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Solution of Fuzzy Multi-Objective Fractional Linear Programming Problem Using Fuzzy Programming Technique based on Exponential Membership Function

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Abstract

A method of solving multi-objective fuzzy fractional linear programming problem (MOFFLPP) is presented. All the coefficients of the fractional multi-objective functions and the constraints are taken to be fuzzy numbers. In this paper a MOFFLPP is first reduced to crisp multi-objective linear programming problem and then the crisp MOLPP is solved by Zimmerman's technique using exponential membership functions.

Key Words: Multi-Objective Fuzzy Fractional Programming Problem, Multi-objective Fuzzy Programming, Exponential membership Function, Crisp LPP.

1. INTRODUCTION

While taking decisions in case of problems like financial and corporate planning, production planning , marketing and media selection, university planning and student admission, health care and hospital planning, air force maintenance units, bank branches, etc. we are frequently confronted with situations such as to decide to optimize department/ equity ratio, profit / cost, inventory / sales, actual cost / standard cost, output / employee, student / cost, nurse / patient ratio etc. with respect to some constraints (Lai and Hwang 1996) [14]. This type of problem is referred to as fractional programming problem. Most of the real world problems of above category are characterized by multiple, conflicting and incommensurate aspects of evaluation. These problems are optimized in the framework of multiple objective fractional programming models. Furthermore while addressing some real world problems, frequently the parameters are imprecise numerical quantities. Fuzzy quantities are very adequate for modelling this situation. Under this situation the above category of problems are termed as multi objective fuzzy fractional programming problems (MOFFPP).

Bellmen and Zadeh [1] introduced the concept of fuzzy quantities and also the concept of fuzzy decision making. L. Campos and A. Munoz [5] and Zimmermann [2] have introduced fuzzy programming approach to solve crisp multi objective linear programming problem. Campos and Munoz [5], Fortemps and Roubens [7] reduced fuzzy multi objective linear programming to crisp problems using ranking function. In literature, different types of solutions of fractional programming problems have been suggested by many authors such as (Lai and Hwang 1994) [14], (Charnes and Cooper 1962) [13], (Chakrabarty and Gupta 2002) [15], etc. Charnes and Cooper [13] solved linear fractional programming problem by resolving it in to two linear programming problems. Later, Kanti Swarup [16] gave an algorithm for the solution of LFPP without reducing it to LPP.

In this paper, we develop a method for solving multi objective fuzzy fractional programming problem where all the parameters of the objective functions and the constraints are fuzzy in nature. In this method, firstly the MOFFPP is reduced to MOFLPP. Similarly the MOFLPP is reduced to MOLPP using the ranking function of Roubens [7] then the MOLPP is solved by using fuzzy programming approach of Zimmermann [2, 3]. A numerical example is given at the end to illustrate the method of solution.

2. LINEAR FRACTIONAL PROGRAMMING PROBLEM

A linear fractional programming problem is given by

$$\max/\min \frac{c^T x + \alpha}{d^T x + \beta} \qquad (2.1)$$

subject to constraints

$Ax \leq = \geq b,\ x \geq 0$

where x, c and d are $n \times 1$ vectors
A is $m \times n$ matrix
b is $m \times 1$ vector
and α, β are scalars.

For maximization problem we have to maximize the fraction $\frac{c^T x+\alpha}{d^T x+\beta}$. That means we have to maximize $c^T x + \alpha$ and minimize $d^T x + \beta$. This implies that we have to maximize $(c^T x + \alpha) - (d^T x + \beta)$ which is linear. To sum up the maximization of fractional objective function subject to given constraints of a FPP ultimately reduced to the maximization of a linear objective function subject to same constraints. Similarly for minimization problem the problem reduces to the problem of minimizing $(c^T x + \alpha) - (d^T x + \beta)$

3. EXPONENTIAL MEMBERSHIP FUNCTION FOR FUZZY NUMBERS

An exponential membership function is defined by

$$\mu^E Z_p(x) = \begin{cases} 1 & \text{if } Z_p \leq L_p \\ \frac{e^{-s\psi_p(x)} - e^{-s}}{1-e^{-s}} & \text{if } L_p < Z_p \leq U_p \\ 0 & \text{if } U_p < Z_p \end{cases} \qquad (3.1)$$

where $\psi_p(x) = \frac{Z_p(x) - L_p}{U_p - L_p}$

s is any parameter, P=1,2,3....p and s is non-zero parameter prescribed by the decision maker

4. RANKING FUNCTION FOR FUZZY NUMBERS

Definition:
Let A be a fuzzy number whose membership function can generally be defined as

$$\mu_A(x) = \begin{cases} \mu_A^L(x) & a^1 \le x \le a^2 \\ 1 & a^2 < x \le a^3 \\ \mu_A^R(x) & a^3 < x \le a^4 \\ 0 & \text{otherwise} \end{cases}$$

where $\mu_A^L(x): [a^1, a^2] \to [0,1]$ and $\mu_A^R(x): [a^3, a^4] \to [0,1]$ are strictly monotonic and continuous mappings. Then it is considered as left right fuzzy number. If the membership function $\mu_A(x)$ is piecewise linear, then it is referred to as a trapezoidal fuzzy number and is usually denoted by $A = (a^1, a^2, a^3, a^4)$.

If $a^2 = a^3$ then trapezoidal fuzzy number is turned into a triangular fuzzy number $A = (a^1, a^3, a^4)$.
A fuzzy number A = (a, b, c) is said to be a triangular fuzzy number if its membership function is given by

$$\mu_A^E(x) = \begin{cases} \frac{x-a}{b-a} & a \le x < b \\ 1 & x = b \\ \frac{x-b}{c-b} & b < x \le c \\ 0 & \text{otherwise} \end{cases}$$

Assume that $R : F(\mathcal{R}) \to \mathcal{R}$. R is linear ordered function that maps each fuzzy number into the real number, in which F denotes the whole fuzzy numbers. Accordingly for any two fuzzy numbers $\tilde{a}$ and $\tilde{b}$ we have.

$\tilde{a} \underset{R}{\geq} \tilde{b}$ iff $R(\tilde{a}) \geq R(\tilde{b})$
$\tilde{a} \underset{R}{>} \tilde{b}$ iff $R(\tilde{a}) > R(\tilde{b})$
$\tilde{a} \underset{R}{=} \tilde{b}$ iff $R(\tilde{a}) = R(\tilde{b})$

We restrict our attention to linear ranking function, that is a ranking function R such that
$R(k\tilde{a} + \tilde{b}) = k R(\tilde{a}) + R(\tilde{b})$
for any $\tilde{a}$ and $\tilde{b}$ in F and any $k \in \mathcal{R}$.

5. ROUBENS RANKING FUNCTION

The ranking function suggested by M. Roubens [7] is defined by
$R(\tilde{a}) = \frac{1}{2}\int_0^1 (\inf \tilde{a}_\alpha + \sup \tilde{a}_\alpha) d\alpha$

This reduces to
$R(\tilde{a}) = \frac{1}{2}(a^L + a^U + \frac{1}{2}(\beta - \alpha))$

for a trapezoidal number
$\tilde{a} = (a^L - \alpha, a^L, a^U, a^U + \beta)$.

6. SOLVING FUZZY MULTI OBJECTIVE LINEAR FRACTIONAL PROGRAMMING PROBLEM
A fuzzy multi objective linear fractional programming problem is defined as follows

max/ min $\tilde{z} = (\tilde{z}_1, \tilde{z}_2, \dots, \tilde{z}_p)$
where $\tilde{z}_i = \frac{\tilde{c}_i^T + \tilde{\alpha}_i}{\tilde{d}_i^T + \tilde{\beta}_i}$, $\quad i = 1,2, \dots p$ (6.1)
subject to $\tilde{A}_i x \ \leq = \geq \tilde{b}_i, \quad x \geq 0$

where
$\tilde{c}_i, \tilde{d}_i$ are (n × 1) fuzzy vectors
$\tilde{A}_i$ are (m × n) matrices on the fuzzy numbers
$\tilde{b}_i$ are (m × 1) fuzzy vectors
$\tilde{\alpha}_i$ and $\tilde{\beta}_i$ are fuzzy numbers.

Step-1:
The fuzzy multi objective linear fractional programming problem (6.1) is first reduced to multi objective fuzzy linear programming problem

max/ min $\tilde{y} = (\tilde{y}_1, \tilde{y}_2, \dots, \tilde{y}_p)$
subject to $\tilde{A}_i x \ \leq = \geq \tilde{b}_i, \quad x \geq 0$
$\tilde{y}_i = (\tilde{c}_i^T + \tilde{\alpha}_i) - (\tilde{d}_i^T + \tilde{\beta}_i)$
It is reduced to the form $\tilde{y}_i = \sum_{i=1}^{n} \tilde{e}_{ij} x_j + \tilde{\gamma}_i$

Thus the problem (6.1) is reduced to the multi objective linear programming problem as follows
max/ min $\tilde{Y} = \left(\sum_{j=1}^{n} \tilde{e}_{1j} x_j + \tilde{\gamma}_1, \sum_{j=1}^{n} \tilde{e}_{2j} x_j + \tilde{\gamma}_2, \sum_{j=1}^{n} \tilde{e}_{pj} x_j + \tilde{\gamma}_p\right)$ (6.2)
subject to $\sum_k \tilde{a}_{jk}^i x_k \lesssim \tilde{b}_j^i, \ x_k \geq 0$

In trapezoidal form
$\tilde{e}_{ij} = \left(e_{ij}^1, e_{ij}^2, e_{ij}^3, e_{ij}^4\right)$
$\tilde{a}_{jk}^i = \left(a_{jk}^{i1}, a_{jk}^{i2}, a_{jk}^{i3}, a_{jk}^{i4}\right)$
$\tilde{\gamma}_i = \left(\tilde{\gamma}_i^1, \tilde{\gamma}_i^2, \tilde{\gamma}_i^3, \tilde{\gamma}_i^4\right)$
$\tilde{b}_j^i = \left(b_j^{i1}, b_j^{i2}, b_j^{i3}, b_j^{i4}\right)$

Step-2:
Using linear Ranking function R as suggested by Roubens [7], the problem (6.2) reduces to
Crisp multi-objective linear programming problem as follows max/ min $R(\tilde{Y}) = R(\tilde{y}_1), R(\tilde{y}_2), \dots, R(\tilde{y}_p)$
where $R(\tilde{y}_i) = \sum_{j=1}^{n} R(\tilde{e}_{ij}) x_j + R(\tilde{\gamma}_i)$
subject to
$\sum R(\tilde{a}_{jk}^i) x_k \lesssim R(\tilde{b}_j^i), \ x_k \geq 0$

This again can be written as max/ min $Y = (y_1, y_2, \dots, y_p)$
where $y_i = \sum_{j=1}^{n} e_{ij} x_j + \gamma_i$
subject to
$\sum \tilde{a}_{jk}^i x_k \lesssim \tilde{b}_j^i, \ x_k \geq 0$ (6.3)
where $e_{ij}, \gamma_i, a_{jk}^i, b_j^i, Y, y_i$ are real numbers corresponding to the fuzzy numbers $\tilde{e}_{ij}, \tilde{\gamma}_i, \tilde{a}_{jk}^i, \tilde{b}_j^i, \tilde{Y}, \tilde{y}_i$ with respect to the ranking function respectively.

Step-3:
If we use the exponential membership function as defined as (3.1) then an equivalent crisp model for the fuzzy model can be formulated as follows:
$Min \ \lambda$
$\lambda \leq \frac{e^{-s\psi_p(x)} - e^{-s}}{1 - e^{-s}}$, $\quad p = 1,2, \dots q$
s.t.

$\sum c'_{pk}\, x_k + (U_p - L_p)\lambda \geq U_p$
$\sum a'_{jk}\, x_k \leq b_j^i, \qquad \lambda \geq 0, x_k \geq 0$
Then it is solved by using fuzzy programming technique of Zimmermann [2]. The optimal solution thus obtained shall be optimal solution of problem (6.1).

7. LEMMA

The optimal solutions of (6.2) and (6.3) are equivalent.
Proof
Let M1, M2 be set of all feasible solutions of (6.2) and (6.3) respectively.
Then $x \in M_1$ iff $\sum_k (\tilde{a}_{jk}^i) x_k \lesssim (\tilde{b}_i)$ $\quad i = 1, 2\ldots m$

By considering R as a linear ranking function, we have
$\sum_k R(\tilde{a}_{jk}^i) x_k \lesssim R(\tilde{b}_i) \qquad$ i=1, 2…m
$\Rightarrow \sum_k a_{jk}^i\, x_k \leq b_j$
Hence $x \in M_2$
Thus $M_1 = M_2$

Let $x^* \in X$ be the complete optimal solution of (6.2).
Then $\tilde{y}_i(x^*) \geq \tilde{y}_i(x) \quad \forall\ x \in X$
where 'X' is a feasible set of solutions.
$R(\tilde{y}_i(x^*)) \geq R(\tilde{y}_i(x))$ (applying ranking function R)
$\Rightarrow \sum_{i=1}^{n} R(\tilde{e}_{ij}) x_j + R(\tilde{\gamma}_j) \geq \sum_{i=1}^{n} R(\tilde{e}_{ij}) + R(\tilde{\gamma}_j) \qquad \forall$ j=1, 2..q
$\Rightarrow \sum_{j=1}^{n} \tilde{e}_{ij}\, x_j^* + \gamma_j \geq \sum_{j=1}^{n} \tilde{e}_{ij}\, x_j + \gamma_j \qquad \forall$ j=1, 2..q
$\Rightarrow y_i(x^*) \geq y_j(x) \qquad \forall\ x$

8. NUMERICAL EXAMPLE

$$\text{Max} \left\{ \frac{\tilde{6}x_1 + \tilde{5}x_2}{\tilde{2}x_1 + \tilde{7}}, \frac{\tilde{2}x_1 + \tilde{3}x_2}{\tilde{1}x_1 + \tilde{1}x_2 + \tilde{7}} \right\} \qquad (8.1)$$
s.t.
$\tilde{1}x_1 + \tilde{2}x_2 \leq \tilde{3}$
$\tilde{3}x_1 + \tilde{2}x_2 \leq \tilde{6}$
$x_1, x_2 \geq 0 \qquad (8.2)$
where
$\tilde{6} = (5.6, 5.7, 6.2)$
$\tilde{5} = (4.7, 4.9, 5.5)$
$\tilde{2} = (1.7, 1.9, 2.5)$
$\tilde{7} = (6.6, 6.8, 7.2)$
$\tilde{2} = (1.6, 1.7, 2.2)$
$\tilde{3} = (2.7, 2.8, 3.1)$
$\tilde{1} = (0.7, 0.9, 1.5)$
$\tilde{1} = (0.7, 0.8, 1.1)$
$\tilde{7} = (6.7, 6.9, 7.5)$
$\tilde{1} = (0.8, 0.9, 1.4)$
$\tilde{2} = (1.8, 1.9, 1.4)$
$\tilde{3} = (2.7, 2.8, 3.1)$
$\tilde{3} = (2.7, 2.9, 3.5)$
$\tilde{2} = (1.7, 1.8, 2.1)$
$\tilde{6} = (5.7, 5.8, 6.1)$

The MOFFLPP (8.1) using step-3 reduces to MOFLPP
$\max \tilde{y}_1 = \tilde{6}x_1 + \tilde{5}x_2 - \tilde{2}x_1 - \tilde{7}$
$\max \tilde{y}_2 = \tilde{2}x_1 + \tilde{3}x_2 - \tilde{1}x_1 - \tilde{1}x_2 - \tilde{7} \qquad (8.3)$
Subject to

$\tilde{1}x_1 + \tilde{2}x_2 \leq \tilde{3}$
$\tilde{3}x_1 + \tilde{2}x_2 \leq \tilde{6}$
$x_1, x_2 \geq 0$ (8.4)

Using ranking function we have
$y_1 = 5.9x_1 + 5.1x_2 - 2.1x_1 - 6.9$ (8.5)
$y_2 = 1.9x_1 + 2.9x_2 - 1.1x_1 - 0.9x_2 - 7.1$ (8.6)

Subject to
$1.1x_1 + 2.1x_2 \leq 2.9$
$3.1x_1 + 1.9x_2 \leq 5.9$
$x_1, x_2 \geq 0$ (8.7)

Solving (8.5) and (8.6) we get
$x_1 = \frac{344}{211}, \; x_2 = \frac{125}{211}$

Solving (8.6) and (8.7) we get
$x_1 = 0, \; x_2 = \frac{29}{21}$

The lower bound (L.B) and upper bound (U.B) of objective functions $z_1^{'}$ and $z_2^{'}$ have been computed as follows

Function	LB	UB
$y_1^{'}$	8.7995	7.0428
$y_2^{'}$	2.7619	2.3765

If we use exponential membership function with the parameter s=1, an equation crisp model can be formulated as
Min λ
s.t.
$s[y_1(x)] + \lambda(U_1 - L_1) \geq s(U_1)$
$s[y_2(x)] + \lambda(U_2 - L_2) \geq s(U_2)$
$\sum a_{ij}x_j \leq b_i^{'}$ $\quad i = 1, 2, \dots, m$
$\lambda \geq 0, x_j \geq 0,$ $\quad j = 1, 2, \dots, n$

Now the problem becomes
min λ = −max λ
subject to
$3.8x_1 + 5.1x_2 + 1.7567\lambda \geq 8.7995$
$0.8x_1 + 2.0x_2 + 0.3854\lambda \geq 2.7619$
$1.1x_1 + 2.1x_2 \geq 2.9$
$3.1x_1 + 1.9x_2 \geq 5.9$
$x_1, x_2 \geq 0$

Solving the above problem, the optimal solution is obtained as:
$X_1^* = 1.1266$
$X_2^* = 0.9731$
Using the above value we get
$\tilde{x}_1^* = (10.8825, 11.1898, 2.3370)$
$\tilde{x}_2^* = (8.5152, 8.9405, 10.0165)$
$\tilde{y}_1^* = (4.4299, 4.6398, 5.4951)$
$\tilde{y}_2^* = (8.1697, 8.6924, 10.2603)$
Let $\tilde{f}_1^* = \frac{\tilde{X}_1^*}{\tilde{X}_2^*}$
$\tilde{f}_2^* = \frac{\tilde{Y}_1^*}{\tilde{Y}_2^*}$

The membership functions are defined as follows:

$$\mu_{\tilde{X}_1}(x) = \begin{cases} 0 & \text{for } x \le 10.8825 \text{ and } x > 12.3370 \\ \frac{x-10.8825}{0.3073} & \text{for } 10.8825 < x \le 11.1898 \\ \frac{12.3370-x}{1.1472} & \text{for } 11.1898 < x \le 12.3370 \end{cases}$$

$$\mu_{\tilde{X}_2}(x) = \begin{cases} 0 & \text{for } x \le 8.5152 \text{ and } x > 10.0165 \\ \frac{x-8.5152}{0.4253} & \text{for } 8.5152 < x \le 8.9405 \\ \frac{10.0165-x}{1.076} & \text{for } 8.9405 < x \le 10.0165 \end{cases}$$

$$\mu_{\tilde{Y}_1}(x) = \begin{cases} 0 & \text{for } x \le 4.4299 \text{ and } x > 5.4951 \\ \frac{x-4.4299}{0.2099} & \text{for } 4.4299 < x \le 4.6398 \\ \frac{5.4951-x}{0.8553} & \text{for } 4.6398 < x \le 5.4951 \end{cases}$$

$$\mu_{\tilde{Y}_2}(x) = \begin{cases} 0 & \text{for } x \le 8.1698 \text{ and } x > 10.2603 \\ \frac{x-8.1698}{0.5226} & \text{for } 8.1698 < x \le 8.6924 \\ \frac{10.2603-x}{1.5679} & \text{for } 8.6924 < x \le 10.2603 \end{cases}$$

a **- cuts :**

$\alpha\tilde{X}_1 = [0.3073\alpha + 10.8825, 12.3370 - 1.1472\alpha]$

$\alpha\tilde{X}_2 = [0.4253\alpha + 8.5152, 10.0165 - 1.076\alpha]$

$\alpha\tilde{Y}_1 = [0.2099\alpha + 4.4299, 5.4951 - 0.8553\alpha]$

$\alpha\tilde{Y}_2 = [0.5226\alpha + 8.1698, 10.2603 - 1.5679\alpha]$

$\alpha(\tilde{X}_1/\tilde{X}_2) = \left[\frac{0.3073\alpha + 10.8825}{10.0165 - 1.076\alpha}, \frac{12.3370 - 1.1472\alpha}{0.4253\alpha + 8.5152}\right] \quad \alpha \in (0,1]$

$\alpha(\tilde{Y}_1/\tilde{Y}_2) = \left[\frac{0.2099\alpha + 4.4299}{10.2603 - 1.5679\alpha}, \frac{5.4951 - 0.8553\alpha}{0.5226\alpha + 8.1698}\right] \quad \alpha \in (0,1]$

Membership functions for optimal objective fractions

$$\mu\tilde{f}_1^*(x) = \begin{cases} 0 & \text{for } x \le 1.0864 \text{ and } x > 1.4438 \\ \frac{10.0165x-10.8825}{0.3073+1.076x} & \text{for } 1.0864 < x \le 1.2516 \\ \frac{12.3370-8.5152x}{1.1472+0.4253x} & \text{for } 1.2516 < x \le 1.4438 \end{cases}$$

$$\mu\tilde{f}_2^*(x) = \begin{cases} 0 & \text{for } x \le 0.4318 \text{ and } x > 0.6726 \\ \frac{10.2603x-4.4299}{0.2099+1.5679x} & \text{for } 0.4318 < x \le 0.5338 \\ \frac{5.4951-8.1698x}{0.8553+0.5226x} & \text{for } 0.5338 < x \le 0.6726 \end{cases}$$

where $\tilde{f}_1^* = \frac{\tilde{X}_1^*}{\tilde{X}_2^*}$, $\tilde{f}_2^* = \frac{\tilde{Y}_1^*}{\tilde{Y}_2^*}$

9. CONCLUSIONS

In this paper, the fuzzy multi objective linear fractional programming problem (FMOLFPP) is reduced to FMOLPP then the FMOLPP is reduced to crisp one using ranking function. Then the resulting problem is solved by applying exponential membership function in Zimmermann's [2] method.

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