

On Sequential Graphs

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Abstract

A labeling or valuation of a graph G is an assignment f of labels to the vertices of G that induces for each edge xy a label depending on the vertex labels $f(x)$ and $f(y)$. In this paper we study some classes of graphs which admit sequential labeling.

Keywords: Labeling, Sequential graph, Braid graph, Total graph, Cyclic graph.

1. INTRODUCTION

Unless mentioned a graph in this paper shall mean a simple finite graph without isolated vertices. For all terminology and notations in Graph Theory, we follow Harary [3] and for all terminology regarding Sequential labeling we follow Grace [2].

Let G be a (p, q) graph. Let $V(G)$, $E(G)$ denote respectively the vertex set and edge set of G . Consider an injective function $g : V(G) \rightarrow X$ where $X = \{0, 1, 2, \dots, q\}$ if G is a tree and $X = \{0, 1, 2, \dots, q-1\}$ otherwise. Define the function $g^* : E(G) \rightarrow \mathbb{N}$, by $g^*(uv) = g(u) + g(v)$ for all edges uv , where \mathbb{N} is the set of all natural numbers. If $g^*(E(G))$ is a sequence of distinct two integers say $\{k, k+d, k+2d, \dots, k+(q-1)d\}$ for some k and d , then the function g is said to be (k, d) -Sequential labeling and the graph which admits such a labeling is called as a Sequential graph.

If $d = 1$, G is a called k - sequential graph

Definition1.1: A chord of a cycle C_n is called a P_k - chord if it divides the cycle into two cycles C_k and C_{n-k+2} .

Theorem1.2: Let G be a graph obtained from C_{2t+1} with a P_4 - chord. Then G is a $(t+2, 1)$, $t \geq 1$ Sequential graph.

Proof: Let $C_{2t+1} = (u_1 u_2, \dots, u_{2t+1} u_1)$ and let $u_2 u_{2t}$ be a P_4 -chord of C_{2t+1} . The graph G consists of $2t + 1$ vertices and $2t + 2$ edges.

Define $f : V(G) \rightarrow \{0, 1, 2, \dots, 2t + 1\}$ as follows:

$$f(u_{2i-1}) = i, \quad 1 \leq i \leq t + 1$$

$$f(u_{2i}) = (t + 1) + i, \quad 1 \leq i \leq t$$

Also $f(u_i) < f(u_j)$ for all $i \neq j$

Clearly vertex labels are distinct.

Then the induced edge labels are given by $f(u_i v_i) = f(u_i) + f(v_i)$ and are as follows:

$$f(u_i u_{i+1}) = t + i + 2, \quad 1 \leq i \leq 2t$$

$$f(u_1 u_{2t+1}) = t + 2$$

$$f(u_2 u_{2t}) = 3t + 3$$

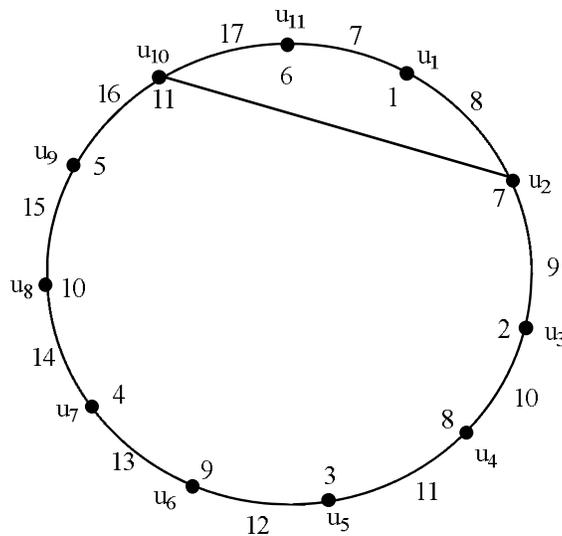
Thus the set of labels of the edges of the cycle = $\{t + 2, t + 3, \dots, 3t + 1, 3t + 2\}$ and the label of the chord is $3t + 3$.

Clearly edge values are distinct and is of the form $\{t + 2, t + 3, \dots, 3t + 2, 3t + 3\}$.

Thus the induced edge labels are given by $f^*(E(G)) = \{k, k + d, k + 2d, \dots, k + (q - 1)d\}$ where $k = t + 2$ and $d = 1$.

Therefore G is a k -Sequential graph.

Illustration 1.3: $(7, 1)$ - Sequential labeling of C_{11} with P_4 -chord is given below.



Theorem 1.4: The graph $K_{1,n} + K_{1,m}$ is a $(2t + 1, 1)$ - Sequential graph for all m, n and for any non-negative integer t .

Proof: Let $V(K_{1,n}) = \{u_0, u_1, \dots, u_n\}$ where $\deg u_0 = n$ and $V(K_{1,m}) = \{v_0, v_1, \dots, v_m\}$ where $\deg v_0 = m$. Join each vertices of $K_{1,n}$ to every vertices of $K_{1,m}$, then we get the graph $K_{1,n} + K_{1,m}$.
Let $G = K_{1,n} + K_{1,m}$

The edge set of G is $E(G) = \{u_0u_i ; 1 \leq i \leq n\} \cup \{v_0v_i ; 1 \leq i \leq m\} \cup \{u_iv_j ; 1 \leq i \leq n, 1 \leq j \leq m\}$

Note that G has $n + m + 2$ vertices and $n + m + (n + 1)(m + 1)$ edges.

Let t be a non-negative integer. Define $f : V(G) \rightarrow \{0, 1, 2, \dots, (n + m + (n + 1)(m + 1)) - 1\}$ as follows:

$$\begin{aligned} f(u_i) &= i + t, & 0 \leq i \leq n \\ f(v_0) &= n + t + 1 \\ f(v_i) &= i(n + 2) + n + t, & 1 \leq i \leq m \end{aligned}$$

Then the induced edge labels are given by $f(u_iv_i) = f(u_i) + f(v_i)$ and are as follows:

$$\begin{aligned} f(u_0u_i) &= 2t + i, & 1 \leq i \leq n \\ f(v_0v_i) &= 2t + i(n + 2) + 2n + 1, & 1 \leq i \leq n \\ f(v_0u_i) &= 2t + (n + 1) + i, & 1 \leq i \leq n \\ f(v_ju_i) &= 2t + (j + 1)(n + 1) + i + (j - 1), & 0 \leq i \leq n, 1 \leq j \leq m \end{aligned}$$

Thus the set of labels of the edges of $K_{1,n} = \{2t + 1, 2t + 2, 2t + 3, \dots, 2t + n\}$

The set of labels of the edges $v_0u_i, 0 \leq i \leq n = \{2t + n + 1, 2t + n + 2, \dots, 2t + 2n + 1\}$

The set of labels of the edges v_0v_i and $v_1u_i, 0 \leq i \leq n$
 $= \{2t + 2(n + 1)\} \cup \{2t + 2(n + 1) + 1, 2t + 2(n + 1) + 2, \dots, 2t + 2(n + 1) + n\}$

The set of labels of the edges v_0v_2 and $v_2u_i, 0 \leq i \leq n$
 $= \{2(n + 1) + n + 1\} \cup \{2t + 3(n + 1) + 2, 2t + 3(n + 1) + 3, \dots, 2t + 3(n + 1) + n\}$

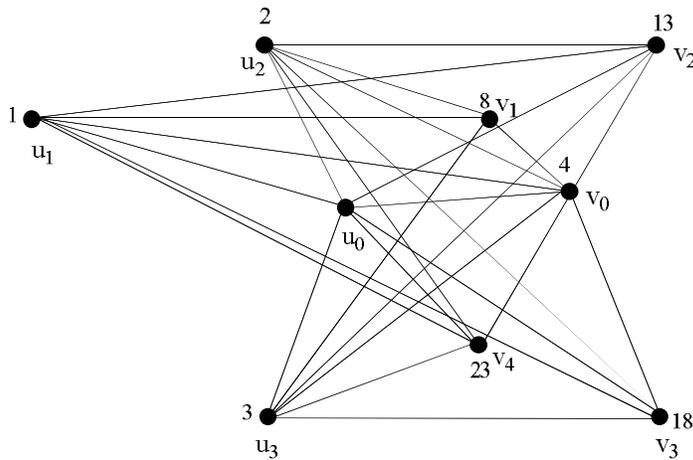
The set of labels of the edges v_0v_m and $v_mu_i, 0 \leq i \leq n$
 $= \{2t + (m + 2)(n + 1) + m - 2\} \cup \{2t + (m + 2)(n + 1) + m - 1, 2t + (m + 2)(n + 1) + m, \dots, 2t + (m + 1)(n + 1) + m + n\}$

Thus the edges values of the graph G is distinct and is of the form
 $2t + 1, 2t + 2, 2t + 3, \dots, 2t + n, 2t + n + 1, 2t + n + 2, \dots, 2t + 2n + 1, \dots,$
 $2t + 2(n + 1) + n, \dots, 2t + 3(n + 1) + n, \dots, 2t + (m + 2)(n + 1) + m - 2, \dots,$
 $2t + (m + 1)(n + 1) + m + n.$

Thus the induced edge labels are given by $f(E(G)) = \{k, k + 1, \dots, k + (q - 1)\}$ where $k = 2t + 1$ and $d = 1$

Therefore f is a Sequential labeling. Thus G is a k -Sequential graph.

Illustration: 1.5: $K_{1,3} + K_{1,4}$ is a sequential graph.



Theorem 1.6: The graph $P_2 + mK_1$ is a $(2t + 1, 1)$ - Sequential graph for all m and for any non-negative integer t .

Proof: Consider a path P_2 with two vertices v_1, v_2 . Let y_1, y_2, \dots, y_m be the m isolated vertices. Join v_1, v_2 with $y_i, 1 \leq i \leq m$. The graph obtained is $P_2 + mK_1$

Let $G = P_2 + mK_1$. The vertex set of G is $V(G) = \{v_1, v_2, y_1, y_2, \dots, y_m\}$
 $E(G) = \{(v_1, v_2)\} \cup \{(v_1, y_i), 1 \leq i \leq m\} \cup \{(v_2, y_i), 1 \leq i \leq m\}$

Then $|V(G)| = 2 + m$ and $|E(G)| = 2m + 1$

Let t be an integer such that $t \geq 0$.

Define labeling $f : V(G) \rightarrow \{0, 1, 2, \dots, 2m\}$ as follows:

$$\begin{aligned} f(v_1) &= t \\ f(v_2) &= t + 1 \\ f(y_i) &= t + 2i, \quad 1 \leq i \leq m \end{aligned}$$

Then the induced edge labels are given by $f^*(u_i v_i) = f(u_i) + f(v_i)$ and are as follows:

$$\begin{aligned} f^*(v_1 v_2) &= 2t + 1 \\ f^*(v_1 y_i) &= 2t + 2i, \quad 1 \leq i \leq m \\ f^*(v_2 y_i) &= 2t + 2i + 1, \quad 1 \leq i \leq m \end{aligned}$$

Thus the label of the edge $v_1 v_2$ is $2t + 1$.

The set of labels of the edges $v_1 y_i, 1 \leq i \leq m$ are $\{2t + 2, 2t + 4, 2t + 6, \dots, 2t + 2m\}$

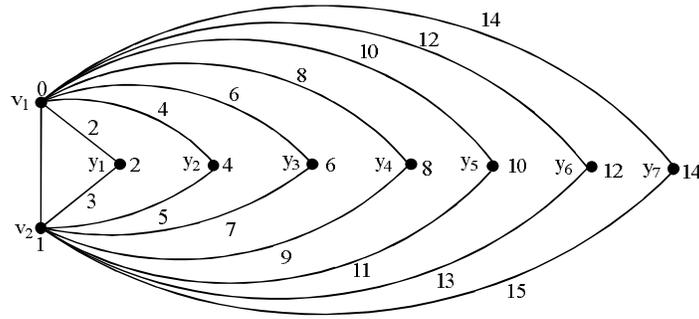
The set of labels of edges $v_2 y_i, 1 \leq i \leq m$ are $\{2t + 3, 2t + 5, \dots, 2t + 2m - 1, 2t + 2m + 1\}$

Therefore the values of the edges form the set $\{2t + 1, 2t + 2, 2t + 3, \dots, 2t + 2m, 2t + 2m + 1\}$

Thus the induced edge labels are given by $f^*(E(G)) = \{k, k + d, k + 2d, \dots, k + (q - 1)d\}$ where $k = 2t + 1$ and $d = 1$.

Thus f is a Sequential labeling of G . Hence $P_2 + m K_1$ is a k -Sequential graph.

Illustration 1.7: $(1, 1)$ - Sequential labeling of $P_2 + 7K_1$ is given below.



Theorem 1.8: The total graph of the path $G = T(P_n)$ is a $(2t + 1, 1)$ - Sequential graph for all n and for any non-negative integer t .

Proof: The vertex set of G is $V(G) = \{u_i, 1 \leq i \leq n, v_i, 1 \leq i \leq n-1\}$
 The edge set of G is $E(G) = \{u_i u_{i+1}, 1 \leq i \leq n-1\} \cup \{v_i v_{i+1}, 1 \leq i \leq n-2\}$
 $\cup \{v_i u_{i+1}, 1 \leq i \leq n-1\} \cup \{u_i v_i; 1 \leq i \leq n-1\}$

Then $|V(G)| = 2n - 1$ and $E(G) = 4n - 5$

Let t be a non-negative integer such that $t \geq 0$. Define $f: V(G) \rightarrow \{0, 1, 2, \dots, 4n - 6\}$ as follows:

$$f(u_i) = 2i + t - 2, \quad 1 \leq i \leq n$$

$$f(v_i) = 2i + t - 1, \quad 1 \leq i \leq n - 1$$

Then the induced edge labels are given by $f(u_i v_i) = f(u_i) + f(v_i)$ and are as follows :

$$f(v_i v_{i+1}) = 2t + 4i, \quad 1 \leq i \leq n - 2$$

$$f(u_i u_{i+1}) = 2t + 4i - 2, \quad 1 \leq i \leq n - 1$$

$$f(u_i v_i) = 2t + 4i - 3, \quad 1 \leq i \leq n - 1$$

$$f(v_i u_{i+1}) = 2t + 4i - 1, \quad 1 \leq i \leq n - 1$$

The set of labels of the edges $v_i v_{i+1}, 1 \leq i \leq n - 2$
 $= \{2t + 4, 2t + 8, 2t + 12, \dots, 2t + 4n - 12, 2t + 4n - 8\}$

The set of labels of the edge $u_i u_{i+1}, 1 \leq i \leq n - 2$
 $= \{2t + 2, 2t + 6, 2t + 10, \dots, 2t + 4n - 10, 2t + 4n - 6\}$

The set of labels of the edges $u_i v_i, 1 \leq i \leq n - 1, v_i u_{i+1}, 1 \leq i \leq n - 1$
 $= \{2t + 1, 2t + 3, 2t + 5, \dots, 2t + 4n - 7, 2t + 4n - 5\}$

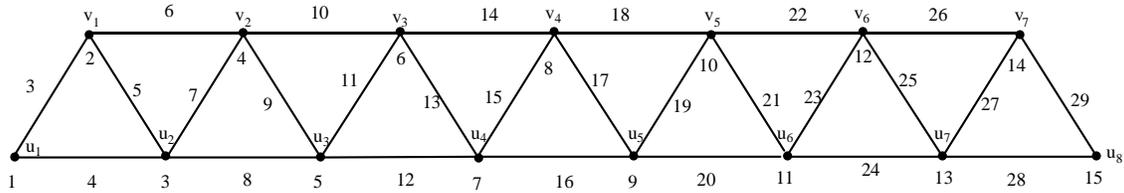
Thus the values of the edges form the set
 $= \{2t + 1, 2t + 3, 2t + 5, \dots, 2t + 4n - 7, 2t + 4n - 6, 2t + 4n - 5\}$

Thus the induced edge labels are given by $f(E(G)) = \{k, k + d, k + 2d, \dots, k + (q - 1)d\}$ where $k = 2t + 1$ & $d = 1$

Hence the above defined labeling pattern f admits Sequential labeling.

Therefore, $T(P_n)$ is a k -Sequential graph.

Illustration: 1.9: (3,1) is a Sequential labeling of $T(P_8)$



Theorem 1.10: The graph $G = T(P_n) \otimes K_m^c$ is a $(2t + 1, 1)$ - Sequential graph for all m, n and for any non-negative integer t .

Proof: The vertex set of G is

$$V(G) = \{u_i ; 1 \leq i \leq n\} \cup \{v_i ; 1 \leq i \leq n - 1\} \cup \{u_{ij} ; 1 \leq i \leq n, 1 \leq j \leq m\} \cup \{v_{ij} ; 1 \leq i \leq n - 1, 1 \leq j \leq m\}$$

The edge set of G is

$$E(G) = \{(u_i u_{i+1}) ; 1 \leq i \leq n - 1\} \cup \{(v_i v_{i+1}) ; 1 \leq i \leq n - 2\} \cup \{(u_i v_i) ; 1 \leq i \leq n - 1\} \cup \{(v_i u_{i+1}) ; 1 \leq i \leq n - 1\} \cup \{(u_i u_{ij}) ; 1 \leq i \leq n, 1 \leq j \leq m\} \cup \{(v_i v_{ij}) ; 1 \leq i \leq n - 1, 1 \leq j \leq m\}$$

$$\text{Then } |V(G)| = 2n(m + 1) - m - 1$$

$$|E(G)| = 2n(m + 2) - m - 5 = q \text{ (say)}$$

Let t be an integer such that $t \geq 0$.

Define a labeling $f : V(G) \rightarrow \{0, 1, 2, \dots, 2n(m + 2) - m - 6\}$ as follows:

$$f(v_i) = 2i + t - 1, \quad 1 \leq i \leq n - 1$$

$$f(u_i) = 2i + t - 2, \quad 1 \leq i \leq n$$

$$f(v_{ij}) = (n - 1)(4 + j - 1) - i + t, \quad 1 \leq i \leq n - 1, 1 \leq j \leq m$$

$$f(u_{ij}) = (n - 1)(4 + m) + n(j - 1) + t - i + 1, \quad 1 \leq i \leq n, 1 \leq j \leq m$$

The vertex labels are distinct.

Then the induced edge labels are given by $f(u_i v_i) = f(u_i) + f(v_i)$ and are as follows:

$$f(v_i v_{i+1}) = 2t + 4i, \quad 1 \leq i \leq n - 2$$

$$f(u_i u_{i+1}) = 2t + 4i - 2, \quad 1 \leq i \leq n - 1$$

$$f(u_i v_i) = 2t + 4i - 3, \quad 1 \leq i \leq n - 1$$

$$f(v_i u_{i+1}) = 2t + 4i - 1, \quad 1 \leq i \leq n - 1$$

$$f(v_i v_{ij}) = 2t + i + (n - 1)(4 + j - 1) - 1, \quad 1 \leq i \leq n - 1, 1 \leq j \leq m$$

$$f(u_i u_{ij}) = 2t + i + (n - 1)(4 + m) + n(j - 1) - 1, \quad 1 \leq i \leq n, 1 \leq j \leq m$$

Thus the set of labels of the edges $v_i v_{i+1}, 1 \leq i \leq n - 2$

$$= \{2t + 4, 2t + 8, 2t + 12, \dots, 2t + 4n - 12, 2t + 4n - 8\}$$

The set of labels of the edges $u_i u_{i+1}, 1 \leq i \leq n - 1$

$$= \{2t + 2, 2t + 6, 2t + 10, \dots, 2t + 4n - 10, 2t + 4n - 6\}$$

The set of labels of the edges $u_i v_i, v_i u_{i+1}, 1 \leq i \leq n - 1$

$$= \{2t + 1, 2t + 3, 2t + 5, \dots, 2t + 4n - 7, 2t + 4n - 5\}$$

The set of labels of the edges $v_1 v_{1j}, v_2 v_{2j}, \dots, v_{n-1} v_{n-1j}, u_1 u_{1j}, u_2 u_{2j}, \dots, u_n u_{nj}, 1 \leq j \leq m$

$$= \{2t + 4n - 4, 2t + 4n - 3, 2t + 4n - 2, \dots, 2t + 2n(m + 2) - m - 6, 2t + 2n(m + 2) - m - 5\}$$

Therefore the set of labels of the edges of G is $\{2t + 1, 2t + 2, 2t + 3, \dots, 2t + 4n - 6,$

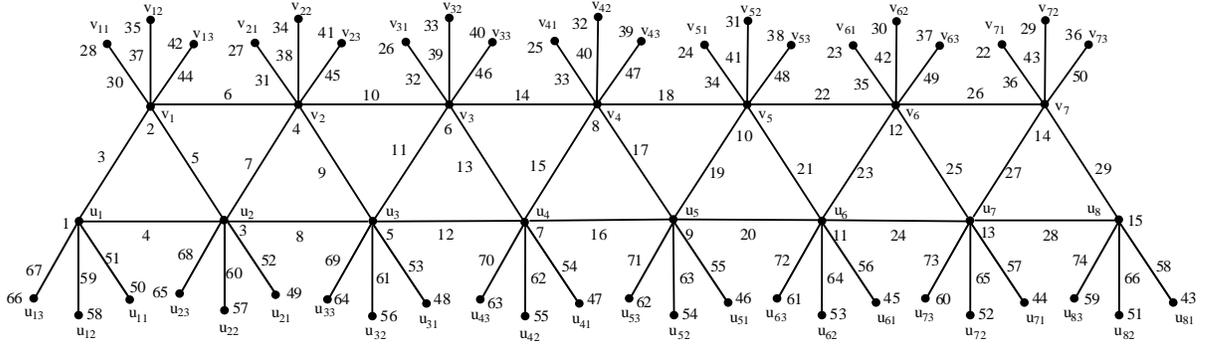
$$2t + 4n - 5, \dots, 2t + 2n(m + 2) - m - 6, 2t + 2n(m + 2) - m - 5\}$$

Thus the induced edge labels are given by $f^*(E(G)) = \{k, k + d, k + 2d, \dots, k + (q - 1)d\}$ where $k = 2t + 1$ and $d = 1$

Thus the above defined labeling pattern f admits Sequential labeling for G .

Therefore $T(P_n) \otimes K_m^c$ is a k -Sequential graph.

Illustration 1.11: $(3,1)$ - Sequential labeling of $T(P_8) \otimes K_3^c$



Theorem 1.12: The braid graph $B(n)$ is a $(2t + 1, 1)$ - Sequential graph for all $n \geq 3$ and for any non-negative integer t .

Proof: The vertex set of $B(n)$ is $V(B(n)) = \{x_1, x_2, \dots, x_n, y_1, y_2, \dots, y_n\}$

The edge set of $B(n)$ is

$$E(B(n)) = \{x_i x_{i+1}; i = 1, 2, 3, \dots, n - 1\} \cup \{y_i y_{i+1}; i = 1, 2, \dots, n - 1\} \cup \{x_i y_{i+1}; i = 1, 2, \dots, n - 1\} \cup \{y_i x_{i+2}; i = 1, 2, \dots, n - 2\}$$

Then $|V(B(n))| = 2n$ and $|E(B(n))| = 4n - 5$

Let t be an integer such that $t \geq 0$. Define labeling $f : V(B(n)) \rightarrow \{0, 1, 2, \dots, 4n - 4\}$ as follows:

$$f(x_i) = 2i + t - 2, \quad i = 1, 2, \dots, n$$

$$f(y_i) = 2i + t - 1, \quad i = 1, 2, \dots, n$$

Then the induced edge labels are given by $f^*(u_i v_i) = f(u_i) + f(v_i)$ and are as follows:

$$f^*(x_i x_{i+1}) = 2t + 4i - 2, \quad 1 \leq i \leq n - 1$$

$$f^*(y_i y_{i+1}) = 2t + 4i, \quad 1 \leq i \leq n - 1$$

$$f^*(x_i y_{i+1}) = 2t + 4i - 1, \quad 1 \leq i \leq n - 1$$

$$f^*(y_i x_{i+2}) = 2t + 4i + 1, \quad 1 \leq i \leq n - 2$$

The set of labels of edges $x_i x_{i+1}$, $1 \leq i \leq n - 1$

$$= \{2t + 2, 2t + 6, \dots, 2t + 4n - 10, 2t + 4n - 6\}$$

The set of labels of the edges $y_i y_{i+1}$, $1 \leq i \leq n - 1$

$$= \{2t + 4, 2t + 8, \dots, 2t + 4n - 8, 2t + 4n - 4\}$$

The set of labels of the edges $x_i y_{i+1}$, $i = 1, 2, \dots, n - 1$ and $y_i x_{i+2}$, $i = 1, 2, \dots, n - 2$

$$= \{2t + 3, 2t + 5, \dots, 2t + 4n - 7, 2t + 4n - 5\}$$

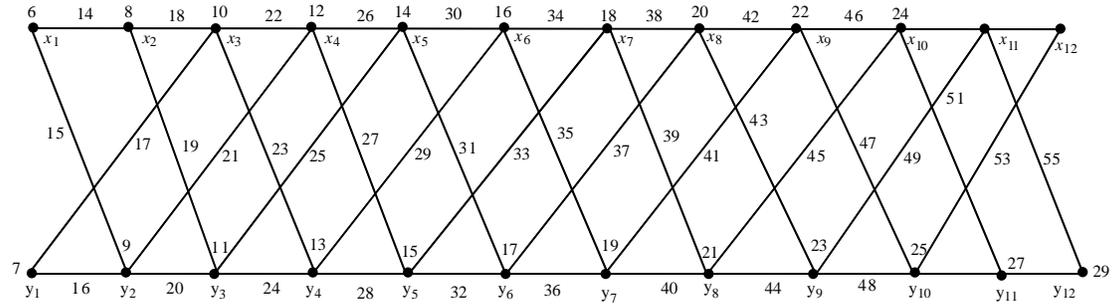
Hence the values of the edges form the set $\{2t + 2, 2t + 3, \dots, 2t + 4n - 5, 2t + 4n - 4\}$

Thus the induced edge labels are given by $f^*(E(G(B(n)))) = \{k, k + d, k + 2d, \dots, k + (q - 1)d\}$ where $k = 2t + 2$ and $d = 1$

So that f is a Sequential labeling of $B(n)$.

Hence $B(n)$ is a k -Sequential graph.

Illustration 1.13: $(14, 1)$ - Sequential labeling of $B(12)$



Theorem 1.14: The graph $G^* = B(n) \odot K_m^c$ is a $(2t + 1, 1)$ - Sequential labeling for all $m, n \geq 3$ and for any non-negative integer t .

Proof: Let $x_i, y_i, 1 \leq i \leq n$ be the vertices of $B(n)$.
Join x_i and $y_i, 1 \leq i \leq n$ to new vertices x_{ij} and $y_{ij}, 1 \leq j \leq m$ respectively.

The resultant graph is $G^* = B(n) \odot K_m^c$

$$E(G^*) = \{x_i x_{i+1} ; 1 \leq i \leq n - 1\} \cup \{y_i y_{i+1} ; 1 \leq i \leq n - 1\} \cup \{x_i y_{i+1} ; 1 \leq i \leq n - 1\} \\ \cup \{y_i x_{i+2} ; 1 \leq i \leq n - 2\} \cup \{x_i x_{ij} ; 1 \leq i \leq n, 1 \leq j \leq m\} \cup \{y_i y_{ij} ; 1 \leq i \leq n, 1 \leq j \leq m\}$$

G^* has $2n(m + 1)$ vertices and $4n + 2nm - 5$ edges.

Let t be a non-negative integer. Define a labeling $f : V(G^*) \rightarrow \{0, 1, 2, \dots, 4n + 2nm - 6\}$ as follows:

$$f(x_i) = 2i + t - 2, \quad 1 \leq i \leq n$$

$$f(y_i) = 2i + t - 1, \quad 1 \leq i \leq n$$

$$f(x_{ij}) = 4(n - 1) + mn - (i - 1) + (j - 1)n + t + 1, \quad 1 \leq i \leq n, 1 \leq j \leq m.$$

$$f(y_{ij}) = 4(n - 1) - (i - 1) + (j - 1)n + t, \quad 1 \leq i \leq n, 1 \leq j \leq m.$$

Then the induced edge labels are given by $f^*(u_i v_i) = f(u_i) + f(v_i)$ and are as follows:

$$f^*(y_i y_{i+1}) = 2t + 4i, \quad 1 \leq i \leq n - 1$$

$$f^*(x_i x_{i+1}) = 2t + 4i - 2, \quad 1 \leq i \leq n - 1$$

$$f^*(x_i y_{i+1}) = 2t + 4i - 1, \quad 1 \leq i \leq n - 1$$

$$f^*(y_i x_{i+2}) = 2t + 4i + 1, \quad 1 \leq i \leq n - 2$$

$$f^*(x_i x_{ij}) = 2t + i + n(m + j + 3) - 4, \quad 1 \leq i \leq n - 1, 1 \leq j \leq m$$

$$f^*(y_i y_{ij}) = 2t + i + n(j + 3) - 4, \quad 1 \leq i \leq n, 1 \leq j \leq m$$

Thus the set of labels of edges $x_i x_{i+1}, 1 \leq i \leq n - 1$
 $= \{2t + 2, 2t + 6, 2t + 10, \dots, 2t + 4n - 10, 2t + 4n - 6\}$

The set of labels of edges $y_i y_{i+1}, 1 \leq i \leq n - 1$
 $= \{2t + 4, 2t + 8, 2t + 12, \dots, 2t + 4n - 8, 2t + 4n - 4\}$

The set of labels of edges $x_i y_{i+1},$ and $y_i x_{i+2}, 1 \leq i \leq n - 1$

$$= \{2t + 3, 2t + 5, 2t + 7, \dots, 2t + 4n - 7, 2t + 4n - 5\}$$

The set of labels of edges $x_i x_{ij}, y_i y_{ij}, 1 \leq i \leq n, 1 \leq j \leq m$
 $= \{2t + 4n - 3, 2t + 4n - 2, 2t + 4n - 1, \dots, 2t + 4n + 2n m - 5, 2t + 4n + 2n m - 4\}$

Therefore the set of labels of the edges of G^*
 $= \{2t + 2, 2t + 3, 2t + 4, \dots, 2t + 4n - 5, 2t + 4n - 4, 2t + 4n - 3, 2t + 4n - 2, 2t + 4n - 1, \dots, 2t + 4n + 2n m - 5, 2t + 4n + 2n m - 4\}$

Thus the induced edge labels are given by $f^*(E(G^*)) = \{k, k + d, k + 2d, \dots, k + (q - 1)d\}$
 where $k = 2t + 2$ and $d = 1$

Therefore the vertex function f defined above is a Sequential labeling for G^* .

Thus G^* is a k -Sequential graph.

Illustration 1.15: $(14, 1)$ - Sequential labeling of $B(4) \otimes K_4^c$

