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ON DIVISOR CORDIAL GRAPH

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Abstract

In this paper we prove that some known graphs such as the Herschel graph and some graphs constructed in this paper are divisor cordial graphs.

Keywords: Herschel graph, Splitting graph, Shell graph, Book graph, Wheel graph, Fan graph, Parachute graph, divisor cordial labeling.

1. INTRODUCTION

In 2011 Varatharajan and others [1] defined a divisor cordial labeling of a graph G with vertex set V(G) as a bijection f from V(G) to $\{1,2,\ldots,|V(G)|\}$ so that each edge uv is assigned the label 1 if f(u) divides f(v) or f(v) divides f(u) and 0 otherwise, such that the number of edges labelled with 0 and the number of edges labelled with 1 differ by atmost 1. If a graph admits this labeling, then it is called a divisor cordial graph. In Varatharajan and others [5] the authors proved some results on divisor cordial graph. Also some work has been done in this area Lawrence and others, Vaidya and others[2,3,4]. In this paper we consider a simple finite graph without isolated vertices. We prove that a number of graphs including the Hershel graph, and some constructed graphs in this paper are divisor cordial graphs.

2. SPLITTING GRAPH

Definition 1.1:

Let G be a graph, for each point v of a graph G take a new point v'. Join v' to those points of G adjacent to v. The graph thus obtained is called the splitting graph of G.

Theorem 1.1: G = G' ∪ P_n is a divisor cordial graph where G' = $Spl(K_{1,n})$

Proof: Let $G = Spl(K_{1,n}) \cup P_n$. Let $v_1, v_2, \ldots, v_n$ be the pendant vertices, v be the opex vertex of $K_{1,n}$ and $u, u_1, u_2, \ldots, u_n$ be the vertices corresponding to $v, v_1, v_2, \ldots, v_n$ in $Spl(K_{1,n})$ also $w_1, w_2, \ldots, w_n$ be the vertices of path P_n. Then $V(G)=\{v, v_1, v_2, \ldots, v_n, u, u_1, u_2, \ldots, u_n, w_1, w_2, \ldots, w_n\}$ and

$$E(G) = \{vv_i / 1 \le i \le n\} \cup \{uu_i / 1 \le i \le n\} \cup \{uv_i / 1 \le i \le n\} \cup \{(w_i w_{i+1} / 1 \le i \le n-1\}.$$

Thus $Spl(K_{1,n})$ has 2n+2 vertices, 3n edges and P_n has n vertices, n-1 edges.
The graph G has $3n+2$ vertices and $4n-1$ edges.
Define $f: V(Spl(K_{1,n}) \cup P_n) \to \{1,2,\ldots,3n+2\}$ as follows

$f(u) = 1$

$f(v)$ = Highest Prime number $\le 2n+2$

$f(v_i) = n+i+1$, for $1 \le i \le n+1$ (except the highest prime number).

$f(u_i) = i+1$, for $1 \le i \le n$

$f(w_i) = 2n+2+i$, for $1 \le i \le n$

The edge labels are

$f(vv_i) = 0$, $1 \le i \le n$

$f(v_i u) = 1$, $i = 1,2,\ldots,n$.

$f(u_i u) = 1$, $i = 1,2,\ldots,n$.

$f(w_i w_{i+1}) = 0$, $1 \le i \le n-1$

Now, we obtain the following
$e_f(1) = 2n$ and

$e_f(0) = n + n - 1 = 2n - 1$.

Hence $|e_f(0) - e_f(1)| = 1$ which satisfies the required condition.

Hence G'∪ P_n is a divisor cordial graph.

Illustration: Spl$(K_{1,3}) \cup P_3$ is a divisor Cordial graph.

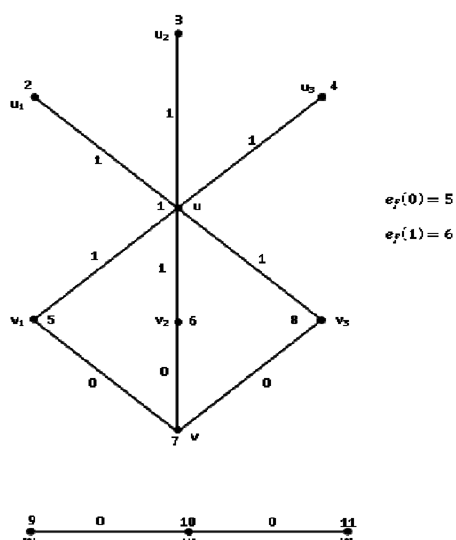

Figure 1:

3. HERSCHEL GRAPH

Definition 1.2:

The Herschel graph (H) is a bipartite graph with 11 vertices and 18 edges, the smallest non-Hamiltonian polyhedral graph.

Theorem 1.2: The Herschel graph is a divisor cordial graph.

Proof: Let H be a Herschel graph. The vertex set H is $V(H) = \{v_i / 1 \le i \le 11\}$. The edge of H is E(H)

$$E(H) = \{v_1 v_{2i} / 1 \le i \le 4\} \cup \{v_i v_n / i = 3,5,7\} \cup \{v_i v_{10} / i = 3,7\} \cup \{(v_2 v_9)\} \cup \{(v_i v_{i+1}) / 2 \le i \le 9\}.$$

The graph H has 11 vertices and 18 edges. The vertices are labeled in this order 1, 2, 4, 8, 3, 6, 5, 10, 7, 9, 11 but the labels of V_{10} and V_{11} are interchanged.

Define $f : V(H) \to \{1, 2, ..., 11\}$

Thus the vertex labels are

$f(v_{10}) = 11$ $\quad f(v_{11}) = 9$

$f(v_1) = 1,$ $\quad f(v_2) = 2$

$f(v_3) = 4$ $\quad f(v_4) = 8$

$f(v_5) = 3$ $\quad f(v_6) = 6$

$f(v_7) = 5$ $\quad f(v_8) = 10$

$f(v_9) = 7$

Now, the corresponding edge labels are

$f(v_1 v_{2i}) = 1 \quad 1 \le i \le 4$

$f(v_i, v_{11}) = 0, \quad i = 3,7$

$f(v_5, v_{11}) = 1$

$f(v_i, v_{10}) = 0, \quad i = 3,7,9$

$f(v_2, v_9) = 0$ $\quad f(v_2, v_3) = 1$

$f(v_3, v_4) = 1$ $\quad f(v_5, v_6) = 1$

$f(v_7, v_8) = 1$ $\quad f(v_4, v_5) = 0$

$f(v_6, v_7) = 0$ $\quad f(v_8, v_9) = 0$

Thus we obtain

$e_f(1) = 9$

$e_f(0) = 9$

Hence $|e_f(1) - e_f(0)| = 0$. It satisfies the condition. Hence H is a divisor cordial graph.

Illustration: Herschel graph is a divisor cordial graph.

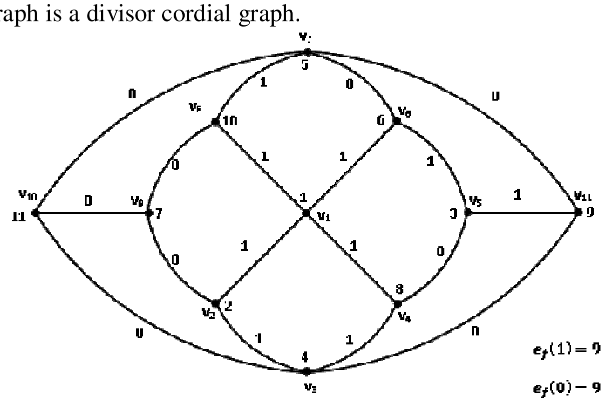

Figure 2:

Definition 1.3:
Let (v_1,a_i,b_i,v_2) be the i^{th} page of B_n, $1 \le i \le n$ with v_1,v_2 as the common vertices of B_n, B_n has 2(n+1) vertices and 3n+1 edges.

Construction

Consider the complete graph K_6 attached with the book graph B_n where the vertices v_1 and v_2 are common. Then the resulting graph is denoted by G*.

Theorem 1.3: The Constructed graph G* is a divisor cordial graph (if n is even only).

Proof: Let $\{ v_1,v_2, \dots ,v_n\}$ be the vertices of K_6 and $\{ u_1,u_2,a_i,b_i, / 1\le n\}$ be the vertices of B_n

Now , the vertex set of G* be $V(G^*) = \{v_i / 1 \le i \le n \text{ where } u_1 = v_1 \ \& \ u_2 = v_2\} \cup / \{a_i / 1 \le i \le n\} \cup \{b_i / 1 \le i \le n\}$

and the edge set be $E(G^*) = \{(v_1 v_i) / 2 \le i \le 6\} \cup \{(v_2 v_j) / 3 \le j \le 6\} \cup \{(v_3 v_j) / 4 \le j \le 6\} \cup$

$\{(V_4 V_j) / 5 \le j \le 6\} \cup \{V_5 V_6\} \cup \{(V_1 a_i) / 1 \le i \le n\} \cup \{(V_2 b_i) / 1 \le i \le n\} \cup \{(a_i b_i) / 1 \le i \le n\}$

Note that the Graph G* has 2n+6 vertices and 3n+15 edges.

Define f: V (G*) $\rightarrow \{1, 2,\dots, 2n+6\}$ as follows:

Case (i) If n is odd

The vertex labels are

$f(v_i) = i, \quad 1 \le i \le 6$

$f(a_i) = 6 + (2i - 1), \quad 1 \le i \le n$

$f(b_i) = 6 + 2i, \quad 1 \le i \le n$

Then the correspondent edge labels are

$f(v_1, v_j) = 1 \quad 2 \le j \le 6$

$f(v_2, v_j) = 1 \quad j = 4,6$

$f(v_2, v_j) = 0 \quad j = 3,5$

$f(v_3, v_j) = 0 \quad j = 4,5$

$f(v_3, v_6) = 1$

$f(v_4, v_j) = 0 \quad 5 \le j \le 6$

$f(v_5, v_6) = 0$

$f(v_1, a_i) = 1 \quad 1 \le i \le n$

$f(a_i, b_i) = 0 \quad 1 \le i \le n$

$f(v_2, b_i) = 1 \quad 1 \le i \le n$

Case (ii) If n is even

The vertex labels are

$f(a_i) = 4 + (2i + 2), \quad 1 \le i \le n$

$f(b_i) = 4 + (2i + 1), \quad 1 \le i \le n$

The corresponding edge labels are

$f(v_1, v_j) = 1 \quad 2 \le j \le 6$

$f(v_2, v_j) = 1 \quad j = 4,6$

$f(v_2, v_j) = 0 \quad j = 3,5$

$f(v_3, v_j) = 0 \quad j = 4,5$

$f(v_3, v_6) = 1$

$f(v_4, v_j) = 0 \quad 5 \le j \le 6$

$f(v_5, v_6) = 0$

$f(v_1, a_i) = 1 \quad 1 \le i \le n$

$f(a_i, b_i) = 0 \quad 1 \le i \le n$

$f(v_2, b_i) = 0 \quad 1 \le i \le n$

Now we obtain

$$e_f(1) = 8 + n + \left\lceil \frac{n}{2} \right\rceil \ \& \quad e_f(0) = 7 + n + \left\lfloor \frac{n}{2} \right\rfloor$$

Therefore, $\left|e_f(0)-e_f(1)\right|\le 1$. It satisfies the condition. Hence G* is a divisor cordial graph.

Illustration: K_6 attach with B_4

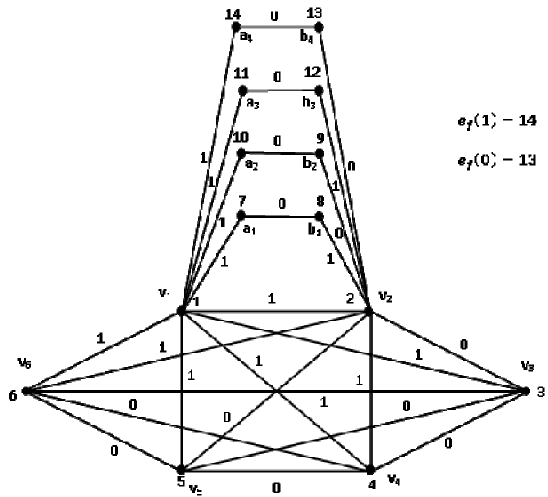

Figure 3:

4. SHELL GRAPH

Definition 1.4: A shell graph is defined as a cycle C_n with (n - 3) chords sharing a common end point called the apex.

Double Shell

Definition1.5: A double shell is one vertex union of two shells.

5. BOW GRAPH

Definition1.6: A Bow graph is defined to be a double shell in which each shell has any order.

Theorem 1.4: All Bow graph with shell orders m and 2m union P_m is a divisor cordial graph.

Proof: Let G be a bow graph with shells of order m and 2m excluding the apex. Let the number of vertices in G be n and the number of edges in G be q, the shell that is present to the left of the apex is called as the left wing and the shell that is present to the right of the apex is considered as the right wing. Let m be the order of the right wing of G and (2m) be the order of the right wing. G the apex of the bow graph is denoted as v_0. Denote the vertices in the right wing of the bow graph from the bottom to the top by v_1, v_2…. v_m ; the vertices in the left wing of the bow graph are denoted from top to bottom by $v_{m+1...}$, v_{3m} and w_1, w_2, … w_m be the vertices of path p_m.

$V(G)=\{v_i / 1\le i\le 3m\}\cup\{w_i / 1\le i\le m\}$

$E(G)=\{v_iv_{i+1} / 1\le i\le m-1\}\cup\{v_0v_i / 1\le i\le 3m\}\cup\{v_iv_{i+1} / m+1\le i\le 3m-1\}\cup\{w_iw_{i+1} / 1\le i\le m-1\}$

The graph G has $4m + 1$ vertices and $7m - 3$ edges.

Define f: $V(G) \to \{1,2,\ldots, 4m + 1\}$ as follows.

$f(v_0) = 1$

$f(v_i) = i + 1, \quad 1\le i\le 3m$

$f(w_i) = 3m + i \quad 1\le i\le m$

Clearly vertex labels are distinct.

The edge labels are

$f(v_0v_i) = 1, \quad 1\le i\le 3m$

$f(v_iv_{i+1}) = 0, \quad 1\le i\le m-1$

$f(v_i\, v_{i+1}) = 0, \quad m+1\le i\le 3m$

$f(w_iw_{i+1}) = 0, \quad 1 \le i \le m-1$

We obtain

$e_f(0) = 4m - 3$

$e_f(1) = 3m$

Hence $|e_f(1) - e_f(0)| \le 1$

Hence G is a divisor cordial graph.

Illustration: (A bow graph of m=4) $\cup P_4$ is divisor cordial graph.

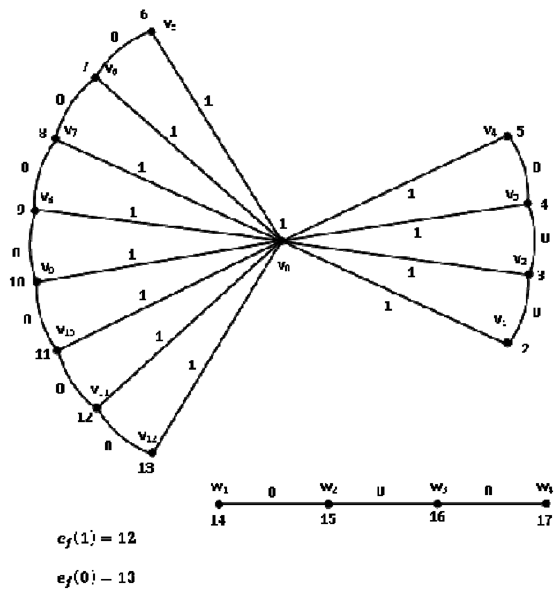

Figure 4:

Construction

Consider the wheel graph $W_n = K_1 + C_n$. Let the vertices of W_n be $\{v_0, v_1, \ldots, v_n\}$. Let $\{v_1', v_2', \ldots, v_n'\}$ be isolated vertices, where v_i' is adjacent to v_n and v_0. Now, the constructed graph W_n^* has $2n$+1 vertices and 4n edges.

Theorem 1.5: The constructed graph W_n^* is a divisor cordial graph, when n+1 is prime.

Proof: Let $\{v_0, v_1, v_2, \ldots, v_n\}$ be the vertices of w_n and $\{v_1', v_2', \ldots, v_n'\}$ be the isolated vertices where v_i' is adjacent to v_0 and v_n.

Note that the graph W_n^* has $2n$+1 vertices and $4n$ edges.

Now the vertex set be $V(W_n^*) = \{v_i / 0 \le i \le n\} \cup \{v_i' / 1 \le i \le n\}$ and the edge set be $E(W_n^*) = \{v_0 v_i / 1 \le i \le n\} \cup \{v_i v_{i+1} / 1 \le i \le n-1\} \cup \{v_n v_1\} \cup \{v_0 v_i' / 1 \le i \le n\} \cup \{v_n v_i' / 1 \le i \le n\}$

The vertex labels are

$f(v_0) = 1$

$f(v_i) = i + 1, 1 \le i \le n$ where $n + 1$ is prime; $f(v_i') = (n + 1) + i, 1 \le i \le n$

Clearly the vertex labels are distinct.

Now, the edge labels are

$f(v_0 v_i) = 1, \quad 1 \le i \le n$

$f(v_i v_{i+1}) = 0, \quad 1 \le i \le n - 1$

$f(v_0 v_i') = 1, \quad 1 \le i \le n$

$f(v_n v_i') = 0, \quad 1 \le i \le n$

Thus we obtain

$e_f(0) = 2n$

$e_f(1) = 2n$

Therefore, $|e_f(1) - e_f(0)| = 0$. It satisfies the condition.

Hence W_n*is a divisor cordial graph.

Illustration: W_{12}* graph is a divisor cordial graph .

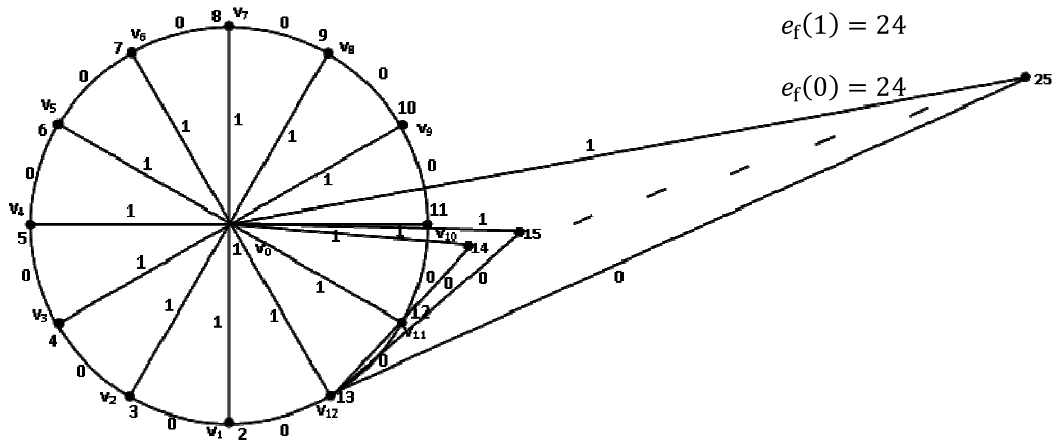

Figure 5:

Definition 1.7: Amal $\{(G_n, x_i)/i = 1, 2, \ldots, k\}$ is the amalgamation of k-copies of the fan graph f_n.

Theorem 1.6: The graph G= Amal $\{(G_n, x_i)/ i = 1, 2, \ldots, k\} \cup P_k$ is a divisor cordial graph.

Proof: Let Amal $\{(G_n, x_i)/ i = 1, 2, \ldots, k\}$ be the amalgamation of k-copies of the fan graph f_n. Let x_1 be the central vertex and v_{ij} be the vertices of the i^{th}fan where j=1,2,…n and i=1,2, …,k. Let $y_1, y_2, \ldots, y_k$ be the vertices of the path P_k. The graph G has k(n+1)+1 vertices and 2kn-1 edges.

Let the vertex set be

$V(G)=\{v_{ij} / 1 \le i \le k \text{ and } 1 \le j \le n\} \cup \{y_i / 1 \le i \le k\}$and the edge set be

$E(G) = \{(v_{ij} v_{i(j+1)})/ 1 \le j \le n-1 \text{ and } i = 1, 2, \ldots, k\}$

$\cup \{x_1 v_{ij} / 1 \le i \le k \text{ and } 1 \le j \le n\} \cup \{(y_i y_{i+1}) / 1 \le i \le k-1\}$

Define $f : V(G) \to \{1, 2, \ldots, k(n+1)+1\}$

$f(x_1) = 1$

$f(v_{1j}) = 1 + j, \quad 1 \le j \le n$

$f(v_{2j}) = (n+1) + j \quad 1 \le j \le n$

$f(v_{3j}) = 2n + 1 + j, \quad 1 \le j \le n$

$f(v_{kj}) = (k-1)n + 1 + j, \quad 1 \le j \le n$

$f(v_{ij}) = (i-1)n+1+j, \quad 1 \le i \le k, 1 \le j \le n$

$f(y_j) = (nk + 1 + i), \quad 1 \le i \le k$

$f(x_1 v_{ij}) = 1, \quad 1 \le i \le k \text{ and } 1 \le j \le n$

$f(v_{ij} v_{i(j+1)}) = 0, \quad 1 \le j \le n-1, \quad i = 1, 2 \ldots k$

$f(y_i y_{i+1}) = 0, \quad 1 \le i \le k-1$

Thus we obtain

$e_f(1) = kn, \quad e_f(0) = kn-1$

$|e_f(1) - e_f(0)| \le 1$. It satisfies the desired condition.

Hence G is a divisor cordial graph.

Illustration: (Amalgamation of three copies of the fan graph f_5) ∪ p_3 is a divisor cordial graph

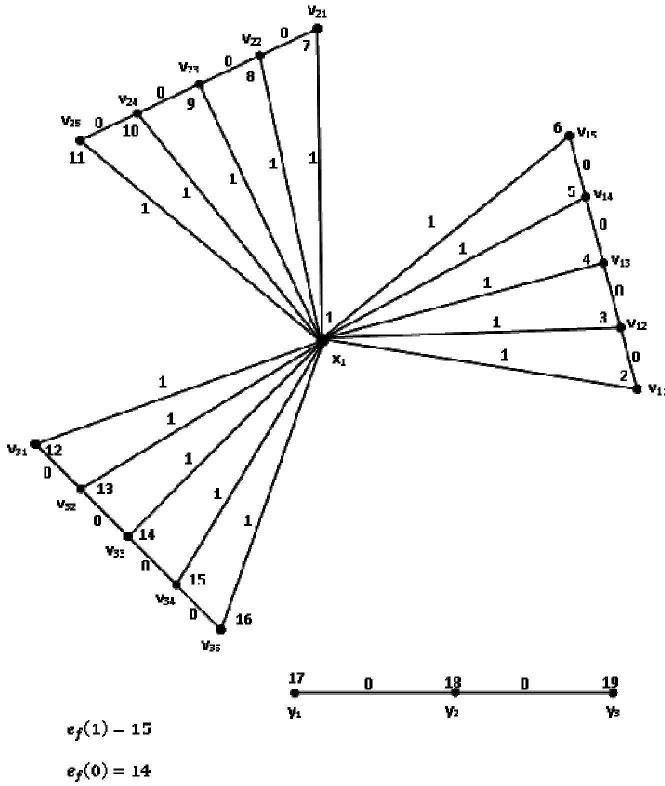

Figure 6:

Definition: 1.8 Let g, b positive integers such that $g \geq 3$, and P_g denote a path of order g with the vertex set $\{v_1, v_2, \ldots, v_g\}$ and the edge set $E(P_g) = \{v_i v_{i+1} \mid i = 1,2,\ldots, g-1\}$. Then the graph $1*P_g$ has the vertex set $V(1*P_g) = \{v\} \cup V(P_g)$ and let the edge set = $E(1*P_g) = E(P_g) \cup \{uv_i \mid i = 1,2,\ldots, g\}$. Finally ($C_{g+b}$) denote the cycle of order ($g+b$) with vertex set $V(C_{g+b}) = \{v_1, v_2, \ldots, v_g, v_1^1, v_2^1, \ldots, v_b^1\}$ and the edge set $E(C_{g+b}) = E(P_g) \cup \{(v_1 v_1^1), (v_g c_b^1)\} \cup \{v_i^1 v_{i+1}^1\}$ for i=1, 2, …, b-1. Then the resultant graph is defined as a parachute [2] denoted by $P_{g,b}$ and is given by $P_{g,b} = (V_{g,b}, E_{g.b})$ $g, b \in N, g \geq 3$ where $P_{g,b}$ is the amalgamation of $(1*P_g) \cup C_{g+b}$, obtained from the union of $1*P_g$ and C_{g+b} by pasting them along P_g such that the intersection $(1*P_g) \cup C_{g+b}$ is equal P_g.

Now $P_{g.b} = (V_{g.b}, E_{g.b})$, $g, b \in N$. Let $v, v_1, v_2, \ldots, v_g$, $v_1^1, v_2^1, \ldots, v_b^1$ be the set of vertices of the graph $P_{g,b}$. Note that $P_{g,b}$ has (g+b+1) vertices and (2g+b) edges.

Construction

Consider the Parachute graph $P_{g,b}$ with g + b + 1 vertices where g vertices are attached with a single vertex V_0. Let as attached b isolated vertices with vertex v_0. The resulting graph is denoted by G* clearly G* contains g + b + 1 + b = g + 2b + 1 vertices and g + g – 1 + b + 1 + b = 2g + 2b edges.

Theorem 1.7: The constructed graph G* is a divisor graph.

Proof:

The graph G* contains g + 2b + 1 vertices and 2g + 2b edges.

V(G*) = $\{v_i / 1 \le i \le g\} \cup \{v_i^1 / 1 \le i \le b\} \cup \{v_0\} \cup \{w_i / 1 \le i \le b\}$ where g = b + 2.

Define f: V(G*) $\rightarrow$ {1, 2, …, g + 2b + 1} as follows:

$f(v_0) = 1$

$f(v_i) = i + 1$ for $1 \le i \le g$

$f(v_i^1) = (g + b + 1) - (i - 1)$ $1 \le i \le b$

$f(w_i) = g + b + (i + 1)$ for $1 \le i \le b$

Clearly vertex labels are distinct.

The edge labels are

$f(v_0v_i) = 1$, $1 \le i \le g$

$f(v_iv_{i+1}) = 0$, since they are consecutive integers $1 \le i \le g-1$

$f(v_i^1v_{i+1}^1) = 0$, $1 \le i \le b-1$

Since v_i^1 and v_{i+1}^1 are consecutive integers $f(v_1 v_1^1) = 0$ & $f(v_nv_n^1) = 0$

$f(v_0 w_i) = 1$ $1 \le i \le b$

Edges labeled with 1 are v_0v_1, v_0v_2,…, v_0v_g, v_0w_1, v_0w_2, …, v_0w_b,

Therefore, $e_f(1) = g + b$

Edge labeled with 0 are

$f(v_iv_{i+1})$ for $1 \le i \le g-1$

$f(v_i^1v_{i+1}^1)$ for $1 \le i \le b-1$ and

$f(v_1v_1^1)$, $f(v_nv_n^1)$

Therefore,

$e_f(0) = g - 1 + b - 1 + 2$

$e_f(0) = g + b$

$|e_f(1) - e_f(0)| = 0.$

Hence G* is a divisor cordial graph.

Illustration: The parachute graph $P_{6,4}$ with 11 vertices where 6 vertices are attached with a single vertex V_0. Let as attached 4 isolated vertices with vertex v_0.

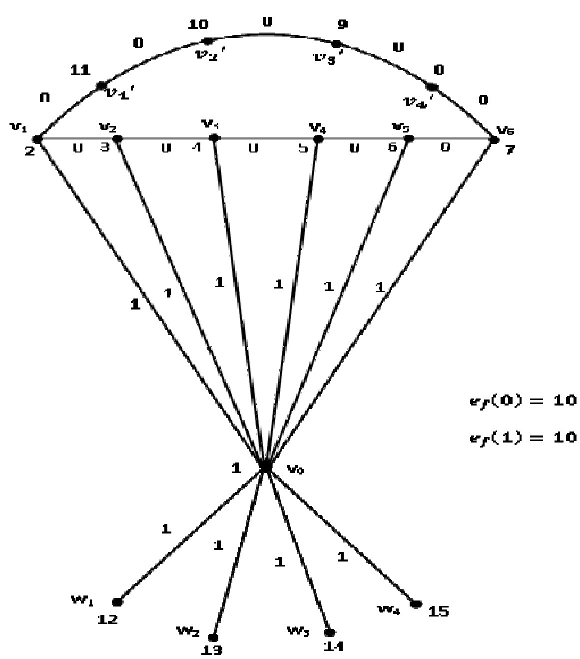

Figure 7:

Illustration Explanation:
The parachute graph $P_{6,4}$ with 11 vertices where 6 vertices are attached with a single vertex V_0. Let as attached 4 isolated vertices with the vertex v_0.

G* contains g + 2b + 1 = 6 + 8 + 1 = 15 vertex and 2g + 2b = 20 edges

$f(v_0v_1) = 1$, $f(v_0, v_2) = 1$ …
$f(v_0v_6) = 1$ and $f(v_0w_1) = 1$ …
$f(v_0v_4) = 1$
Now, $e_f(1) = 6 + 4 = 10$
$f(v_1v_2) = 0$ $f(v_2v_3) = 0$
$f(v_5v_6) = 0$
$f(v_1^1v_2^1) = 0$ $f(v_3^1v_4^1) = 0$
$f(v_1v_1^1) = 0$
$f(v_6v_4^1) = 0$
Now, $e_f(0) = 10$
Therefore $|e_f(1) - e_f(0)| = 0$. Hence G* is a divisor cordial graph.

Theorem 1.8: P_n+2k_1 is a divisor cordial graph.

Proof: Let the vertex set of G be V(G) = $\{x_1, x_2,\dots, x_m, y_1, y_2\}$ and the edge set be
E(G) = $\{(x_ix_{i+1})/1 \le i \le m-1\} \cup \{(y_1x_i)/1 \le i \le m\} \cup \{(y_2x_i)/1 \le i \le m\}$.
Therefore, $|V(G)| =$ m + 2 and $|E(G)| = 3m-1$.
Define f : V(G) $\to$ {1, 2, …, m +2} as below:
$f(y_1) = 1$
$f(y_2) = 2$
$f(x_i) = 2 + i$ for $1 \le i \le m$
Clearly the vertex labels are distinct. Also, here
$f(x_i\, x_{i+1}) = 0$, $1 \le i \le m-1$, since x_i and x_{i+1} are consecutive
$f(y_1x_i) = 1$ for i=1,2,…m
$f(y_2x_i) = 1$ if i is even
$f(y_2x_i) = 0$ if i is odd
The Edges labeled with 1 are $y_1x_1, y_1x_2,\dots y_1x_m, y_2x_2, y_2x_4,\dots,y_2x_m$ if m is even.

$e_f(1) = m + 2$ if m is odd, and
$e_f(1) = m + 3$ if m is even.

Edges labeled with 0 are $x_1x_2, x_2x_3\dots x_{m-1}x_m$ and $y_2x_1, y_2x_3\dots y_2x_m$, if m is odd.

$e_f(0) = m + 2$

Hence $|e_f(0) - e_f(1)| \le 1$. Hence G is a divisor cordial graph.

Illustration: $P_5 + 2K_1$ is a divisor cordial graph.

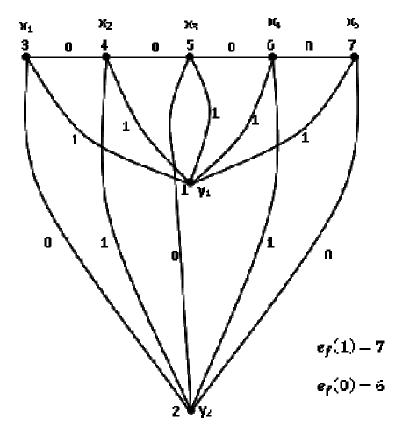

Figure 8:

Definition 1.9: Let G be a graph with n vertices and e edges. A graph H is called a super subdivision of G if H is obtained from G by replacing every edge e_i of G by a complete bipartite graph K_{2,m_i} for some m_i, $1 \le i \le q$ in such a way that the ends of each e_i are merged with the two vertices of 2-vertices part of K_{2,m_i} after removing the edge e_i from the graph G [see, 1].

Definition 1.10: A super subdivision H of G is said to be an arbitrary super subdivision of G if every edge of G is replaced by an arbitrary $K_{2,m}$ where m may vary for each edge arbitrarily.

Theorem 1.9: An arbitrary super subdivision of $K_{1,n}$ is a divisor cordial graph.

Proof: Let $v_1, v_2, \ldots, v_n$ be the pendant vertices of $K_{1,n}$, v_o be its apex vertex and $e_i = v_o v_i$ for $1 \le i \le n$. Let G be the graph obtained by arbitrary super subdivision of $K_{1,n}$, in which each edge e_i of $K_{1,n}$, is replaced by a complete bipartite graph K_{2,m_i} and u_{ij} be the vertices of m_i vertices part where $1 \le i \le n, 1 \le j \le m_i$. The graph has n+m+1 vertices and 2m edges where $m = \sum_{i=1}^{n} m_i$.

We define f: $V(G) \to \{1, 2, \ldots, n+m+1\}$ as follows:

$f(v_o) = 1$

$f(v_i)$ = the highest 'n' prime numbers $\le n+m+1$.

$f(u_{1j}) = 1+j$ for $1 \le j \le m_1$.

$f(u_{2j}) = 1+m_1+j$ for $1 \le j \le m_2$.

$f(u_{3j}) = 1+m_1+m_2+j$ for $1 \le j \le m_3$.

$f(u_{nj}) = n+1+m_1+\ldots+m_{n-1}+j$ for $1 \le j \le m_n$, (except the highest 'n' prime numbers).

$f(v_o u_{ij}) = 1$ for $1 \le i \le n,\ 1 \le j \le m_i$.

$f(v_i u_{ij}) = 0$ for $1 \le i \le n,\ 1 \le j \le m_i$.

Thus the edge labels are

$e_f(1) = m$

$e_f(0) = m$

Therefore, $|e_f(1) - e_f(0)| = 0$. Hence the graph G is a divisor Cordial Graph.

Illustration: An arbitrary Super Subdivision of $K_{1,4}$ and its divisor cordial labeling is shown where $m_1=2, m_2=4, m_3=3$ and $m_4=5$.

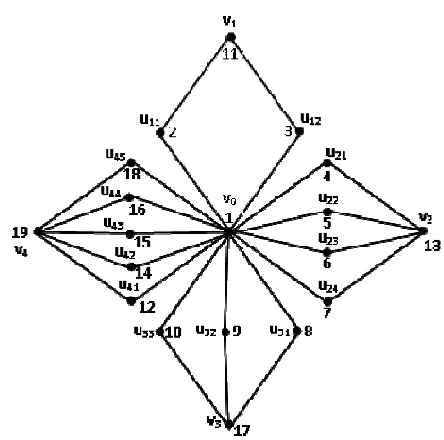

Figure 9: Cordial labeling for arbitrary super subdivision of $K_{1,4}$

6. HELM GRAPH

Definition: 1.11: The helm graph H_n is obtained from the wheel graph W_n by attaching a pendent edge at each vertex of the n - cycle of the wheel.

Theorem: 1.10: The union of Helm graph and star graph $(H_n \cup K_{1,n})$ is a divisor cordial graph if n<7.

Proof: Let $c, v_1, v_2, \ldots, v_n, v_1^1, v_2^1, \ldots, v_n^1$ be the vertices of H_n. Let $c_1^{\ 1}, w_1, w_2, \ldots, w_n$ be the vertices of $K_{1,n}$. $V(H_n) = \{c, v_1, v_2, \ldots, v_n, v_1^1, v_2^1, \ldots, v_n^1\}$ and

$E(H_n)=\{cv_i /1 \le i \le n\} \cup \{v_i v_i^1 /1 \le i \le n\} \cup \{(v_i v_{i+1})/1 \le i \le n-1\}$, $V(K_{1,n}) = \{c_1^{\ 1}, w_1, w_2, \ldots, w_n\}$

$E(K_{1,n}) = \{c_1^{\ 1} w_i /,1 \le i \le n\}, V(H_n U K_{1,n}) = V(H_n) U V(K_{1,n})$,

$E(H_n U K_{1,n}) = E(H_n) U E(K_{1,n})$

$$|V(H_n U K_{1,n})| = 3n+2 \ \& \ |E(H_n U K_{1,n})| = 4n$$

Define $f : V(G) \to \{1, 2, \ldots 3n+2\}$ as follows:

$f(c) = 2$

$f(c_1^{\ 1}) = 1$

$f(v_i) = 2i \qquad 2 \le i \le n+1$

$f(v_i^1) = 2i+1 \qquad 1 \le i \le n$

$f(w_i) = 2(n+1)+i \qquad 1 \le i \le n$

The edge labels are

$f(v_1 v_n) \equiv 0 (\mathrm{mod}\ 4)$

$f(cv_i) = 1 \qquad 1 \le i \le n$

$f(v_i v_i^1) = 0 \qquad 1 \le i \le n$

$f(c^1 w_i) = 1 \qquad 1 \le i \le n$

$f(v_i v_{i+1}) = 0 \qquad 1 \le i \le n-1$

$e_f(1) = 2n$

$e_f(0) = 2n$

$|e_f(1) - e_f(0)| = 0$

Hence the union of Helm graph and star graph is a divisor cordial graph.

Illustration: $H_6 \cup K_{1,6}$ is a divisor cordial graph.

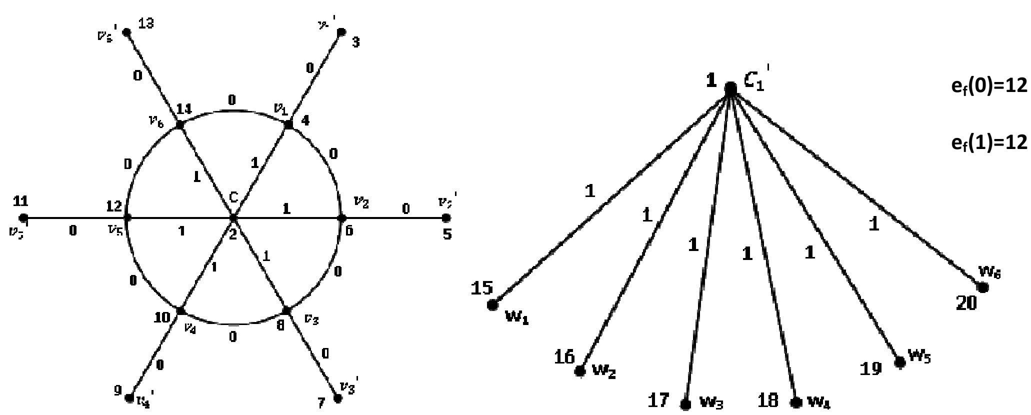

Figure 10:

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