

On Euler-Beta Transform of I-function of two variables

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Abstract

In this paper, the authors establish the Euler-Beta transform of various products involving a General Class of Polynomials, Struve's function and I-function of two variables. Some similar formulae are also derived as special cases.

Keywords: Euler-Beta transform, General Class of Polynomials, Struve's Function, I-function of two variables, H-function of two variables.

1. INTRODUCTION

Recently, the Mellin Transform and Laplace Transform of product of extended general class of polynomials with I-function of two variables [4] and Mellin and Laplace transforms involving product of Struve's function and I-function of two variables [5] were evaluated . In the present paper we establish the Euler-Beta transform involving product of a General Class of Polynomials with I-function of two variables and Struve's Function with I-function of two variables.

We shall utilize the following formulae in the present investigation.

The I-function of two variables given by Shantha Kumari et.al [10]

$$I[z_1, z_2] = I_{p_1, q_1; p_2, q_2; p_3, q_3}^{o, n_1; m_2, n_2; m_3, n_3} \left[\begin{matrix} z_1 \\ z_2 \end{matrix} \left| \begin{matrix} (a_j; \alpha_j, A_j; \varepsilon_j)_{1, p_1} : (c_j, C_j; U_j)_{1, p_2} ; (e_j, E_j; P_j)_{1, p_3} \\ (b_j; \beta_j, B_j; \eta_j)_{1, q_1} : (d_j, D_j; V_j)_{1, q_2} ; (f_j, F_j; Q_j)_{1, q_3} \end{matrix} \right. \right]$$

$$= \frac{1}{(2\pi i)^2} \int_{L_s} \int_{L_t} \phi(s, \tau) \theta_1(s) \theta_2(\tau) z_1^s z_2^\tau ds d\tau \tag{1.1}$$

where $\phi(s, \tau) = \frac{\prod_{j=1}^{n_1} \Gamma^{\varepsilon_j} (1 - a_j + \alpha_j s + A_j \tau)}{\prod_{j=n_1+1}^{p_1} \Gamma^{\varepsilon_j} (a_j - \alpha_j s - A_j \tau) \prod_{j=1}^{q_1} \Gamma^{\eta_j} (1 - b_j + \beta_j s + B_j \tau)}$

$$\theta_1(s) = \frac{\prod_{j=1}^{n_2} \Gamma^{U_j} (1 - c_j + C_j s) \prod_{j=1}^{m_2} \Gamma^{V_j} (d_j - D_j s)}{\prod_{j=n_2+1}^{p_2} \Gamma^{U_j} (c_j - C_j s) \prod_{j=m_2+1}^{q_2} \Gamma^{V_j} (1 - d_j + D_j s)}$$

$$\theta_2(\tau) = \frac{\prod_{j=1}^{n_3} \Gamma^{P_j} (1 - e_j + E_j \tau) \prod_{j=1}^{m_3} \Gamma^{Q_j} (f_j - F_j \tau)}{\prod_{j=n_3+1}^{p_3} \Gamma^{P_j} (e_j - E_j \tau) \prod_{j=m_3+1}^{q_3} \Gamma^{Q_j} (1 - f_j + F_j \tau)}$$

where $n_j, p_j, q_j (j=1,2,3), m_j (j=2,3)$ are non-negative integers such that $0 \leq n_j \leq p_j, q_j > 0, 0 \leq m_j \leq q_j (j=2,3)$ (not all zero simultaneously) $\alpha_j, A_j (j=1, \dots, p_1), \beta_j, B_j (j=1, \dots, q_1), C_j (j=1, \dots, p_2), D_j (j=1, \dots, q_2), E_j (j=1, \dots, p_3), F_j (j=1, \dots, q_3)$ are positive quantities. $a_j (j=1, \dots, p_1), b_j (j=1, \dots, q_1), c_j (j=1, \dots, p_2), d_j (j=1, \dots, q_2), e_j (j=1, \dots, p_3), f_j (j=1, \dots, q_3)$ are complex numbers. The exponents $\varepsilon_j, \eta_j, U_j, V_j, P_j, Q_j$ may take non integer values. L_s and L_t are suitable contours of Mellin-Barnes type. Moreover, the contour L_s is in the complex s-plane and runs from $\sigma_1 - i\infty$ to $\sigma_1 + i\infty (\sigma_1 \text{ real})$, so that all the poles of $\Gamma^{V_j} (d_j - D_j s) (j=1, \dots, m_2)$ lie to the right of L_s and all poles of $\Gamma^{U_j} (1 - c_j - C_j s) (j=1, \dots, n_2) \Gamma^{\varepsilon_j} (1 - a_j + \alpha_j s + A_j \tau) (j=1, \dots, n_1)$ lie to the left of L_s . Similar conditions for L_t follow in complex t-plane. The detailed conditions of this function can be found in [10]. The Euler-Beta transform of the function f (z) (see [1]) is defined as

$$B\{f(z); a, b\} = \int_0^1 z^{a-1} (1-z)^{b-1} f(z) dz \text{ where } \text{Re}(a) > 0 \text{ and } \text{Re}(b) > 0 \tag{1.2}$$

The general class of polynomials [7, 8] is

$$S_n^m [x] = \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} x^k, n = 0, 1, 2, \dots \tag{1.3}$$

where m is an arbitrary positive integer and the coefficients $A_{n,k} (n, k \geq 0)$ are arbitrary constants. The Struve's function [3] is defined as

$$H_{v,y,u}^{\lambda,k} [x] = \sum_{m=0}^{\infty} \frac{(-1)^m (z/2)^{v+2m+1}}{\Gamma(km+y)\Gamma(v+\lambda m+u)}, \tag{1.4}$$

where $\text{Re}(k) > 0, \text{Re}(\lambda) > 0, \text{Re}(y) > 0, \text{Re}(v+u) > 0$.

2. MAIN RESULTS

Theorem 2.1:

$$\int_0^t x^{\rho-1} (t-x)^{\sigma-1} S_n^m (bx^h) I_{p_1,q_1;p_2,q_2;p_3,q_3}^{o,n_1;m_2,n_2;m_3,n_3} \left[\begin{matrix} \gamma x^\delta \left(a_j; \alpha_j, A_j; \varepsilon_j \right)_{1,p_1} : \\ \eta x^\theta \left(b_j; \beta_j, B_j; \eta_j \right)_{1,q_1} : \\ (c_j, C_j; U_j)_{1,p_2} ; (e_j, E_j; P_j)_{1,p_3} \\ (d_j, D_j; V_j)_{1,q_2} ; (f_j, F_j; Q_j)_{1,q_3} \end{matrix} \right] dx$$

$$= \Gamma(\sigma) \sum_{r=0}^{[n/m]} F(r) t^{\rho+\sigma+hr-1} I_{p_1,q_1+1;p_2,q_2;p_3,q_3}^{o,n_1+1;m_2,n_2;m_3,n_3} \left[\begin{matrix} \gamma t^\delta \left(a_j; \alpha_j, A_j; \varepsilon_j \right)_{1,p_1} ; (1-\rho-hr; \delta, \theta) \\ \eta t^\theta \left(b_j; \beta_j, B_j; \eta_j \right)_{1,q_1} ; (1-\rho-\sigma-hr; \delta, \theta) \\ (c_j, C_j; U_j)_{1,p_2} ; (e_j, E_j; P_j)_{1,p_3} \\ (d_j, D_j; V_j)_{1,q_2} ; (f_j, F_j; Q_j)_{1,q_3} \end{matrix} \right] \tag{2.1}$$

where $F[r] = \frac{(-n)_{mr}}{r!} A_{n,r} b^r$ provided $\text{Re}(\rho) > 0, \text{Re}(\sigma) > 0, \delta > 0, \theta > 0$ and h, b are complex numbers, m is an arbitrary positive integer and the coefficients $A_{n,r} (n, r \geq 0)$ are arbitrary constants.

Proof.

Express integral form of I function and general class of polynomials as series using (1.1) and (1.3) in L.H.S we get

$$\int_0^t x^{\rho-1} (t-x)^{\sigma-1} \sum_{r=0}^{[n/m]} \frac{(-n)_{mr}}{r!} A_{n,r} (bx^h)^r \frac{1}{(2\pi i)^2} \int_{L_s} \int_{L_\tau} \phi(s,\tau) \theta_1(s) \theta_1(\tau) (\gamma x^\delta)^s (\eta x^\theta)^\tau ds d\tau dx$$

Interchanging the order of integration and evaluating the inner integral, we get the result. The change of order of integration is justifiable due to convergence of integrals.

Theorem 2.2:

$$\int_0^t x^{\rho-1} (t-x)^{\sigma-1} H_{v,y,u}^{\lambda,k} [ax^g] I_{p_1,q_1;p_2,q_2;p_3,q_3}^{o,n_1;m_2,n_2;m_3,n_3} \left[\begin{matrix} \gamma x^\delta \left(a_j; \alpha_j, A_j; \varepsilon_j \right)_{1,p_1} : \\ \eta x^\theta \left(b_j; \beta_j, B_j; \eta_j \right)_{1,q_1} : \\ (c_j, C_j; U_j)_{1,p_2} ; (e_j, E_j; P_j)_{1,p_3} \\ (d_j, D_j; V_j)_{1,q_2} ; (f_j, F_j; Q_j)_{1,q_3} \end{matrix} \right] dx$$

$$\begin{aligned}
 &= \Gamma(\sigma) \sum_{m=0}^{\infty} G(m) t^{\rho+\sigma+g(v+2m+1)-1} I_{p_1, q_1+1; p_2, q_2; p_3, q_3}^{o, n_1+1; m_2, n_2; m_3, n_3} \left[\begin{matrix} \gamma t^\delta (a_j; \alpha_j, A_j; \varepsilon_j)_{1, p_1} : \\ \eta t^\theta (b_j; \beta_j, B_j; \eta_j)_{1, q_1} : \\ (1-\rho-g(v+2m+1); \delta, \theta), (c_j, C_j; U_j)_{1, p_2} ; (e_j, E_j; P_j)_{1, p_3} \\ (1-\rho-\sigma-g(v+2m+1); \delta, \theta), (d_j, D_j; V_j)_{1, q_2} ; (f_j, F_j; Q_j)_{1, q_3} \end{matrix} \right] \quad (2.2)
 \end{aligned}$$

where $G[m] = \frac{(-1)^m (a/2)^{v+2m+1}}{\Gamma(km+y)\Gamma(v+\lambda m+u)}$ provided $\text{Re}(k) > 0, \text{Re}(\lambda) > 0, \text{Re}(y) > 0, \text{Re}(v+u) > 0, \text{Re}(\rho) > 0, \text{Re}(\sigma) > 0, \delta > 0, \theta > 0$ and a, g are complex numbers.

Proof.

Using (1.1) and (1.4) representing the integral form of I-function of two variables and Struve’s function in series form we get

$$\begin{aligned}
 &\int_0^t x^{\rho-1} (t-x)^{\sigma-1} \sum_{m=0}^{\infty} \frac{(-1)^m \left(\frac{ax^g}{2}\right)^{v+2m+1}}{\Gamma(km+y)\Gamma(v+\lambda m+u)} \frac{1}{(2\pi i)^2} \\
 &\quad \int_{L_S} \int_{L_T} \phi(s, \tau) \theta_1(s) \theta_1(\tau) (\gamma x^\delta)^s (\eta x^\theta)^\tau ds d\tau dx.
 \end{aligned}$$

Interchanging the order of integration and evaluating the inner integral we get the result. The change of order of integration is justifiable due to convergence of integrals.

3. SPECIAL CASES

(i) Put $b = 1$ and $h = 0$ in (2.1) we get Euler-Beta Transform of I-function of two variables.

$$\begin{aligned}
 &\int_0^t x^{\rho-1} (t-x)^{\sigma-1} I_{p_1, q_1; p_2, q_2; p_3, q_3}^{o, n_1; m_2, n_2; m_3, n_3} \left[\begin{matrix} \gamma x^\delta (a_j; \alpha_j, A_j; \varepsilon_j)_{1, p_1} : (c_j, C_j; U_j)_{1, p_2} ; (e_j, E_j; P_j)_{1, p_3} \\ \eta x^\theta (b_j; \beta_j, B_j; \eta_j)_{1, q_1} : (d_j, D_j; V_j)_{1, q_2} ; (f_j, F_j; Q_j)_{1, q_3} \end{matrix} \right] dx \\
 &= \Gamma(\sigma) t^{\rho+\sigma-1} I_{p_1, q_1+1; p_2, q_2; p_3, q_3}^{o, n_1+1; m_2, n_2; m_3, n_3} \left[\begin{matrix} \gamma t^\delta (a_j; \alpha_j, A_j; \varepsilon_j)_{1, p_1} : (1-\rho; \delta, \theta) \\ \eta t^\theta (b_j; \beta_j, B_j; \eta_j)_{1, q_1} : (1-\rho-\sigma; \delta, \theta) \\ (c_j, C_j; U_j)_{1, p_2} ; (e_j, E_j; P_j)_{1, p_3} \\ (d_j, D_j; V_j)_{1, q_2} ; (f_j, F_j; Q_j)_{1, q_3} \end{matrix} \right] \quad (3.1)
 \end{aligned}$$

(ii) Take $a = 1$ and $g = 0$ in (2.2), we get another Euler-Beta Transform of I-function of two variables.

(iii) Take $\varepsilon_j = \eta_j = U_j = V_j = P_j = Q_j = 1$ in (2.1) we get Euler-Beta Transform of product of a general class of polynomials and H-function of two variables.

$$\begin{aligned}
 & \int_0^t x^{\rho-1} (t-x)^{\sigma-1} S_n^m (bx^h) H_{p_1, q_1; p_2, q_2; p_3, q_3}^{o, n_1; m_2, n_2; m_3, n_3} \left[\gamma x^\delta \left(a_j; \alpha_j, A_j \right)_{1, p_1} : (c_j, C_j)_{1, p_2} ; (e_j, E_j)_{1, p_3} \right. \\
 & \left. \eta x^\theta \left(b_j; \beta_j, B_j \right)_{1, q_1} : (d_j, D_j)_{1, q_2} ; (f_j, F_j)_{1, q_3} \right] dx \\
 & = \Gamma(\sigma) \sum_{r=0}^{[n/m]} F(r) t^{\rho+\sigma+hr-1} H_{p_1, q_1+1; p_2, q_2; p_3, q_3}^{o, n_1+1; m_2, n_2; m_3, n_3} \left[\gamma t^\delta \left(a_j; \alpha_j, A_j \right)_{1, p_1} ; \right. \\
 & \quad \left. (1-\rho-hr; \delta, \theta), (c_j, C_j)_{1, p_2} ; (e_j, E_j)_{1, p_3} \right. \\
 & \quad \left. (1-\rho-\sigma-hr; \delta, \theta) (d_j, D_j)_{1, q_2} ; (f_j, F_j)_{1, q_3} \right] , \tag{3.2}
 \end{aligned}$$

where $F[r] = \frac{(-n)_{mr}}{r!} A_{n,r} b^r$ provided $\text{Re}(\rho) > 0, \text{Re}(\sigma) > 0, \delta > 0, \theta > 0$ and h, b are complex numbers, m is an arbitrary positive integer and the coefficients $A_{n,r} (n, r \geq 0)$ are arbitrary constants.

(iv) Take $\varepsilon_j = \eta_j = U_j = V_j = P_j = Q_j = 1$ in (2.2) we get Euler-Beta Transform of product of Struve's function and H-function of two variables

$$\begin{aligned}
 & \int_0^t x^{\rho-1} (t-x)^{\sigma-1} H_{v, y, u}^{\lambda, k} [ax^g] H_{p_1, q_1; p_2, q_2; p_3, q_3}^{o, n_1; m_2, n_2; m_3, n_3} \left[\gamma x^\delta \left(a_j; \alpha_j, A_j \right)_{1, p_1} : (c_j, C_j)_{1, p_2} ; (e_j, E_j)_{1, p_3} \right. \\
 & \left. \eta x^\theta \left(b_j; \beta_j, B_j \right)_{1, q_1} : (d_j, D_j)_{1, q_2} ; (f_j, F_j)_{1, q_3} \right] dx \\
 & = \Gamma(\sigma) \sum_{m=0}^{\infty} G(m) t^{\rho+\sigma+g(v+2m+1)-1} H_{p_1, q_1+1; p_2, q_2; p_3, q_3}^{o, n_1+1; m_2, n_2; m_3, n_3} \left[\gamma t^\delta \left(a_j; \alpha_j, A_j \right)_{1, p_1} ; \right. \\
 & \quad \left. (1-\rho-g(v+2l+1); \delta, \theta), (c_j, C_j)_{1, p_2} ; (e_j, E)_{1, p_3} \right. \\
 & \quad \left. (1-\rho-\sigma-g(v+2l+1); \delta, \theta), (d_j, D_j)_{1, q_2} ; (f_j, F_j)_{1, q_3} \right] , \tag{3.3}
 \end{aligned}$$

where $G[m] = \frac{(-1)^m (a/2)^{v+2m+1}}{\Gamma(km+y)\Gamma(v+\lambda m+u)}$ provided $\delta > 0, \theta > 0, \text{Re}(k) > 0, \text{Re}(\lambda) > 0,$

$\text{Re}(y) > 0, \text{Re}(v+u) > 0, \text{Re}(\rho) > 0, \text{Re}(\sigma) > 0$ and a, g are complex numbers,

(v) Put $t=1$ in (2.1) we get

$$\begin{aligned}
 & \int_0^1 x^{\rho-1} (1-x)^{\sigma-1} H_{v, y, u}^{\lambda, k} [ax^g] H_{p_1, q_1; p_2, q_2; p_3, q_3}^{o, n_1; m_2, n_2; m_3, n_3} \left[\gamma x^\delta \left(a_j; \alpha_j, A_j; \varepsilon_j \right)_{1, p_1} : \right. \\
 & \left. \eta x^\theta \left(b_j; \beta_j, B_j; \eta_j \right)_{1, q_1} : \right. \\
 & \quad \left. (c_j, C_j; U_j)_{1, p_2} ; (e_j, E_j; P_j)_{1, p_3} \right. \\
 & \quad \left. (d_j, D_j; V_j)_{1, q_2} ; (f_j, F_j; Q_j)_{1, q_3} \right] dx
 \end{aligned}$$

$$\begin{aligned}
 &= \Gamma(\sigma) \sum_{r=0}^{\lfloor \frac{n}{m} \rfloor} \frac{(-n)_{mr}}{r!} A_{n,r} b^r I_{p_1, q_1+1; p_2, q_2; p_3, q_3}^{o, n_1+1; m_2, n_2; m_3, n_3} \left[\gamma \left(a_j; \alpha_j, A_j; \varepsilon_j \right)_{1, p_1}; (1-\rho-hr; \delta, \theta) \right. \\
 &\quad \left. \eta \left(b_j; \beta_j, B_j; \eta_j \right)_{1, q_1}; (1-\rho-\sigma-hr; \delta, \theta) \right. \\
 &\quad \left. (c_j, C_j; U_j)_{1, p_2}; (e_j, E_j; P_j)_{1, p_3} \right. \\
 &\quad \left. (d_j, D_j; V_j)_{1, q_2}; (f_j, F_j; Q_j)_{1, q_3} \right] \tag{3.4}
 \end{aligned}$$

(vi) Put t = 1 in (2.2) we get

$$\begin{aligned}
 &\int_0^1 x^{\rho-1} (1-x)^{\sigma-1} H_{v, y, u}^{\lambda, k} [ax^g] I_{p_1, q_1+1; p_2, q_2; p_3, q_3}^{o, n_1; m_2, n_2; m_3, n_3} \left[\gamma x^\delta \left(a_j; \alpha_j, A_j; \varepsilon_j \right)_{1, p_1} : \right. \\
 &\quad \left. \eta x^\theta \left(b_j; \beta_j, B_j; \eta_j \right)_{1, q_1} : \right. \\
 &\quad \left. (c_j, C_j; U_j)_{1, p_2}; (e_j, E_j; P_j)_{1, p_3} \right. \\
 &\quad \left. (d_j, D_j; V_j)_{1, q_2}; (f_j, F_j; Q_j)_{1, q_3} \right] dx \\
 &= \Gamma(\sigma) \sum_{m=0}^{\infty} \frac{(-1)^m (a/2)^{v+2m+1}}{\Gamma(km+y)\Gamma(v+\lambda m+u)} I_{p_1, q_1+1; p_2, q_2; p_3, q_3}^{o, n_1+1; m_2, n_2; m_3, n_3} \left[\gamma \left(a_j; \alpha_j, A_j; \varepsilon_j \right)_{1, p_1} : \right. \\
 &\quad \left. \eta \left(b_j; \beta_j, B_j; \eta_j \right)_{1, q_1} : \right. \\
 &\quad \left. (1-\rho-g(v+2m+1); \delta, \theta); (c_j, C_j; U_j)_{1, p_2}; (e_j, E_j; P_j)_{1, p_3} \right. \\
 &\quad \left. (1-\rho-\sigma-g(v+2m+1); \delta, \theta); (d_j, D_j; V_j)_{1, q_2}; (f_j, F_j; Q_j)_{1, q_3} \right] \tag{3.5}
 \end{aligned}$$

4. CONCLUSION

On specialization of parameters in I-function of two variables, we get Euler-Beta Transform of various special functions with general class of polynomials and Struve's function as special cases.

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