqrt{a_5}}\right)e^{z\sqrt{a_5}}erfc\left(\frac{z}{2\sqrt{t}}+\sqrt{a_5 t}\right)+\left(t-\frac{z}{2\sqrt{a_5}}\right)e^{-z\sqrt{a_5}}erfc\left(\frac{z}{2\sqrt{t}}-\sqrt{a_5 t}\right)\right] \\ -\frac{b_4}{2}\left[e^{z\sqrt{Kr}}erfc\left(\frac{z}{2\sqrt{t}}+\sqrt{Krt}\right)+e^{-z\sqrt{Kr}}erfc\left(\frac{z}{2\sqrt{t}}-\sqrt{Krt}\right)\right]\end{cases}$$

when $Sc=1, \Pr=1$ (25)

$$\theta(z,t)=\begin{cases}\frac{1}{2}\left[\left(t+\frac{z\sqrt{\Pr}}{2\sqrt{a_4}}\right)e^{z\sqrt{a_4\Pr}}erfc\left(\frac{z\sqrt{\Pr}}{2\sqrt{t}}+\sqrt{a_4t}\right)+\left(t-\frac{z\sqrt{\Pr}}{2\sqrt{a_4}}\right)e^{-z\sqrt{a_4\Pr}}erfc\left(\frac{z\sqrt{\Pr}}{2\sqrt{t}}-\sqrt{a_4t}\right)\right] & \text{when } \Pr\neq 1\\ \frac{1}{2}\left[\left(t+\frac{z}{2\sqrt{a_5}}\right)e^{z\sqrt{a_5}}erfc\left(\frac{z}{2\sqrt{t}}+\sqrt{a_5t}\right)+\left(t-\frac{z}{2\sqrt{a_5}}\right)e^{-z\sqrt{a_5}}erfc\left(\frac{z}{2\sqrt{t}}-\sqrt{a_5t}\right)\right] & \text{when } \Pr=1\end{cases} \tag{26}$$

$$C(z,t)=\begin{cases}\frac{1}{2}\left[e^{z\sqrt{KrSc}}erfc\left(\frac{z\sqrt{Sc}}{2\sqrt{t}}+\sqrt{Krt}\right)+e^{-z\sqrt{KrSc}}erfc\left(\frac{z\sqrt{Sc}}{2\sqrt{t}}-\sqrt{Krt}\right)\right] & \text{when } Sc\neq 1\\ \frac{1}{2}\left[e^{z\sqrt{Kr}}erfc\left(\frac{z}{2\sqrt{t}}+\sqrt{Krt}\right)+e^{-z\sqrt{Kr}}erfc\left(\frac{z}{2\sqrt{t}}-\sqrt{Krt}\right)\right] & \text{when } Sc=1\end{cases} \tag{27}$$

4. Some important characteristics of flow

From the velocity field equation (25), the expression for the dimensionless shear stress (τ^*) at the plate is given by

$$\tau^*=\left(\frac{\partial F}{\partial z}\right)_{z=0}=\begin{cases}\frac{b_1}{2a_2\sqrt{a_1}}erf\left(\sqrt{a_1t}\right)-\sqrt{a_1}\left(1-\frac{b_1}{a_2^2}-\frac{b_2}{a_3}-\frac{b_1t}{a_2}\right)erf\left(\sqrt{a_1t}\right)-\frac{e^{-a_1t}}{\sqrt{\pi t}}\left(1-\frac{b_1}{a_2^2}-\frac{b_2}{a_3}-\frac{b_1t}{a_2}\right)\\ -\frac{b_1e^{a_2t}}{a_2^2}\left[\sqrt{(a_1+a_2)}erf\left(\sqrt{(a_1+a_2)t}\right)+\frac{1}{\sqrt{\pi t}}e^{-(a_1+a_2)t}\right]-\frac{b_2e^{a_3t}}{a_3}\left[\sqrt{(a_3+a_1)}erf\left(\sqrt{(a_3+a_1)t}\right)+\frac{1}{\sqrt{\pi t}}e^{-(a_3+a_1)t}\right]\\ -\left[\frac{b_1\sqrt{\Pr}}{2a_2\sqrt{a_4}}erf\left(\sqrt{a_4t}\right)+\sqrt{a_4\Pr}\left(\frac{b_1}{a_2^2}+\frac{b_1t}{a_2}\right)erf\sqrt{(a_4t)}+\frac{\sqrt{\Pr}}{\sqrt{\pi t}}\left(\frac{b_1}{a_2^2}+\frac{b_1t}{a_2}\right)e^{-a_4t}\right]\\ +\frac{b_1e^{a_2t}}{a_2^2}\left[\sqrt{\Pr(a_2+a_4)}erf\left(\sqrt{(a_2+a_4)t}\right)+\frac{\sqrt{\Pr}}{\sqrt{\pi t}}e^{-(a_2+a_4)t}\right]-\frac{b_2}{a_3}\left[\sqrt{KrSc}erf\left(\sqrt{Krt}\right)+\frac{\sqrt{Sc}}{\sqrt{\pi t}}e^{-Krt}\right]\\ +\frac{b_2e^{a_3t}}{a_3}\left[\sqrt{Sc(Kr+a_3)}erf\left(\sqrt{(Kr+a_3)t}\right)+\frac{\sqrt{Sc}}{\sqrt{\pi t}}e^{-(Kr+a_3)t}\right] \\ \qquad\qquad\text{when } Sc\neq 1, \Pr\neq 1\\ -\frac{b_3}{2\sqrt{a_1}}erf\left(\sqrt{a_1t}\right)-2\sqrt{a_1}\left(1+b_4+b_3t\right)erf\left(\sqrt{a_1t}\right)-\frac{e^{-a_1t}}{\sqrt{\pi t}}\left(1+b_4+b_3t\right)\\ +b_3\left[\frac{1}{2\sqrt{a_5}}erf\left(\sqrt{a_5t}\right)+2t\sqrt{a_5}erf\left(\sqrt{a_5t}\right)-\sqrt{\frac{t}{\pi}}e^{-a_5t}\right]+b_4\left[\sqrt{Kr}erf\left(\sqrt{Krt}\right)-\frac{e^{-Krt}}{\sqrt{\pi t}}\right]\end{cases}$$

when $Sc=1, \Pr=1$ (28)

From the temperature field equation (26) the expression for the dimensionless rate of heat transfer coefficient (Nu) at the plate is given by

$$Nu = -\left(\frac{\partial\theta}{\partial z}\right)_{z=0} = \begin{cases} \left(\frac{\sqrt{\Pr}}{2\sqrt{a_4}} + t\sqrt{a_4 \Pr}\right) erf\left(\sqrt{a_4 t}\right) + \frac{\sqrt{t \Pr}}{\sqrt{\pi}} e^{-a_4 t} & \text{when } \Pr \neq 1 \\ \left(\frac{1}{2\sqrt{a_4}} + t\sqrt{a_4}\right) erf\left(\sqrt{a_4 t}\right) + \frac{\sqrt{t}}{\sqrt{\pi}} e^{-a_4 t} & \text{when } \Pr = 1 \end{cases} \tag{29}$$

From the concentration field equation (27), the expression for the dimensionless rate of mass transfer coefficient (Sh) at the plate is given by

$$Sh = -\left(\frac{\partial C}{\partial z}\right)_{z=0} = \begin{cases} \sqrt{KrSc}\, erf\sqrt{Krt} + e^{-Krt}\sqrt{\frac{Sc}{\pi t}} & \text{when } Sc \neq 1 \\ \sqrt{Kr}\, erf\sqrt{Krt} + e^{-Krt}\sqrt{\frac{1}{\pi t}} & \text{when } Sc = 1 \end{cases} \tag{30}$$

All the constants used above are given in the appendix.

5. RESULTS AND DISCUSSION

To assess the physical properties of the problem the effects of various parameters like magnetic parameter, permeability parameter, Grashof number, modified Grashof number, Prandtl number, radiation parameter, chemical reaction parameter, heat absorption parameter, Schmidt number, skin friction coefficient, Nusselt number and Sherwood number are examined. Then numerically evaluated results are presented in graphical and tabular form. The value of physical parameters such as $M^2 = 5,\ R = 4,\ Gr = 5,\ m = 0.5,\ \Pr = 0.71$ and $t = 0.5$ are taken from literature Das et al. [8]. For the problem to be realistic Schmidt number is chosen for hydrogen $(Sc = 0.22)$ and $Q = 0.2, K = 0.2$ and $Kr = 0.2$ are taken arbitrarily.

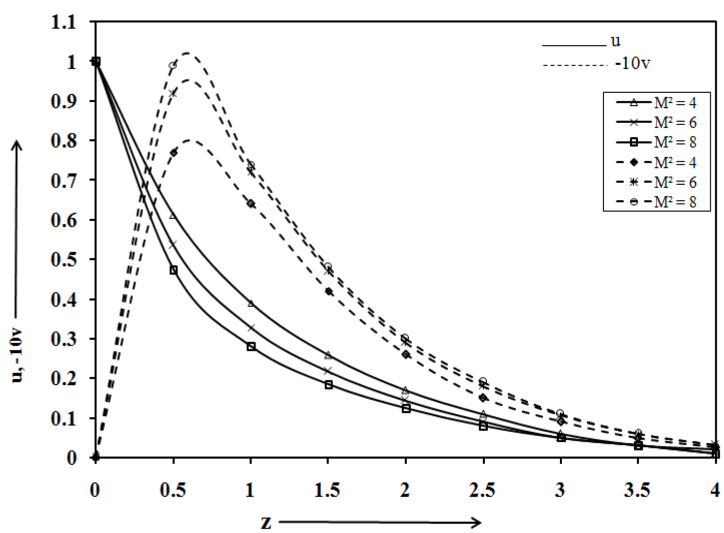


Fig.1: Effect of magnetic parameter on primary and secondary velocities

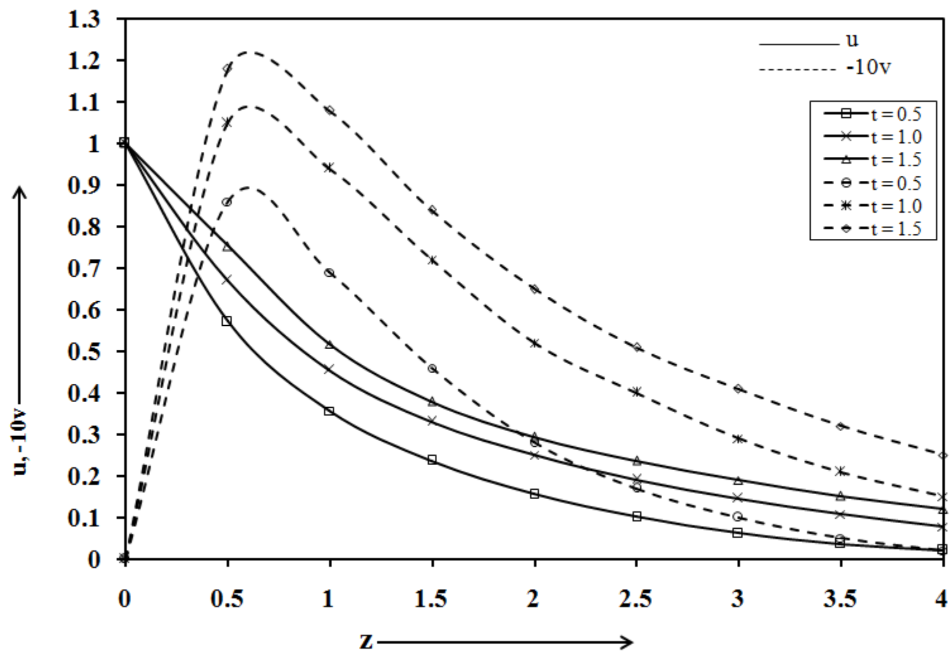


Fig. 2: Effect of time on primary and secondary velocities

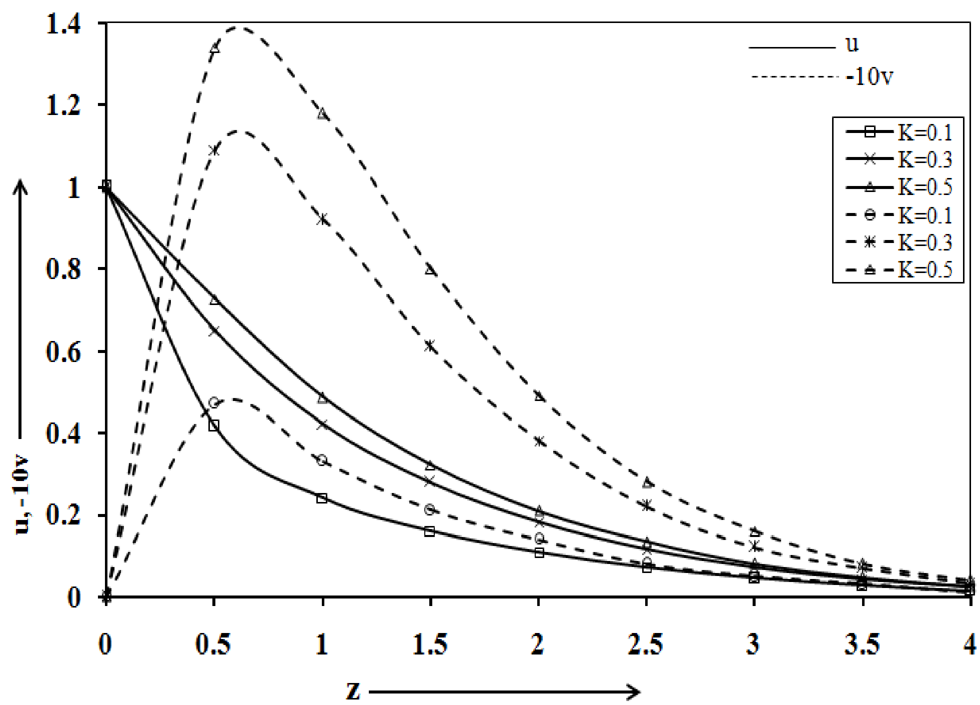


Fig. 3: Effect of permeability parameter on primary and secondary velocities

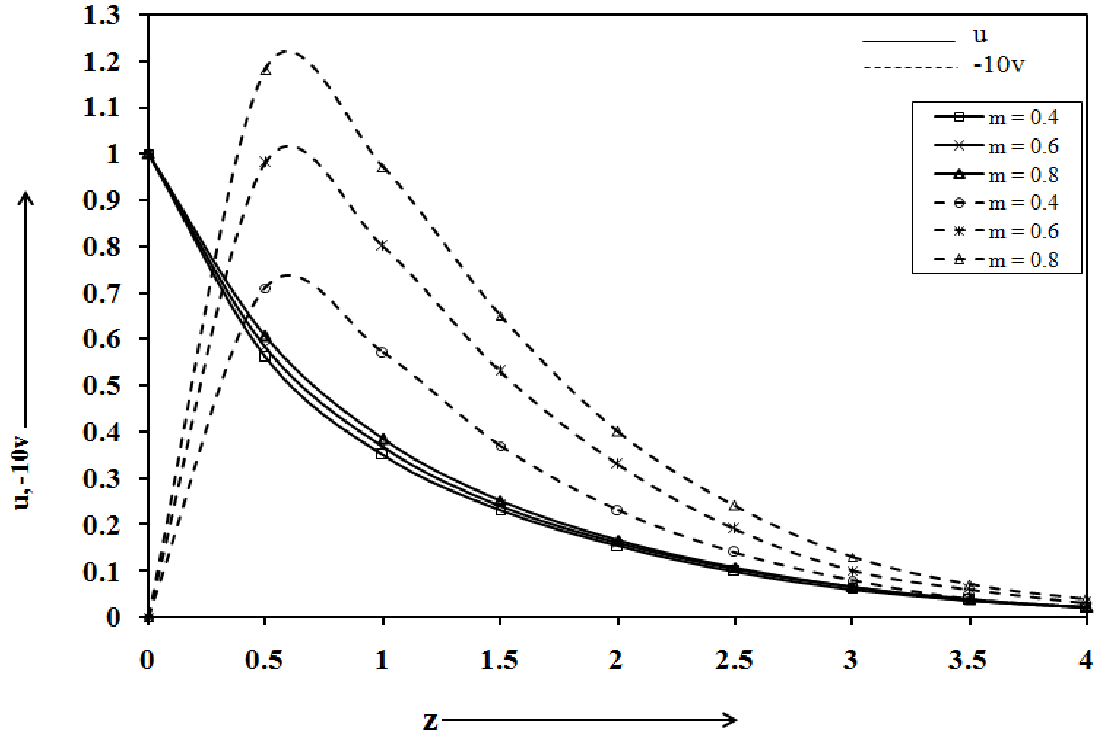


Fig. 4: Effect of Hall parameter on primary and secondary velocities

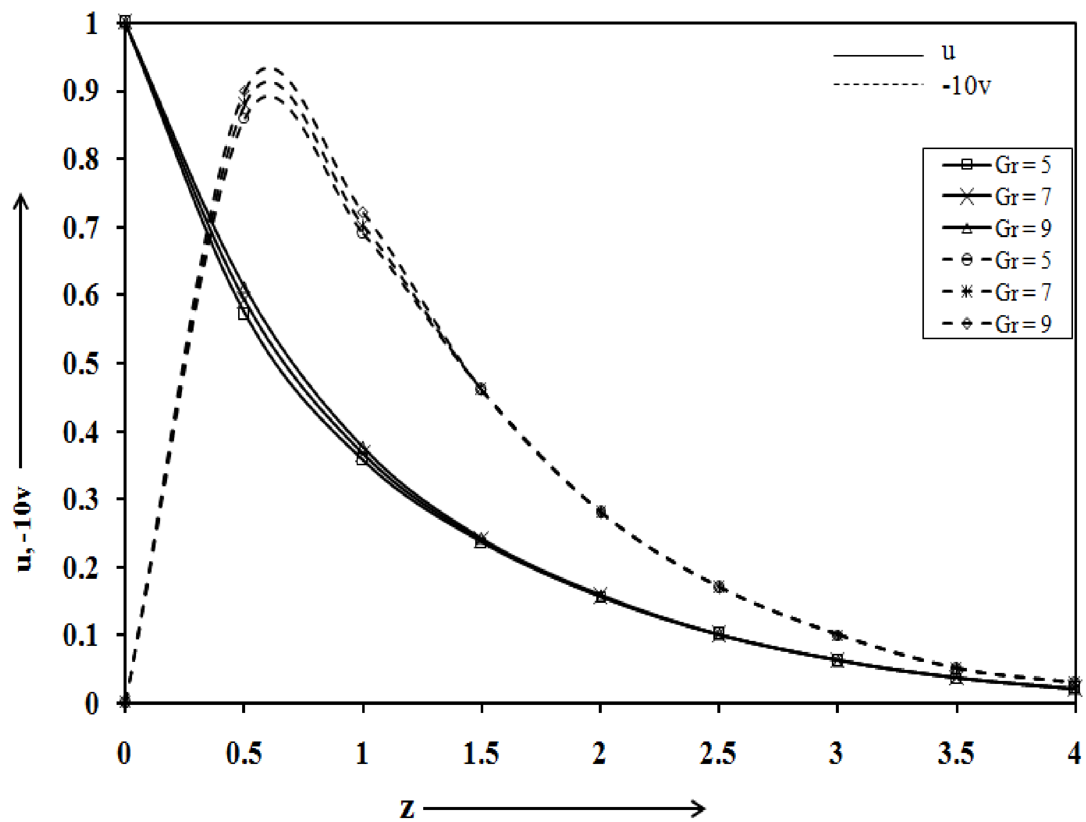


Fig. 5: Effect of Grashof number on primary and secondary velocities

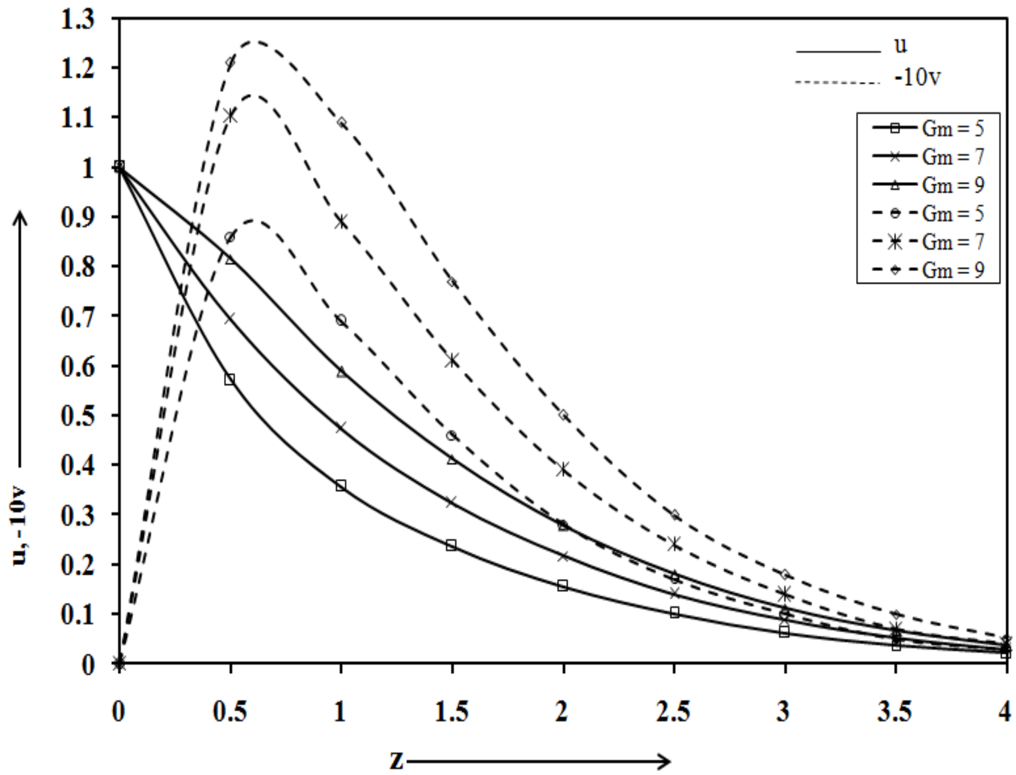
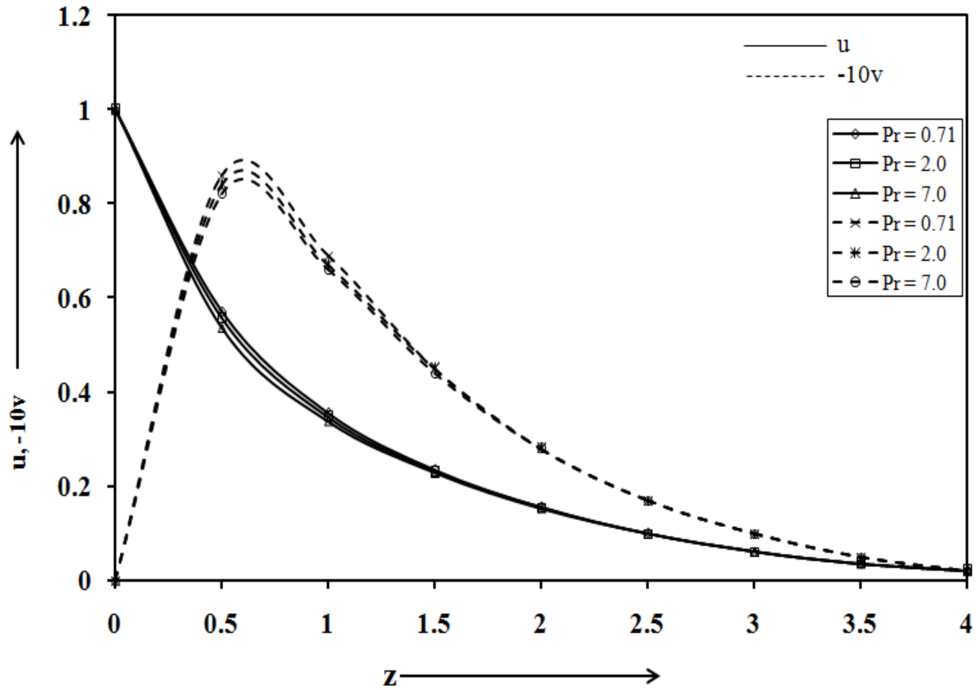
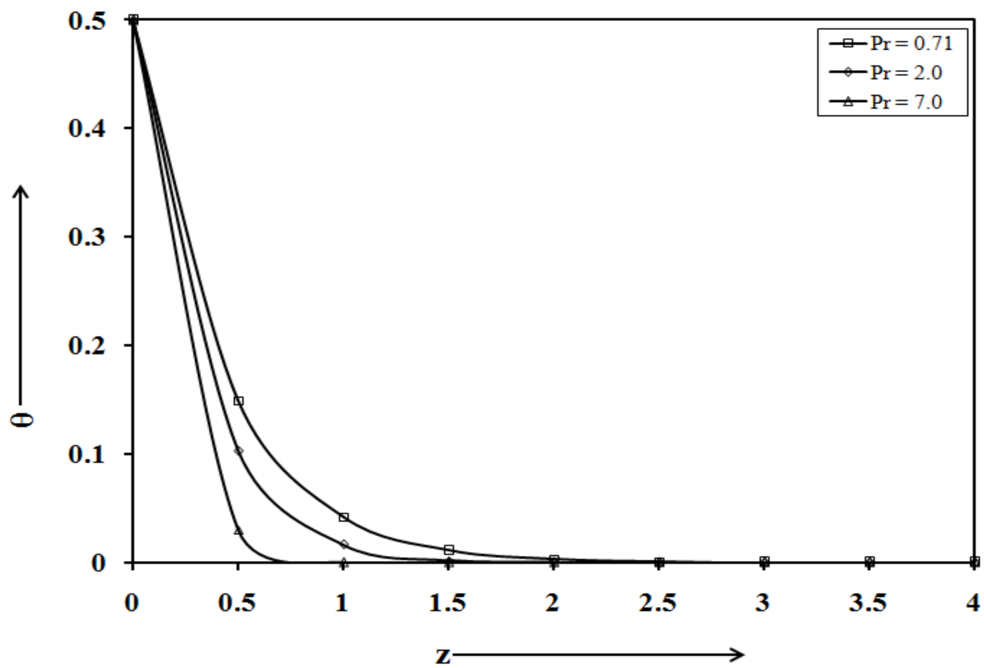


Fig. 6: Effect of modified Grashof number on primary and secondary velocities

Figures 1 to 6, 7(a), 8(a), 9(a), 10(a) and 11(a) represent the primary velocity (u) and secondary velocity (v) against space coordinate (z) for various values of pertinent flow parameters. Fig.1 refects that the primary velocity decreases while magnitude of the secondary velocity increases on increasing value of magnetic parameter. When a transverse magnetic field is applied, the Lorentz force acts in a direction opposite to the flow and slows down the motion of the fluid thereby reducing the primary velocity. In contrast, for the secondary flow this force acts as an aiding force. In fig. 2, it is seen that both the primary and secondary fluid velocities increase with increase in time. Fig. 3 depicts that both the primary and secondary fluid velocities increase with an increase in the permeability parameter. This is due to the reason that the presence of a porous medium decreases the resistance to flow. It is evident from fig. 4 that Hall parameter increases both the primary and secondary fluid velocities. Due to Hall effect an induced current is produced which reduces the resistance offered by the Lorentz force. In fig. 5 and fig. 6 rise in the magnitudes of both the primary and secondary fluid velocities is observed for the increasing values of Grashof number and modified Grashof number. An enhancement in thermal buoyancy force and species buoyancy force is the possible reason of increment in fluid velocities.



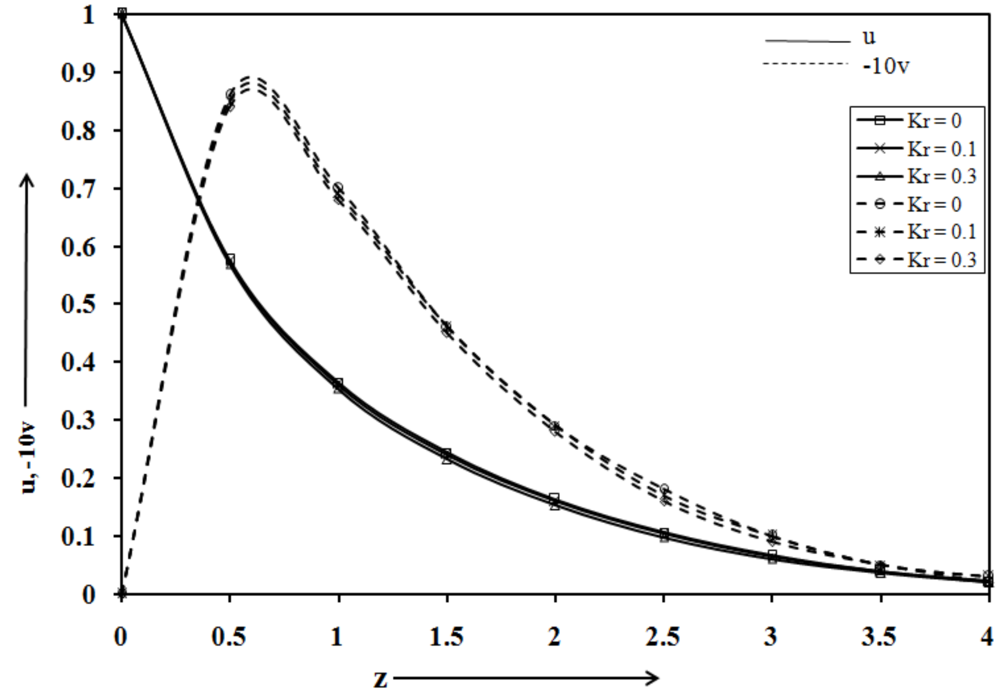
(a)



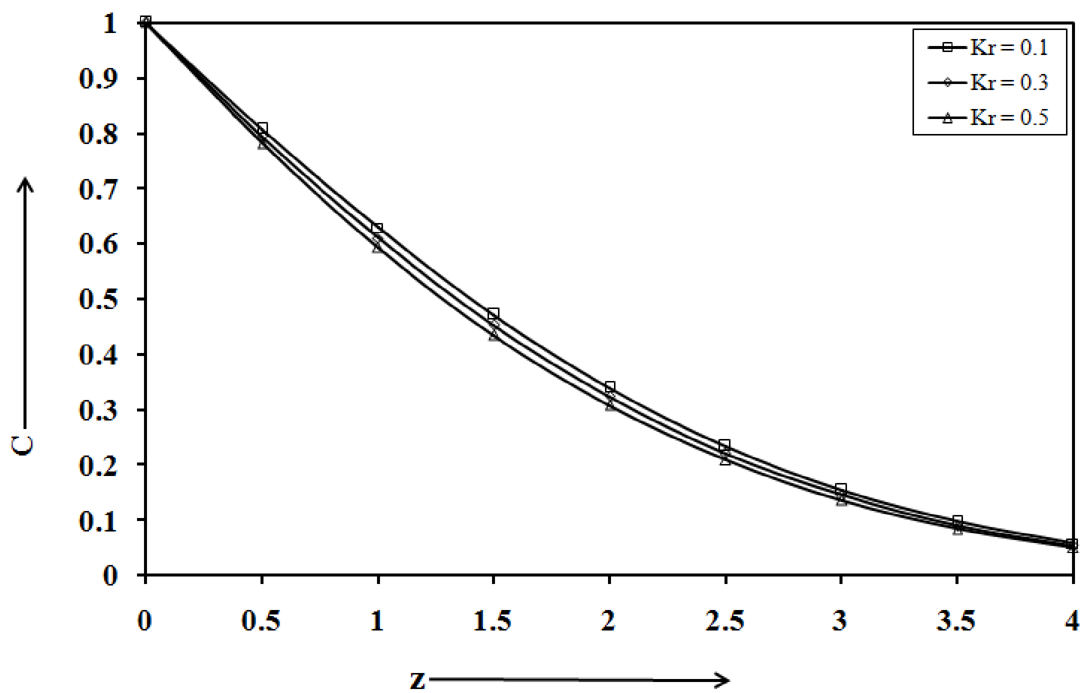
(b)

Fig. 7: Effect of Prandtl number on (a) primary and secondary velocities and (b) temperature

A decrement in both the primary and secondary fluid velocities and temperature for higher values of Prandtl number is noticed in fig. 7(a) and fig 7(b). Higher value of the Prandtl number implies large viscosity and low thermal diffusivity which is the cause of decrement in fluid velocity and temperature in the boundary layer region.

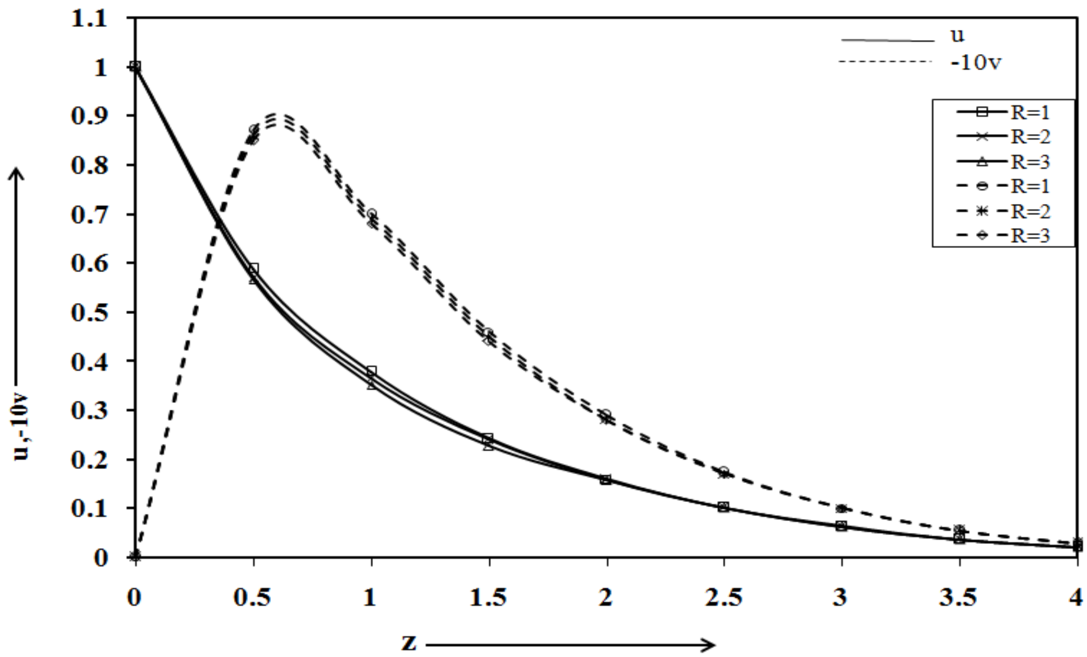


(a)

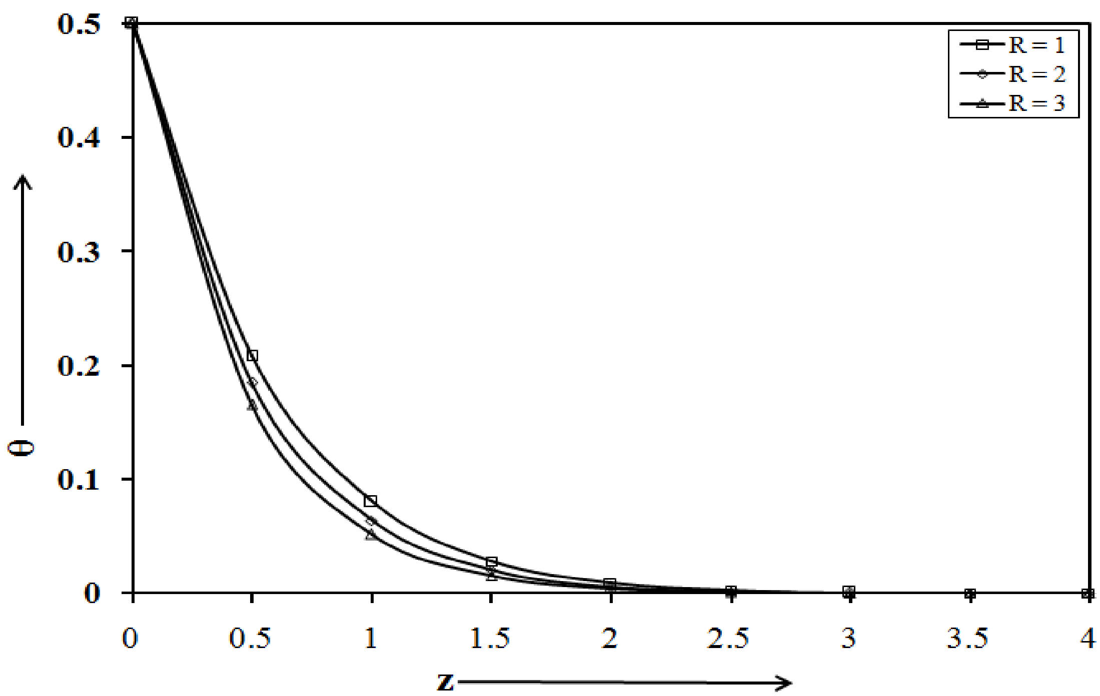


(b)

Fig. 8: Effect of chemical reaction parameter on (a) primary and secondary velocities and (b) concentration

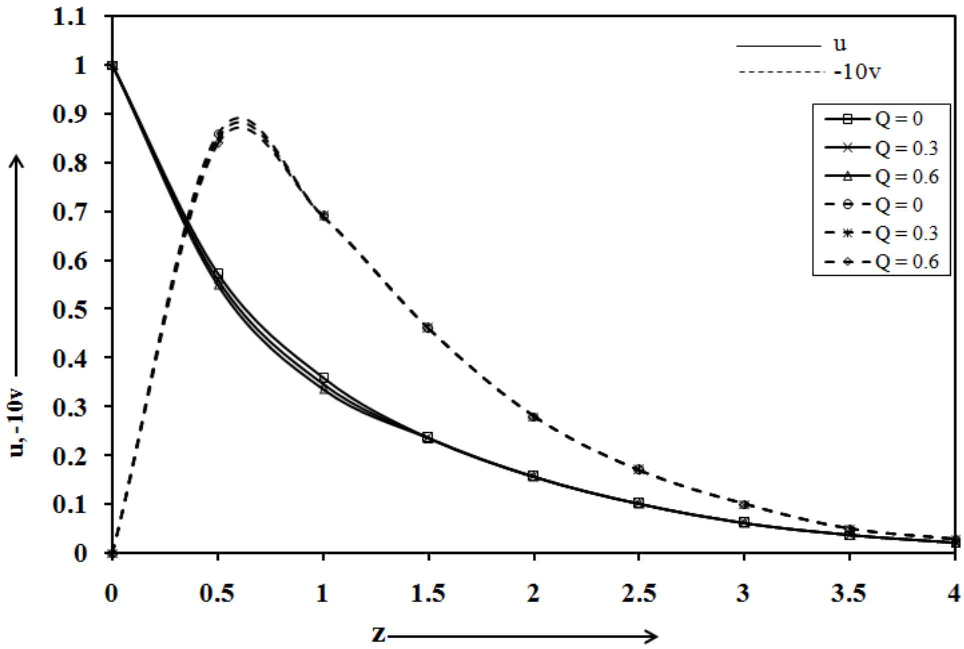


(a)

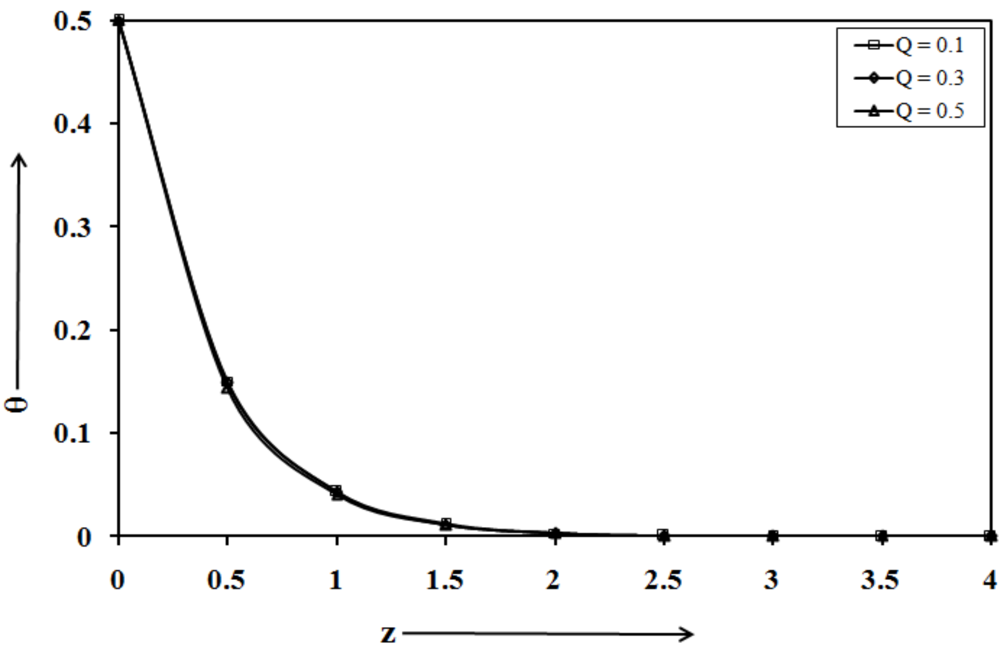


(b)

Fig. 9: Effect of radiation parameter on (a) primary and secondary velocities and (b) temperature



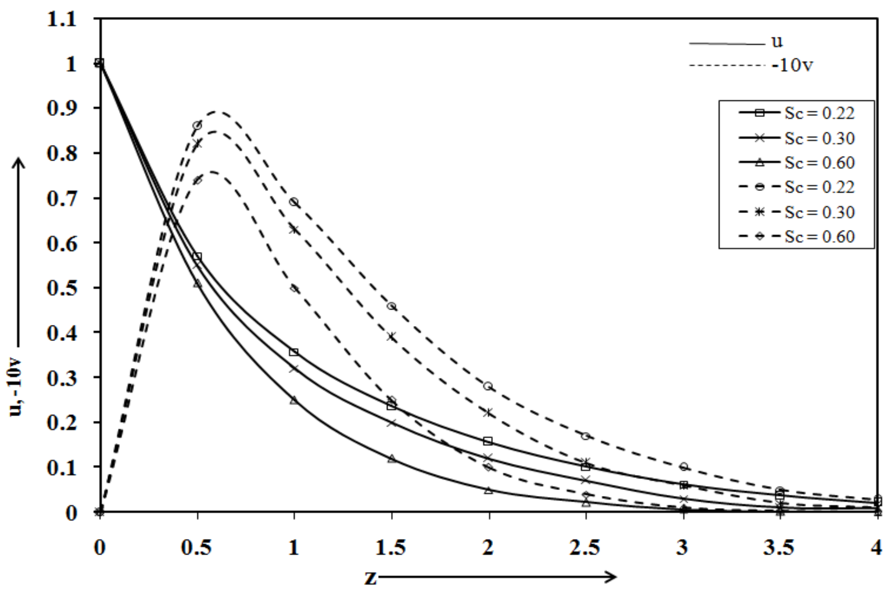
(a)



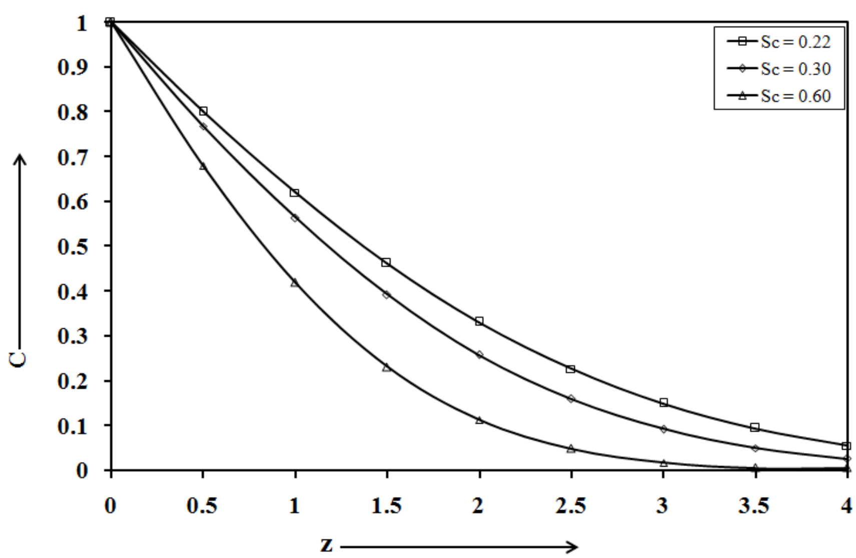
(b)

Fig. 10: Effect of heat absorption parameter on (a) primary and secondary velocities and (b) temperature

It is seen from fig.8 (a) and fig.8 (b) that the primary and secondary fluid velocities get retarded and species concentration gets reduced in the boundary layer region for higher values of chemical reaction parameter. It is due to reduction in species buoyancy force and thinness of concentration boundary layer near the plate. From the figs.9 (a) and 9 (b), it is noticed that increase in the radiation parameter decreases primary and secondary velocities and temperature. In optically thin fluid, thermal radiations penetrate through the fluid without getting absorbed in it. Increase in the radiation parameter reduces the rate of heat transfer through the fluid, which results in the decrease in temperature and hence fluid velocity in the boundary layer. It is inferred from fig. 10(a) and fig.10 (b) that both the primary and secondary fluid velocities and the temperature decrease with increase in heat absorption parameter. The reason behind it is that the heat absorption causes a decrease in the kinetic energy as well as thermal energy of the fluid.



(a)



(b)

Fig. 11: Effect of Schmidt number on (a) primary and secondary velocities and (b) concentration

A decrement in both the primary and secondary fluid velocities and concentration for higher values of Schmidt number is observed in fig. 11(a) and fig. 11(b). The fluid becomes viscous and mass diffusivity decreases when value of Schmidt number increases; this reduces the velocity and species concentration in the boundary layer region. In order to verify the correctness of the present approach, we have made comparisons with available study by Das et al. [8]. The results are found in good agreement and one of the comparisons is shown in Fig.12 for different values of Grashof number (Gr).

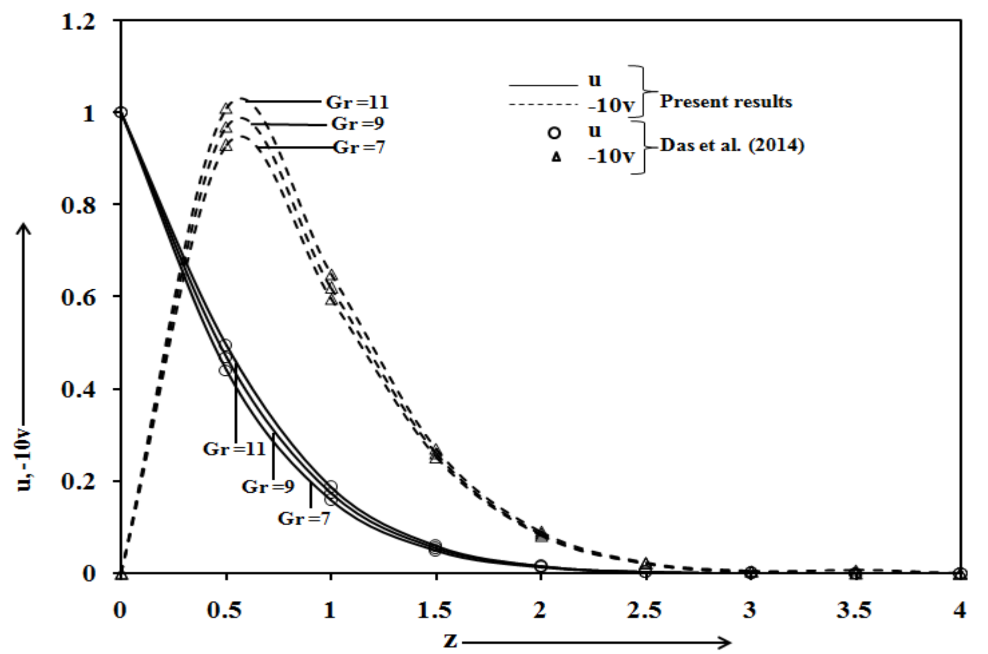


Fig. 12: Primary and Secondary fluid velocities when $Gm=0, \frac{1}{K}=0, Sc=0, Kr=0, Q=0$

Table 1: Effects of flow parameters on shear stress due to primary flow $(-\tau_x)$ and secondary flow $(-\tau_y)$, Nusselt number (Nu) and Sherwood number (Sh) at the plate

Pr	M^2	Kr	N	Q	m	Gr	Gm	K	Sc	t	$-\tau_x$	$-\tau_y$	Nu	Sh
0.71	5	0.2	4	0.2	0.5	5	5	0.2	0.22	0.5	1.365	0.432	1.195	0.411
7.0	5	0.2	4	0.2	0.5	5	5	0.2	0.22	0.5	1.501	0.420	3.216	-
0.71	**6**	0.2	4	0.2	0.5	5	5	0.2	0.22	0.5	1.542	0.494	-	-
0.71	5	**0.3**	4	0.2	0.5	5	5	0.2	0.22	0.5	1.367	0.433	-	0.429
0.71	5	0.3	**5**	0.2	0.5	5	5	0.2	0.22	0.5	1.378	0.431	1.293	-
0.71	5	0.2	4	**0.3**	0.5	5	5	0.2	0.22	0.5	1.366	0.432	1.205	-
0.71	5	0.2	4	0.2	**0.6**	5	5	0.2	0.22	0.5	1.301	0.485	-	-
0.71	5	0.2	4	0.2	0.5	**6**	5	0.2	0.22	0.5	1.279	0.436	-	-
0.71	5	0.2	4	0.2	0.5	5	**6**	0.2	0.22	0.5	1.120	0.448	-	-
0.71	5	0.2	4	0.2	0.5	5	5	**0.3**	0.22	0.5	0.990	0.479	-	-
0.71	5	0.2	4	0.2	0.5	5	5	0.2	**0.60**	0.5	1.610	0.381	-	0.678
0.71	5	0.2	4	0.2	0.5	5	5	0.2	0.22	**1.0**	0.743	0.499	2.220	0.315

Table 1 expresses the numerical results of shear stresses due to the primary and the secondary flow, Nusselt number (Nu) and Sherwood number (Sh) at the plate for different values of parameters occurring in governing equations. It is observed that the absolute value of shear stress at the plate due to primary flow decreases with increase in the Hall parameter, permeability parameter, Grashof number, modified Grashof number and time, whereas absolute value of shear stress due to secondary flow decreases with an increase in the Prandtl number and radiation parameter and Schmidt number. The rate of heat transfer at the plate increases with an increase in the radiation parameter, Prandtl number, heat absorption coefficient and time whereas, the rate of mass transfer at the plate increases with an increase in the chemical reaction parameter and Schmidt number.

6. CONCLUSIONS

The effects of chemical reaction and heat absorption on hydromagnetic flow past a moving plate through porous medium with ramped wall temperature in the presence of Hall current and thermal radiation is investigated. It is found that chemical reaction parameter retards the primary velocity as well as the secondary fluid velocity in the boundary layer region. An increase in the heat absorption parameter leads to fall in primary and secondary velocities. Further, the absolute value of shear stress due to primary and secondary velocities at the moving plate enhances with an increase in chemical reaction parameter and heat absorption parameter. The rate of heat transfer increases with increase in heat absorption parameter and rate of mass transfer increases with increase in chemical reaction parameter at the plate.

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