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Reliability Analysis of Six Unit Bridge and Parallel Series Networks With Critical and Non-Critical Human Errors

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Abstract

Humans play a pivotal role in the design, development and operational phases of engineering systems. Reliability evaluation of systems without taking into consideration the human element does not provide a realistic picture. Hence, there is a definite need for incorporating the occurrence of human errors in system reliability evaluation. This paper presents reliability analysis of bridge and parallel-series networks with critical and non-critical Human errors. A newly developed approach is used to perform the system reliability analysis. The approach demonstrated in this paper, which consists of six unit bridge network, is a modified version of the block diagram approach.

Keywords: Reliability, Critical and Non-Critical human errors, Exponential distribution, Bridge network, MTTF, Hazard rate.

1. INTRODUCTION

A Human error is defined as a failure to perform a prescribed task (or the performance of a prohibited action) which could lead to disruption of scheduled operations or result in damage to property and equipment. Furthermore, depending upon the severity of human error consequences, human errors can be classified into two categories, namely, critical and non-critical. For our purpose the occurrence of a critical human error causes the entire system to fail where as the occurrence of a non-critical human error results in a single unit failure only.

This paper presents reliability analysis of bridge and parallel-series networks with critical and non-critical Human errors [4, 1, 2]. A newly developed approach [3, 1, 2] is used to perform system reliability analysis. This approach is a modified version of the block diagram approach and demonstrated in this paper which consists of six unit bridge network.

2. ASSUMPTIONS

The following assumptions are associated with the analysis in this study:

1. A unit can fail either due to a hardware failure or due to a non-critical human error
2. The occurrence of a critical human error can result in total system failure but the occurrence of a non-human critical error can cause the failure of a single unit only.
3. Each unit failure is independent of others.

3. SYMBOLS

The following symbols are associated with this model

F_j hardware failure probability of the unit for j=1,2,3,4,5,6.

f_i the unit failure probability with respect to non-critical human errors, for j=1,2,3,4,5,6.

f_c critical human error occurrence probability associated with the system

R_{Hj} hard ware reliability of the j^{th} unit

R_{NCj} reliability of the j^{th} unit with respect to non-critical human errors

R_j reliability of the j^{th} unit with respect to hardware failures and non-critical human errors

R_c system reliability with respect to critical human errors

$R_{H, NC}$ system reliability with respect to hardware failure and non-critical human errors

R_h bridge system reliability with respect to hardware failure, critical and non-critical human errors.

S Laplace transform variable

T time

The time-independent reliability analyses are developed for the following two cases.

4. GENERAL MODEL

Bridge Network:

This paper represents a six unit bridge network with critical and non-critical human errors as shown in figure 1.

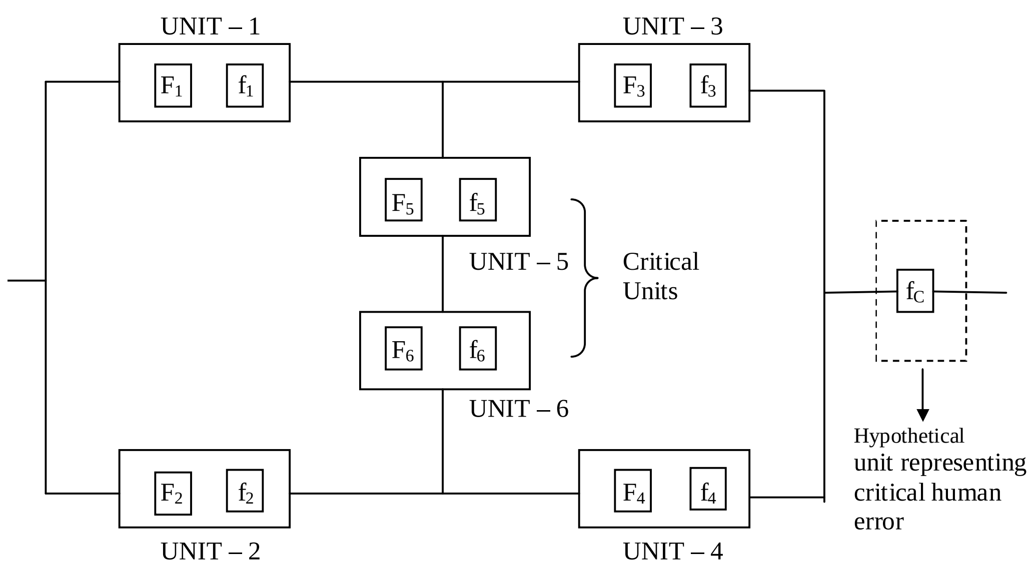

Figure 1: Block diagram for Bridge Network

In this figure, each real unit is represented by a rectangle. The failure probability of each unit is divided in to two components, namely, hardware failure probability and non-critical human error probability.

These failure probabilities are represented by blocks connected in series as shown in each rectangle in figure 1. A hypothetical unit representing critical human errors is connected in series with the bridge network. The total system can fail due to the failure of this hypothetical unit.

5. SPECIAL CASE MODELS – RELIABILITY AND MTTF

Case 1 : Non-identical units :

The hardware reliability of j^{th} unit is given by

$$R_{Hj} = 1 - F_j, \text{ for } j = 1,2,3,4,5,6 \quad (1)$$

The reliability for j^{th} unit respect to non-critical human error is

$$R_{NCj} = 1 - f_j, \text{ for } j = 1,2,3,4,5,6 \quad (2)$$

The reliability for the j^{th} unit with respect to hardware failures and non-critical human errors is

$$R_j = R_{Hj} * R_{NCj}, \text{ for } j = 1,2,3,4,5,6 \quad (3)$$

The bridge network's reliability with respect to hardware failures and non-critical human errors is :

$$R_{H,NC} = 2R_1R_2R_3R_4R_5R_6 - R_2R_4R_3R_5R_6 - R_2R_4R_1R_5R_6 - R_1R_2R_3R_5R_6 - R_1R_3R_4R_5R_6 + R_2R_5R_6R_3 + R_2R_4 + R_1R_3 \quad (4)$$

The reliability of the bridge network with respect to critical human errors only is

$$R_C = 1 - f_C \quad (5)$$

Finally, using equations (4) and (5), we get

$$R_6 = R_C * R_{H,NC} \quad (6)$$

Case – II : Identical Units:

By using $R_j = R$ (i.e., $F_j = F$ and $f_j = f$), for j = 1,2,3,4,5,6 in equation (6) yields

$$R_b = R_C\,(2R^6 - 4R^5 + R^4 + 2R^2) \quad (7)$$

where $R = R_H * R_{NC}$, $R_H = 1 - F$ and $R_{NC} = 1 - f$

The plots of equation (7) are shown in fig. 2 for the specified values of F, f and F_C.

These plots clearly show the impact of varying critical human error probability f on bridge system reliability. It is evident from these plots that the system reliability decreases with increasing values of f and f_C.

Reliability Plots Type – I

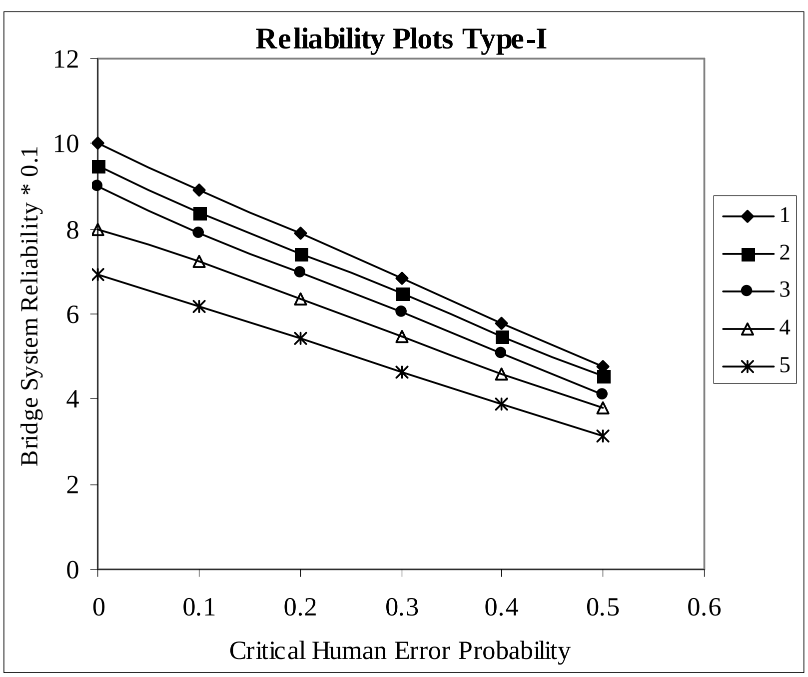

Fig. 2:

F = 0.1

Series	1	2	3	4	5
	f = 0	0.1	0.2	0.3	0.4

Time dependent analysis for the following two cases is developed.

Case A : Exponentially distributed failure times

For exponentially distributed hardware failure, critical and non-critical human error times the time dependent equations for R_H, R_{NC}, R and R_C are as follows

$$R_H(t) = e^{-\lambda_H t} \tag{8}$$

where λ_H is the constant hard ware failure rate of a unit

$$R_{NC}(t) = e^{-\lambda_{NC} t} \tag{9}$$

where λ_{NC} is the constant non-critical human error rate associated with a unit.

$$R(t) = e^{-Xt} \tag{10}$$

where $X = \lambda_H + \lambda_{NC}$

$$R_C(t) = e^{-\lambda_C t} \tag{11}$$

where λ_C is the constant critical human error rate associated with the system. Using equation (7) – (11); we get reliability of the seven identical unit network as follows

$$R_b(t) = 2e^{-(6X+\lambda_C)t} - 4e^{-(5X+\lambda_C)t} + e^{-(4X+\lambda_C)t} + 2e^{-(2X+\lambda_C)t} \tag{12}$$

The plots of equations (12) are shown in fig. 3 for the assumed values of the model parameters.

Reliability Plots Type – II

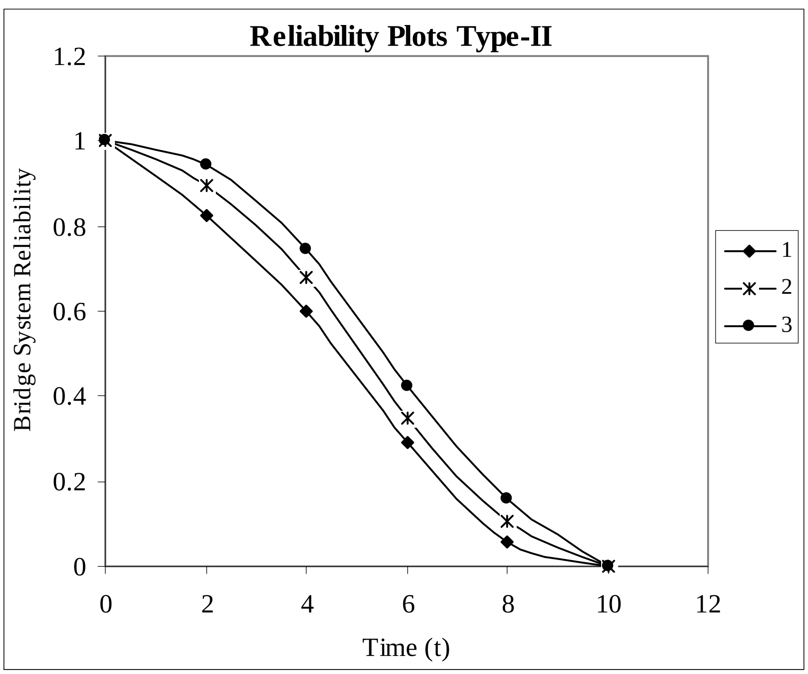

Fig. 3:

$\lambda_H = 0.12$, $\lambda_{NC} = 0.08$

Series	1	2	3
	λ_C=0	λ_C = 0.03	λ_C = 0.07

The mean time to failure of the bridge system is given by

$$MTTF_b = \int_0^\infty R_b(t)dt$$
$$= \frac{2}{6X+\lambda_C} - \frac{4}{5X+\lambda_C} + \frac{1}{4X+\lambda_C} + \frac{2}{2X+\lambda_C} \quad (13)$$

The plots of the above equation are shown in fig. 4.

Mean Time to Failure Plots

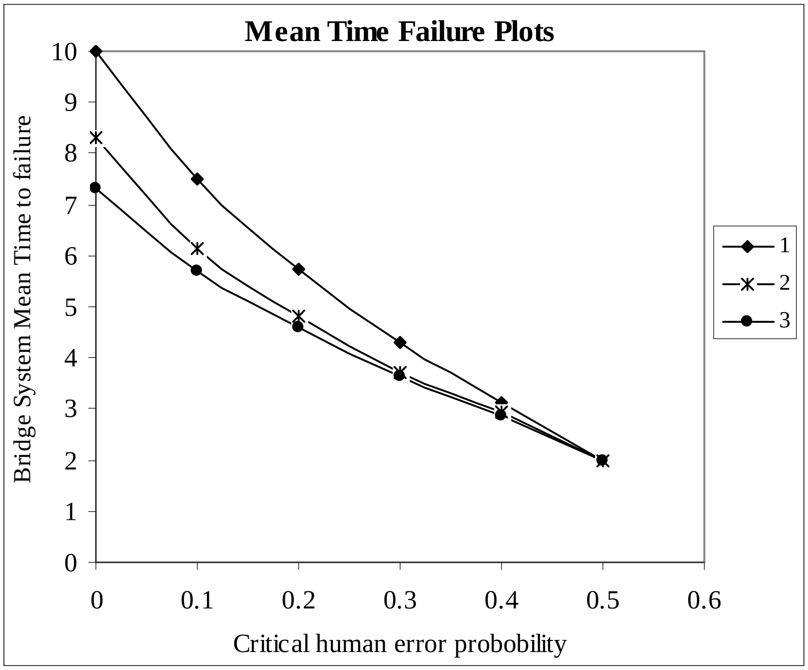

Fig. 4:

$\lambda_H = 0.1$

Series	1	2	3
	λ_{NC}=0	λ_{NC} = 0.02	λ_{NC} = 0.04

These plots clearly demonstrate the effect of varying critical and non-critical human error rates λ_C and λ_{NC} on the bridge system mean time to failure.

The variance of time to failure of the bridge system with human errors is

$$\sigma b^2 = -2 \lim_{s \to 0} R_b^1(s) - (MTTF_b)^2$$
$$= \frac{4}{(6X+\lambda_C)^2} - \frac{8}{(5X+\lambda_C)^2} + \frac{2}{(4X+\lambda_C)^2} + \frac{4}{(2X+\lambda_C)^2} - \left[\frac{2}{2X+\lambda_C} - \frac{4}{5X+\lambda_C} + \frac{1}{4X+\lambda_C} + \frac{2}{2X+\lambda_C}\right]^2 \quad (14)$$

where $R_b^1(t)$ is the derivate of Laplace transform of $R_b(t)$ with respect to s.

The bridge system failure density f_u^n is given by

$$f_b(t) = -R_b^1(t)$$
$$= 2(6X+\lambda_C)e^{-(6X+\lambda_C)t} - 4(5X+\lambda_C)e^{-(5X+\lambda_C)t} + (4X+\lambda_C)e^{-(4X+\lambda_C)t} + 2(2X+\lambda_C)e^{-(2X+\lambda_C)t} \quad (15)$$

where $R_b^1(t) = \frac{d}{dt}R_b(t)$

the hazard rate f_u^n of the bridge system is

$$h_b(t) = \frac{f_b(t)}{R_b(t)}$$
$$= \frac{2(6X+\lambda_C)e^{-6Xt} - 4(5X+\lambda_C)e^{-5Xt} + (4X+\lambda_C)e^{-4Xt} + 2(2X+\lambda_C)e^{-2Xt}}{2e^{-6Xt} - 4e^{-5Xt} + e^{-4Xt} + 2e^{-2Xt}} \quad (16)$$

The plots of hazard rate of function of the bridge system is given below in fig.5.

Hazard Rate Plots Type – I

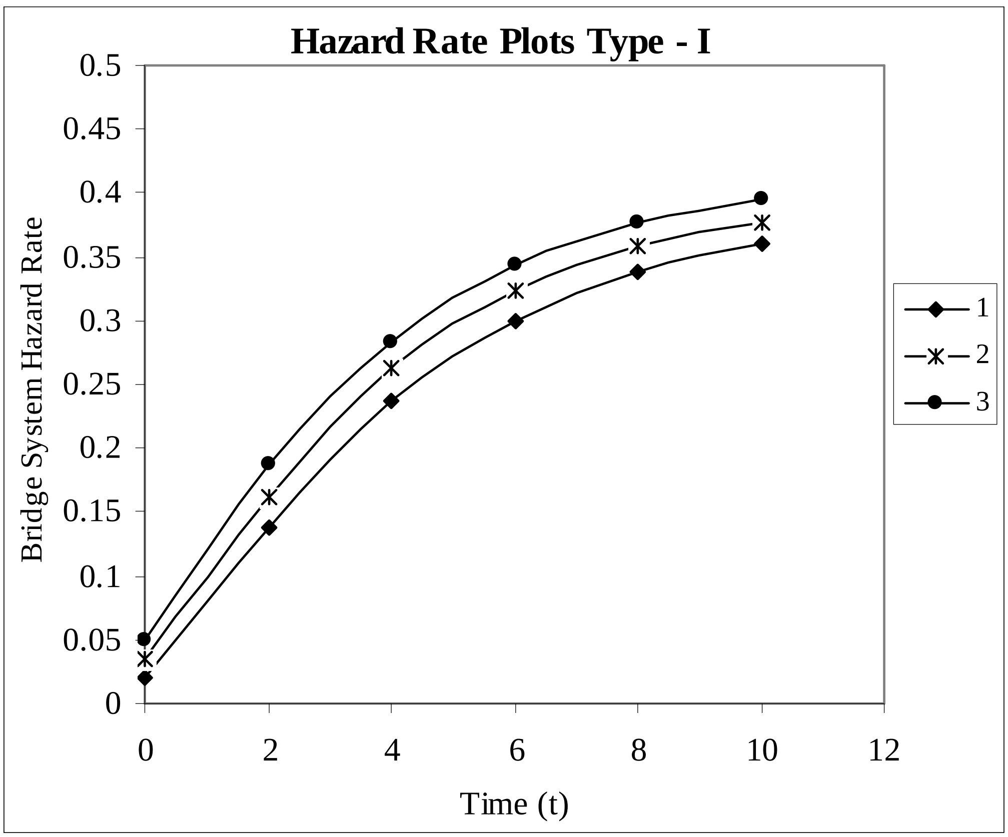

Fig. 5:

$\lambda_H = 0$ $\lambda_{NC} = 0.08$

Series	1	2	3
	$\lambda_C=0$	$\lambda_C = 0.03$	$\lambda_C = 0.07$

These plots clearly show the impact of varying time + and λ_C. It is evident from there plots that the System Hazard rate increases with increasing values of t and λ_C.

Case B : Rayleigh distributed failure times the time dependent equations for R_H, R_{NC}, R and R_C are

$$R_H(t) = e^{-\beta_H t^2} \tag{17}$$

where $\beta_H = \frac{1}{\alpha_H}$;

α_H is the scale parameter associated with the Rayleigh distribution representing hardware failure times of a unit

$$R_{NC}(t) = e^{-\beta_{NC} t^2} \tag{18}$$

where $\beta_{NC} = \frac{1}{\alpha_{NC}}$; α_{NC} is the scale parameter associated with the Rayleigh distribution representing failure times of a unit due to non-critical human errors.

$$R(t) = e^{-yt^2} \tag{19}$$

where $y = \beta_H + \beta_{NC}$

$$R_C(t) = e^{-\beta_C t^2} \tag{20}$$

and $\beta_C = 1/\alpha_C : \alpha_C$ is the scale parameter associated with the Rayleigh distribution representing bridge system failure time with respect to critical human error.

Using equations (7), (17) – (20), the bridge system reliability with human error is

$$R_b(t) = 2e^{-(\beta_C+6y)t^2} - 4e^{-(\beta_C+5y)t^2} + e^{-(\beta_C+4y)t^2} + 2e^{-(\beta_C+2y)t^2} \tag{21}$$

$$\text{MTTF}_b = \int_0^\infty R_b(t)dt = \left(\frac{\pi}{\beta_C+6y}\right)^{1/2} - 2\left(\frac{\pi}{\beta_C+5y}\right)^{1/2} + \frac{1}{2}\left(\frac{\pi}{\beta_C+4y}\right)^{1/2} + \left(\frac{\pi}{\beta_C+2y}\right)^{1/2} \tag{22}$$

The failure density function of the bridge system is

$$f_b(t) = \frac{d}{dt}R_b(t)$$

$$= t[4(\beta_C+6y)e^{-(\beta_C+6y)t^2} - 8(\beta_C+5y)e^{-(\beta_C+5y)t^2} + 2(\beta_C+4y)e^{-(\beta_C+4y)t^2} + 4(\beta_C+2y)e^{-(\beta_C+4y)t^2}] \tag{23}$$

The system hazard rate function is expressed as

$$h_b(t) = \frac{f_b(t)}{R_b(t)} = \frac{A(t)}{B(t)} \tag{24}$$

where $A(t) = 2t[(\beta_C+6y)e^{-6yt^2} - 4(\beta_C+5y)e^{-5yt^2} + (\beta_C+4y)e^{-4yt^2} + 2(\beta_C+2y)e^{-4yt^2}]$

where $B(t) = e^{-6yt^2} - 4e^{-5yt^2} + e^{-4yt^2} + 2e^{-4yt^2}$

The plots of equation (24) are shown in fig. 6.

Hazard Rate plots Type – II

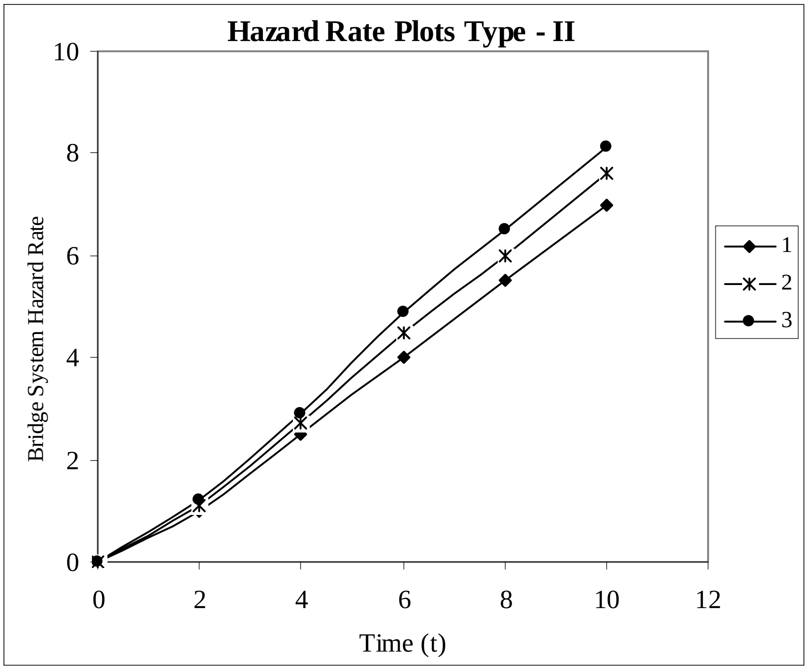

Fig. 6:

$\beta_H = 0.12$ $\beta_{NC} = 0.08$

Series	1	2	3
	$\beta_C=0$	$\beta_C = 0.03$	$\beta_C = 0.07$

These plots clearly show the impact of varying time t and β_C. It is evident from these plots that the system Hazard Rate increases with increasing values of t and β_C.

6. RESULTS

The numerical results pertaining to equation (21) are tabulated in table 1.

Table 1: Reliability values for Bridge system

For the hypothetical values **$\beta_H = 0.13$, $\beta_{NC} = 0.09$** the reliability values for bridge system are computed as follows.

Time (t)	**Bridge System Reliability** **$\beta_H = 0.13$, $\beta_{NC} = 0.09$**				
	$\beta_C = 0.0$	**$\beta_C = 0.02$**	**$\beta_C = 0.04$**	**$\beta_C = 0.06$**	**$\beta_C = 0.08$**
0.0	1.000	1.000	1.000	1.000	1.000
0.3	0.999	0.998	0.996	0.994	0.992
0.6	0.987	0.980	0.973	0.966	0.959
0.9	0.936	0.921	0.906	0.899	0.878
1.2	0.821	0.798	0.775	0.753	0.732
1.5	0.646	0.619	0.591	0.566	0.540
1.8	0.453	0.425	0.399	0.373	0.350
2.1	0.283	0.259	0.237	0.217	0.199
2.4	0.159	0.142	0.126	0.112	0.100
2.7	0.081	0.070	0.061	0.053	0.046
3.0	0.038	0.032	0.027	0.022	0.019

7. CONCLUSIONS:

From the table we observe that as the time increases the reliability decreases for a fixed value of β_C . Also we observe that as β_C increases the reliability decreases with increasing value of time T.

REFERENCES

[1]. Dhillon B.S. and Rayapati S.N.: Reliability modelling of systems with human errors, proceedings of the Twentieth Hawaii International Conference on System Sciences (HICSS-20). Honolulu. Hawaii. USA, Vol.111. pp. 493-501, 1987.

[2]. Dhillon, B.S. (1986): Human Reliability with Human Factors, Pergamon Press, Inc., New York.

[3]. Dhillon, B.S., and Rayapati S.N.: Human Reliability analysis methods, Proceedings of the Third International Conference of Human Factors in Manufacturing (HUMAN-3), Standard – Upon – Avon, U.K., 1986

[4]. Dhillon, B.S., and Rayapati S.N.: Analysis of redundant systems with human errors, Proceedings of the Annual Reliability and Maintainability Symposium, IEEE, New York, U.S.A., pp.315-321, 1985.