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DETERMINATION OF OPTIMAL RESERVE BETWEEN TWO MACHINES IN SERIES WITH THE REPAIR TIME HAS CHANGE OF PARAMETER AFTER THE TRUNCATION POINT

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Abstract

In inventory control, suitable models for various real life systems are constructed with the objective of determining the optimal inventory level. A new type of inventory model using the so-called Setting the Clock Back of Zero (SCBZ) property is analyzed in this paper. There are two machines M_1 and M_2 in series and the output of M_1 is the input of M_2. Hence a reserve inventory between M_1 and M_2 is maintained. The method of obtaining the optimal size of reserve inventory $\hat{S}$, assuming cost of excess inventory, cost of shortage and when the rate of consumption of M_2 is a constant, has already been attempted. In this paper, it is assumed that the repair time of M_1 is a random variable and the distribution of the same undergoes a parametric change after a truncation point X_0, which is taken to be a random variable. The optimal size of the reserve inventory is obtained under the above said assumption . Numerical illustration is also provided.

Keywords: Reserve inventory, Truncation point, SCBZ property.

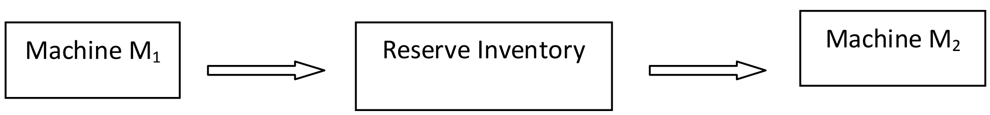
1. INTRODUCTION

A system which has two machines M_1 and M_2 are in series is considered. The output of M_1 is the input of M_2. The breakdown of M_1 results the idle time of M_2, since there is no input to the Machine M_2, whenever the Machine M_1 breaks down it leads to the showdown of M_2 and this state continues till the Machine M_1 gets repaired. The idle time of M_2 is very costly and hence, to avoid the idle time of M_2, a reserve inventory is maintained in between M_1 and M_2.When a huge inventory is kept as a reserve then it takes more carrying cost and when there is less inventory kept as reserve, then it recurs idle time cost of M_2.Since the duration of repair time of M_1 is high then the reserve inventory will be exhausted by M_2. In order to balance these costs the optimal inventory must be maintained. The repair time of M_1 is a random variable and after the repair of M_1 is over, it supplies to the reserve inventory. During the repair time of M_1, the Machine M_2 gets the input from the reserve.

The very basic model of determining optimal reserve inventory between two machines in series is discussed by Hanssman [1]. Ramachandran et al. [2] have discussed the model for three Machines . S. Sachithanantham [3] et al. have discussed the model in which the optimal reserve inventory between the two Machines has been obtained under the assumption that the repair time of Machine M_1 is a random variable with Exponential probability distribution which satisfies the SCBZ Property . The SCBZ property is discussed by Raja Rao and Talwaker [4]. S. Sachithanantham et al. [5] discussed the optimal reserved inventory model between two Machines under the assumption that the probability distribution of repair time of M_1 is exponential, which satisfies the SCBZ property and the truncation point X_0 is taken assumed to be a random variable. In that model the authors assumed that the probability function of the truncation point was exponential.

Ramerthilagam et al. [6] have discussed the same model with the assumption that the truncation point is a random variable and the probability function of the truncation point was a uniform distribution. Venkatesan et al. [7] have discussed the optimal reserved inventory between three Machines. Hentry et al. [8] have discussed model of determining the optimal reserve inventory between two machines with the assumption, that the truncation point of the repair time of Machine M_1 is followed the goal generalized distribution. Sachithanantham et al. [9] have discussed the same model with reference to the truncation point on the repair time, in which the truncation point is assumed to be a random variable and it follows mixed exponential distribution with the assumption, that the probability function of repair time of Machine M_1 follows exponential distribution which satisfies SCBZ Properties.

The following diagram explains the system.



2. NOTATIONS

h : Cost per unit time of holding one unit of reserve inventory
d : Cost per unit time of idle time of machine M_2
μ : Mean time interval between successive breakdowns of machine M_1 , assuming exponential distributions of inter-arrival times.
t : Continuous random variable denoting the repair time of M_1 with probability density function g(.) and CDF G (.).
r : Content consumption rate per unit time of machine M_2
S : Reserve inventory between M_1 and M_2
$\hat{S}$: Optimum reserve inventory
T : Random variable denoting the idle time of M_2

3. RESULTS

Model I:

If T is a random variable denoting idle time of M_2 then it is given by

$$\begin{cases} 0 & if \quad t \le \frac{S}{r} \\ t-\frac{S}{r} & if \quad t > \frac{S}{r} \end{cases}$$

Hence the expected total cost of inventory holding and the idle time of M_2 per unit of time is given by

$$E(C) = hS + \frac{d}{\mu} E(T)$$

$$\Rightarrow E(C) = hS + \frac{d}{\mu} \int_{\frac{S}{r}}^{\infty} \left(t - \frac{S}{r}\right) g(t)dt$$

The optimal reserve size $\hat{S}$can be obtained by solving the equation $\frac{dE(c)}{dS} = 0$.

The expression for optimal reserve inventory is given by $G\left[\frac{s}{r}\right] = 1 - \left[\frac{r\mu h}{d}\right]$.

This is the basic model discussed in [1].

Model II:

In this Model, it is assumed that the repair time of machine M_1 is a random variable and undergoes a parametric change. That is the probability density function of the repair time follows the gamma distribution and it takes Parametric change after the truncation point X_0

ie.,
$$\begin{cases} g(\theta;\beta,t) & if \quad t \le x_0 \\ g(\theta^*;\beta,t) & if \quad t > x_0 \end{cases}$$

$$\begin{cases} g(\theta;\beta,t) = \frac{\theta^\beta}{\Gamma(\beta)} e^{-\theta t} t^{\beta-1} & if \quad t \le x_0 \\ g(\theta^*;\beta,t) = \frac{\theta^\beta}{\Gamma(\beta)} \theta^* e^{-\theta^* t} e^{x_0(\theta^*-\theta)} & if \quad t > x_0 \end{cases}$$

Thus, it can be shown that the distribution of repair time satisfies the so called SCBZ property as discussed in [8].

If x_0 is a random variable denoting that truncation point and it is distributed as an exponential distribution with parameter λ, then the probability density function of the repair time can be written as

$$f(t) = g(\theta,\beta;t)P[t \le x_0] + g(\theta^*,\beta;t)P[t > x_0]$$

$$f(t) = g(\theta,\beta;t)e^{-\lambda t}dt + \lambda\int_0^t g(\theta^*,\beta;t)\,\lambda e^{-\lambda x_0}dx_0$$

It may be observed that the random variable 'T' defined in equation (1) also undergoes a parametric change and the average idle time of M_2 is

$$E(T) = \int_{\frac{S}{r}}^{\infty}\left(t-\frac{S}{r}\right)f(t)dt$$

$$= \int_{\frac{S}{r}}^{\infty}\left(t-\frac{S}{r}\right)g(\theta;\beta,t)e^{-\lambda t}dt + \int_{\frac{S}{r}}^{\infty}\left(t-\frac{S}{r}\right)\left[\int_0^t g(\theta^*;\beta,t)\lambda e^{-\lambda x_0}dx_0\right]dt$$

Thus the expected total cost

$$E(C) = hS + \frac{d}{\mu}\left[\int_{\frac{S}{r}}^{\infty}\left(t-\frac{S}{r}\right)g(\theta;\beta,t)e^{-\lambda t}dt + \int_{\frac{S}{r}}^{\infty}\left(t-\frac{S}{r}\right)\left[\int_0^t g(\theta^*;\beta,t)\lambda e^{-\lambda x_0}dx_0\right]dt\right]$$

$$= hS + \frac{d}{\mu}\left[\int_{\frac{S}{r}}^{\infty}\left(t-\frac{S}{r}\right)\frac{\theta^\beta}{\Gamma(\beta)}e^{-\theta t}e^{-\lambda t}\,dt + \int_{\frac{S}{r}}^{\infty}\left(t-\frac{S}{r}\right)\left[\int_0^t e^{x_0(\theta^*-\theta)}\theta^* e^{-\theta^* t}\lambda e^{-\lambda x_0}dx_0\right]dt\right]$$

$$= hS + \frac{d}{\mu}\left[\frac{\theta^\beta}{\Gamma(\beta)}\int_{\frac{S}{r}}^{\infty}\left(t-\frac{S}{r}\right)e^{-(\lambda+\theta)t}t^{\beta-1}dt + \int_{\frac{S}{r}}^{\infty}\left(t-\frac{S}{r}\right)\left[\int_0^t (e^{x_0(\theta^*-\theta)}\theta^* e^{-\theta^* t}\lambda e^{-\lambda x_0}dx_0)\right]dt\right]$$

$$= hS + \frac{d}{\mu}\left[\frac{\theta^\beta}{\Gamma(\beta)}\int_{\frac{S}{r}}^{\infty}\left(t-\frac{S}{r}\right)e^{-(\lambda+\theta)t}t^{\beta-1}dt + \frac{\lambda\theta^*}{(\lambda+\theta-\theta^*)}\int_{\frac{S}{r}}^{\infty}\left(t-\frac{S}{r}\right)e^{-\theta^* t}\left[1-e^{-i(\lambda+\theta-\theta^*)}\right]dt\right]$$

$$= hS + \frac{d}{\mu}\left[\frac{\theta^\beta}{\Gamma(\beta)}I_1 + \frac{\lambda\theta^*}{(\lambda+\theta-\theta^*)}I_2\right]$$

Where $I_1 = \int_{\frac{S}{r}}^{\infty}\left(t-\frac{S}{r}\right)e^{-(\lambda+\theta)t}t^{\beta-1}dt$

$$I_2 = \int_{\frac{S}{r}}^{\infty}\left(t-\frac{S}{r}\right)e^{-\theta^* t}\left[1-e^{-t(\lambda+\theta-\theta^*)}\right]dt$$

$$\frac{d}{dS}E(C) = 0$$

$$\frac{d}{dS}E(C) = 0 \Rightarrow h + \left(\frac{d\theta^\beta}{\mu\Gamma(\beta)}\right)\left(\frac{d}{dS}I_1\right) + \left(\frac{\lambda}{\mu}\frac{\lambda\theta^*}{(\lambda+\theta-\theta^*)}\right)\frac{d}{dS}I_2 = 0$$

It can seen that

$$\frac{d}{dS}I_1 = \frac{d}{dS}\int_{\frac{S}{r}}^{\infty}\left(t-\frac{S}{r}\right)e^{-(\lambda+\theta)t}t^{\beta-1}dt$$

$$= \left(-\frac{1}{r}\right)\int_{\frac{S}{r}}^{\infty} e^{-(\lambda+\theta)t}t^{\beta-1}dt$$

$$= \left(-\frac{1}{r}\right)[\int_{0}^{\infty} e^{-(\lambda+\theta)t}t^{\beta-1}dt - \int_{0}^{\frac{S}{r}} e^{-(\lambda+\theta)t}t^{\beta-1}dt]$$

$$= \left(-\frac{1}{r}\right)\left[\frac{1}{(\lambda+\theta)^{\beta}}\int_{0}^{\infty} e^{-(\lambda+\theta)t}(\lambda+\theta)^{\beta}\, t^{\beta-1}dt - \int_{0}^{\frac{S}{r}} e^{-(\lambda+\theta)t}\, t^{\beta-1}dt\right]$$

$$= \left(-\frac{1}{r}\right)\left[\frac{1}{(\lambda+\theta)^{\beta}}\Gamma(\beta) - \int_{0}^{\frac{S}{r}} e^{-(\lambda+\theta)t}t^{\beta-1}dt\right]$$

$$= \left(-\frac{1}{r}\right)\left[\frac{1}{(\lambda+\theta)^{\beta}}\Gamma(\beta) - \sum_{j=1}^{\beta}\frac{\prod_{i=1}^{j-1}(\beta-i)}{(\lambda+\theta)^{j}}e^{-(\lambda+\theta)\frac{S}{r}}\left(\frac{S}{r}\right)^{\beta-j}\right]$$

$$\frac{d}{dS}I_1 = \left(-\frac{1}{r}\right)\left[\frac{\Gamma(\beta)}{(\lambda+\theta)^{\beta}} + \sum_{j=1}^{\beta}\frac{\prod_{i=1}^{j-1}(\beta-i)}{(\lambda+\theta)^{j}}e^{-(\lambda+\theta)\frac{S}{r}}\left(\frac{S}{r}\right)^{\beta-j}\right]$$

I_2 is considered as it is

$$\frac{d}{dS}I_2 = \frac{d}{dS}\int_{\frac{S}{r}}^{\infty}\left(t-\frac{S}{r}\right)e^{-\theta^*}\left[1-e^{-t(\lambda+\theta-\theta^*)}\right]dt$$

$$= 0 - \frac{1}{r}f\left(\frac{S}{r},S\right) + \int_{\frac{S}{r}}^{\infty}\left(-\frac{1}{r}\right)\left[e^{-\theta^* t} - e^{-t(\lambda+\theta)}\right]dt$$

$$= -\frac{1}{r}\left\{\left[\frac{e^{-\theta^* t}}{-\theta^*}\right] - \left[\frac{e^{-t(\lambda+\theta)}}{-(\lambda+\theta)}\right]_{\frac{S}{r}}^{\infty}\right\}$$

$$= -\frac{1}{r}\left\{\left[\frac{e^{-\theta^*\frac{S}{r}}}{\theta^*}\right] + \left[0 - \frac{e^{-\left(\frac{S}{r}\right)(\lambda+\theta)}}{(\lambda+\theta)}\right]\right\}$$

$$= -\frac{1}{r}\left\{\frac{e^{\theta^*\frac{S}{r}}}{\theta^*} - \frac{e^{-\frac{S}{r}(\lambda+\theta)}}{(\lambda+\theta)}\right\}$$

$$\frac{d}{ds}E(C) = 0 \Rightarrow h - \left(\frac{d\theta^{\beta}}{\mu\Gamma(\beta)}\right)\left[\frac{\Gamma(\beta)}{(\lambda+\theta)^{\beta}} + \sum_{j=1}^{\beta}\frac{\prod_{i=1}^{j-1}(\beta-i)}{(\lambda+\theta)^{j}}e^{-(\lambda+\theta)\frac{S}{r}}\left(\frac{S}{r}\right)^{\beta-j}\right]$$
$$-\left(\frac{d\lambda\theta^*}{\mu r(\lambda+\theta-\theta^*)}\right)\left\{\frac{e^{-\theta^*\frac{S}{r}}}{\theta^*} - \frac{e^{-\left(\frac{S}{r}\right)(\lambda+\theta)}}{(\lambda+\theta)}\right\} = 0$$

$$\frac{h\mu r(\lambda+\theta-\theta^*)}{d} = \frac{\theta^{\beta}(\lambda+\theta-\theta^*)}{\Gamma(\beta)}\left[\frac{\Gamma(\beta)}{(\lambda+\theta)^{\beta}} + \sum_{j=1}^{\beta}\frac{\prod_{i=1}^{j-1}(\beta-i)}{(\lambda+\theta)^{j}}e^{-(\lambda+\theta)\frac{S}{r}}\left(\frac{S}{r}\right)^{\beta-j}\right]$$
$$-\lambda\theta^*\left\{\frac{e^{-\theta^*\frac{S}{r}}}{\theta^*} - \frac{e^{-\left(\frac{S}{r}\right)(\lambda+\theta)}}{(\lambda+\theta)}\right\}$$

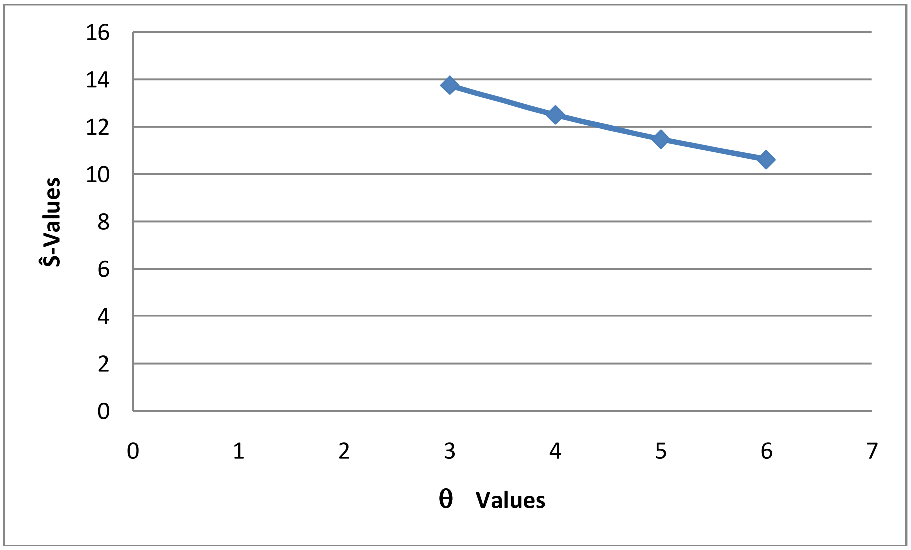
The value of S which satisfies the equation (1.4) for fixed values of $h, \mu, r, d, \lambda, \theta, \theta^*$ and $\beta = 2$ or 3 optimal value of reserve inventory $\hat{S}$ can be obtained numerically. Select one parameter as variable while the others as fixed.

4. NUMERICAL ILLUSTRATION

If $\beta = 1$, the results reduce to the results of [6]. The variations in the values of $\hat{\dot{S}}$ consequent to the changes in $h, \mu, r, d, \lambda, \theta, \theta^*$ has been studied by taking the numerical illustrations. The tables and the corresponding curves are given.

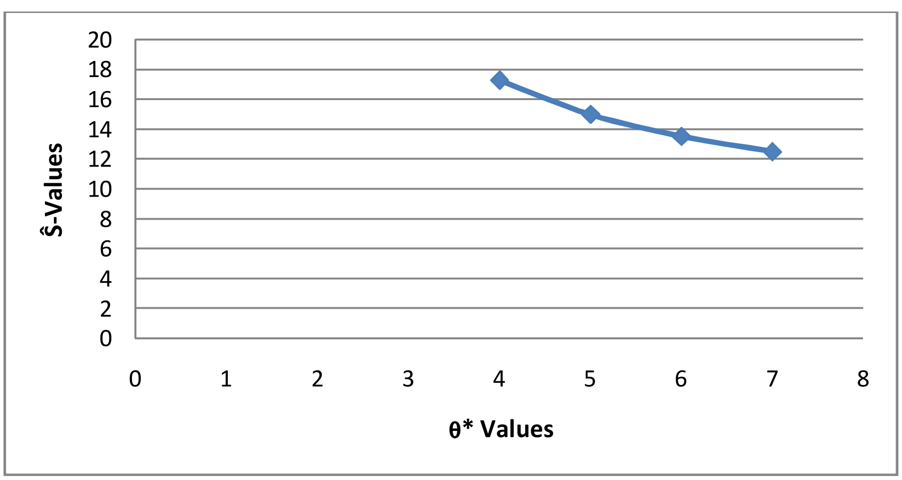
Case (i): The values of the constants are fixed arbitrarily, $h = 5, \mu = 2, r = 30, \theta^* = 7, \lambda = 5$ and $d = 3000$ and the optimal inventory $\hat{\dot{S}}$ for various values of θ.

θ	3	4	5	6
Ŝ	13.74366	12.5004	11.4756	10.6111



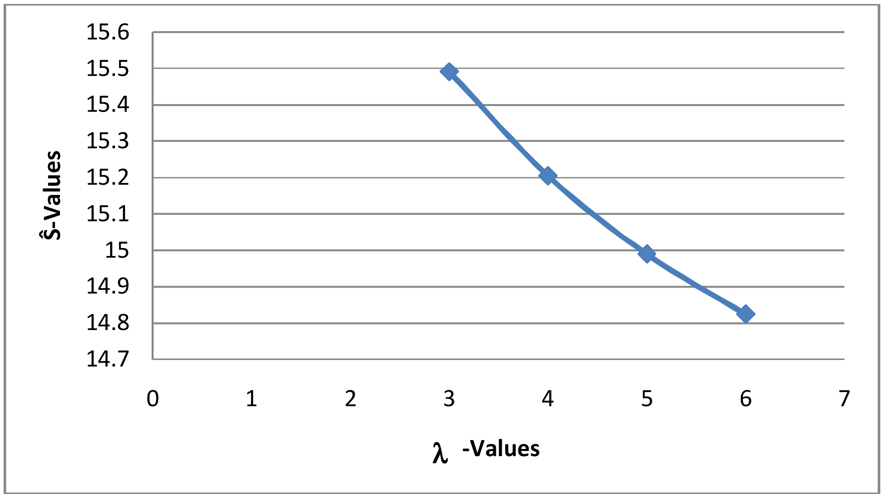
Case (ii): The values of the constants are fixed arbitrarily, h = 5, μ = 2, r = 30, λ= 4, λ = 5 and d = 3000 and the optimal reserve inventory Ŝ for various values of θ *

θ *	4	5	6	7
Ŝ	17.2694	14.9895	13.5209	12.5004



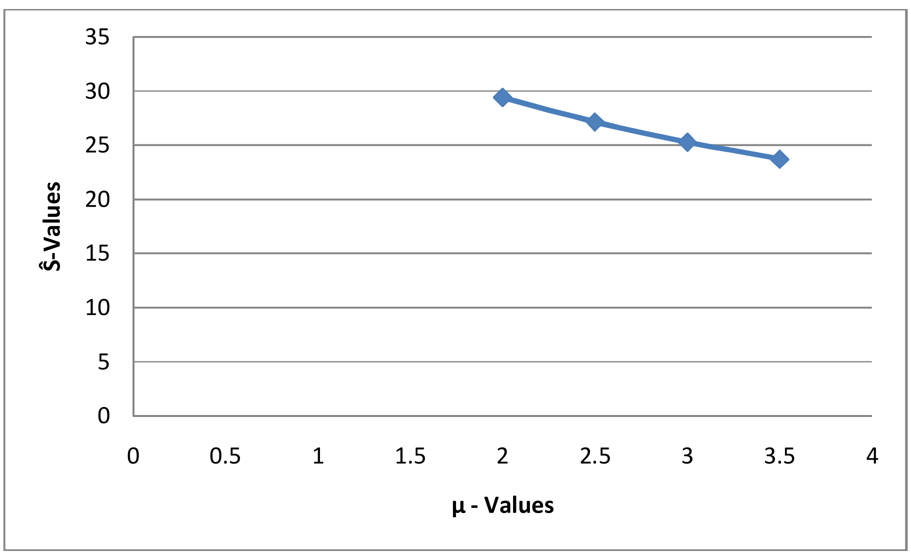
Case (iii): The values of the constants are again fixed arbitrarily as h = 5, μ = 2, r = 30, θ= 4, θ *= 5, d = 3000 and the optimal reserve inventory Ŝ for various values of λ

λ	3	4	5	6
Ŝ	15.4903	15.2044	14.9895	14.8243



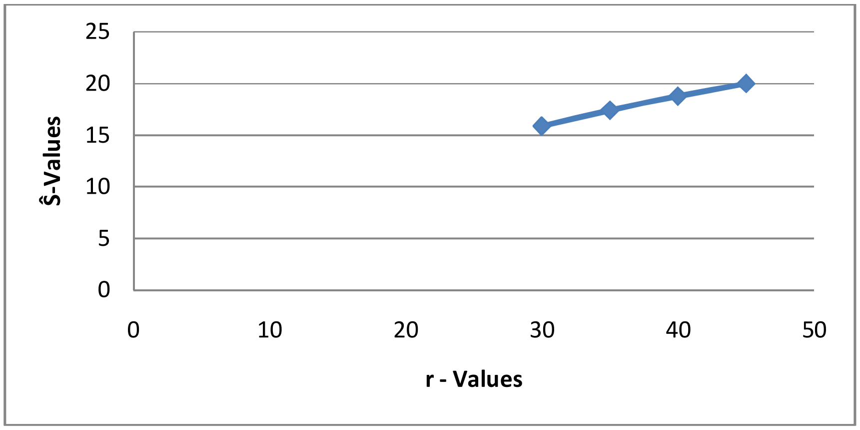
Case (iv): The values of the constants are fixed arbitrarily, h = 4, r = 30, θ= 1.5, θ *= 3, λ = 5, d = 3200 and the optimal reserve inventory Ŝ for various values of μ

μ	2	2.5	3	3.5
Ŝ	29.3715	27.1103	25.2560	23.6821



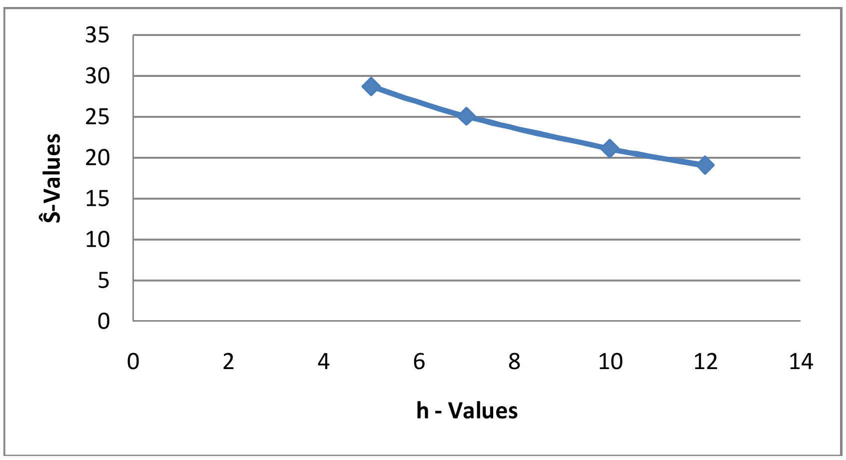
Case (v): The values of the constants are fixed arbitrarily, h = 4, μ = 2, θ = 4, θ *= 5, λ = 3, d = 3200 and the optimal reserve inventory Ŝ for various values of r

r	30	35	40	45
Ŝ	15.8992	17.4082	18.7609	19.9761



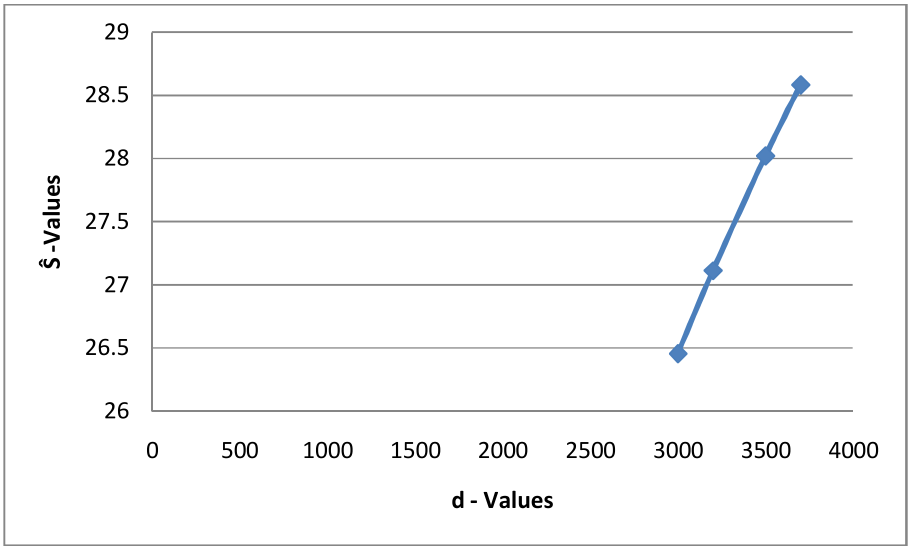
Case (vi): The values of the constants are fixed arbitrarily, μ = 2, r = 30, θ = 1.5, θ *= 3, λ = 3, d = 3000 and the optimal reserve inventory Ŝ for various values of r

h	5	7	10	12
Ŝ	28.6889	25.0507	21.1146	19.0610



Case (vii): The values of the constants are fixed arbitrarily, h = 5, μ = 2, r = 30, θ = 1.5, θ *= 3, λ = 5, and the optimal reserve inventory Ŝ for various values of d.

d	3000	3200	3500	3700
Ŝ	26.4547	27.1103	28.0193	28.5823



6. CONCLUSIONS

From the figures and curves it can be seen that as the value of carrying cost 'h' increases, Ŝ decreases and suggests the smallest inventory. If the idle time cost'd' increases, Ŝ increases which is quite justifiable.

If the rate of consumption of M_2 increases, the Ŝ also increases and it suggests a larger inventory. As the value of μ, the parameter of the distribution of the inter arrival times between successive breakdowns of M_1 increases, then the average number of breakdowns per unit time decreases. Hence there is a decrease in the value of Ŝ and it is quit plausible.

As the value of θ increases, the parameter of the repair time distribution of M_1 increases then the average time to repair the machine M_1 decreases. Thus the repair time of machine M_1 is shorter and hence the optimal reserve inventory Ŝ decreases. A similar behavior in Ŝ is experienced when θ* increases.

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