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HIGH ACCURACY ANALYSIS FOR ACCELERATION MOTION OF VERTICALLY FALLING SPHERICAL PARTICLES IN INCOMPRESSIBLE NEWTONIAN FLUID BY VARINATIONAL ITERATIONS METHOD

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Abstract

In this paper, the acceleration motion of a vertically falling spherical particles made of Glass, Iron, Copper and Silver with diameter (D=1mm) in Newtonian fluid is discussed using Varinational Iteration Method (VIM) & Diagonal Pade' approximant and compare the results with Runge-Kutta 4^{th} order method to verify the accuracy. It observed that the VIM which was used to solve such nonlinear differential equations is more accurate and simpler as compared to Diagonal pade', Collocation Method (CM), Homotopy Perturbation Method (HPM), Akbari- Ganji's Method (AGM)etc. The Conclusions clearly reveals that the time of reaching the particle at terminal velocity in a vertically falling procedure is significantly increased with increasing the density of a particles and the acceleration period for lighter particles is shorter than other. Further from these four particles, the glass's particle has low velocity and reaches early at terminal velocity due to its lowest density as compared to other. To obtain the results for all different methods, the symbolic calculus software MATLAB was used.

Keywords: Acceleration motion, Newtonian fluid, Spherical particle, Terminal velocity, Varinational Iteration Method

1. INTRODUCTION

Description of the motion of immersed bodies in Newtonian and non-Newtonian fluids has been a subject of great interest due to its wide range of applications in industry. Clift et al & Chhabra [1-2] gave the settling mechanism of solid particles, bubbles or drop, both in Newtonian and non-Newtonian fluid. Jalal et al. [7] used HAM and obtained the solution of the one-dimensional non-linear particles equation. Kaur and Garg [8-9] investigate the acceleration motion of vertically falling non-spherical particles both in Newtonian and non-Newtonian fluid and Radiation effect on a velocity of vertically falling non spherical particlesby VIM and Diagonal Pade' approximate. Recently, several attempts have been made to develop analytical tools to solve the equation of motion of falling object in fluid. Kaur and Garg [10] also investigated the effects of density and size on a terminal velocity of a vertically falling spherical particles in Newtonian fluid by Diagonal Pade' and Collocation Method and derived a solution for terminal velocity of the particles over time. They consider a water as liquid and take four types particles made of different materials with different size and gave a reliable results. From literature review, one of the well-known analytical correlations between Reynolds numbers [14] and drag coefficient for sphere in a Newtonian fluid could be expressed as follows:

C_D=f (Re, n) (1)

The drag coefficient could be obtained from Stokes law in following form:

$C_D=\frac{24}{Re}$ X (n), Where $Re = \frac{\rho u^2}{Ku^n D^{-n}}$ is the Reynolds number (1a)

$and X\ (n)\ = 6^{\frac{n-1}{2}}(\frac{3}{n^2+n+1})^{n+1}$is a deviation factor (1b)

was obtained by many researcher with help of numerical or experimental result. From all these, one of the well-known analytical correlated equation of Renaud et al [12] was used. In this work, we study terminal velocity and acceleration motion of vertically falling spherical particles made of glass, iron, copper and silver with diameter (D=1mm).The analysis derived by VIM and were compared with Pade' method & R-K 4th order method[10].

2. PROBLEM DESCRIPTION

Consider a rigid body, spherical particle with equivalent volume diameter D, mass m and density $\rho_s$is falling in an infinite extent of incompressible Newtonian fluid of density ρ and viscosity μ, u represents the velocity of the spherical particle at any instant time t, and g is the acceleration due to gravity [17]. Thus, the Basset – Boussinesq-Ossen (BBO) equation for the unsteady motion of particle in a fluid is given by [10]

$$m\frac{du}{dt} = mg\ (1-\frac{\rho}{\rho_s})\ - \frac{\pi D^2 \rho C_D}{8}u^n - \frac{\pi \rho D^3}{12}\frac{du}{dt} \quad (2)$$

Where $C_D$the drag coefficient. In right hand side of the Eq.(2), the 1st term represent the buoyancy effect, the 2nd term corresponds to drag resistance, 3rd term is associated with the added mass effect which is due to acc. of fluid around the particle. The complexity of the above equation arises due to the non-linear nature of drag coefficient. So by rewriting force balance Eq. (2) of motion of the particle,

$\alpha\frac{du}{dt}$+β (n)u^n-γ=0, $u\ (0) = 0$ (2a)

In which α =m+$\frac{1}{12}\pi D^3$ρ , β (n) =3πKX (n)D^{2-n}, γ = mg (1-$\frac{\rho}{\rho_s}$)

So for Newtonian fluid (n=1) $X(n) = 1$,

$\alpha\frac{du}{dt}$+βu- γ =0, $u\ (0) = 0$ (2b)

α =m+$\frac{1}{12}\pi D^3$ρ , $\beta\ (1)\ = 3\pi D$,γ = mg (1-$\frac{\rho}{\rho_s}$) (2c)

Table 1: Physical properties and values of constants of Eq. (2b)[6]

Particle	d(mm)	Density/(kg/m^3)	V=4/3πr^3	Mass(m)(gram)	α (kg/m^3)	β (kg/m^3)(1)	γ (kg/m^3)
glass	1.0	2590kg/m^3	5.2381x10^1	1.3567x10^{-3}	1.6178x10^{-6}	9.4286x10^{-6}	8.1768x10^{-6}
Iron	1.0	7874kg/m^3	5.2381x10^1	4.1245x10^{-3}	4.3856x10^{-6}	9.4286x10^{-6}	3.5298x10^{-5}
Copper	1.0	8940kg/m^3	5.2381x10^1	4.6829x10^{-3}	4.9440x10^{-6}	9.4286x10^{-6}	4.0770x10^{-5}
Silver	1.0	10490kg/m^3	5.2381x10^1	5.4948x10^{-3}	5.7559x10^{-6}	9.4286x10^{-6}	4.8727x10^{-5}

3. SOLUTIONS OF PROBLEM

Jihuan He was introduced Varinational Iteration Method (VIM) to solve such nonlinear ordinary and partial differential equations in 1997. He's Varinational iteration method (VIM) has been extensively applied as a power tool for solving various kinds of problems [3-6,].Slota [13] obtained results for the Heat equation by VIM which were same as the exact solution.Wang and Liu [11,16] used the VIM for the solution of Integro-Differential Equations and Free Vibration of an Euler–Bernoulli Beam. Recently, Kaur and Garg also used VIM to solve the various kind of problems [8, 9]. To clarify the VIM, we consider the following differential

$Lu\ (t) + Nu(t) = g(t)$ (3)

where L is a linear operator, N is a nonlinear operator and $g(t)$ is a non-homogeneous term. By using the Varinational iteration method, a correction functional can be constructed as

$$u_{n+1}(t)=u_n(t)+\int_0^t ʎ\{Lu_n(\zeta)+N\tilde{u}_n(\zeta)-g(\zeta)\}d\zeta \quad (3a)$$

in which ʎ is a general Lagrange multiplier, which can be determined by the help of Varinational theory, the subscript $_n$ means the nth approximation; u_n is restricted variation and $\delta\tilde{u}_n=0$According to VIM, firstly we will find Lagrange multiplier and then trial function u_0 to get the successive iterations u_{n+1}, $n\geq 0$ which converge to the exact solution. The solution is $u=\lim_{n\to\infty}u_n$

To solve Eq. (2b) using VIM, the correction functional can be constructed as follows:

$$u_{n+1}(t)=u_n(t)+\int_0^t ʎ\{\alpha\frac{du_n(s)}{ds}+\beta u_n(s)-\gamma\}ds \quad (3b)$$

The stationary conditions can be obtained as follows:

$$\frac{\beta}{\alpha}ʎ(t)_{s=t}-ʎ'(t)_{s=t}=0$$

$$1+\alpha ʎ(t)_{s=t}=0 \quad (3c)$$

Subsequently, the Lagrangian multiplier is obtained as:

$$ʎ=\frac{-e^{\frac{\beta}{\alpha}(s-t)}}{\alpha} \quad (3d)$$

$$u_{n+1}(t)=u_n(t)-\int_0^t\frac{-e^{\frac{\beta}{\alpha}(s-t)}}{\alpha}\{\alpha\frac{du_n(s)}{ds}+\beta u_n(s)-\gamma\}ds \text{with initial condition } u_0(t)=0 \quad (3e)$$

The solution of equation (3e) is obtained with the help of MATLAB programming.

4. RESULTS AND DISCUSSION

The applicability of the proposed method for the non-linear equation of motion of settling particles will be discussed in this study. In order to measure the accuracy of the results, R-K 4th order method has been derived for nonlinear differential Eq. (2b). The values of the fluid density (water) and consistency coefficient has been taken $\rho=997kg/m^3$and $K=1$ respectively. The physical properties of particles and corresponding coefficient of Eq. (2b) have been tabulated in Table.1.Terminal velocity of particles made of Glass Iron, Copper and Silver has been depicted in Table.4. It shows that due to increasing the density of particles, the terminal velocity also increasing. Same size (diameter=1mm) particles made of different materials has been compared, the particle of glass has lowest terminal velocity due to lowest density and particle of Silver has highest terminal velocity because of high density. Matlab code was used to find the numerical solution of the present problem.

Table2: Velocity results of vertically falling spherical particles of different materials by VIM, Diagonal Pade' approximant, & R-K 4th order Method for diameter (D=1mm)

t(time)	Glass(u),D=1mm			Iron(u)			Copper(u)			Silver(u)		
sec	VIM	Pade'	R-K	VIM	Pade'	R-K	VIM	Pade'	R-K	VIM	Pade'	R-K
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.05	0.2192	0.2192	0.2192	0.3816	0.3816	0.3816	0.3933	0.3933	0.3933	0.4064	0.4064	0.4064
0.10	0.3830	0.3830	0.3830	0.7242	0.7242	0.7242	0.7508	0.7508	0.7508	0.7809	0.7809	0.7809
0.15	0.5054	0.5052	0.5054	1.0320	1.0320	1.0320	1.0758	1.0758	1.0758	1.1259	1.1259	1.1259
0.20	0.5969	0.5960	0.5969	1.3083	1.3083	1.3083	1.3712	1.3712	1.3712	1.4437	1.4437	1.4437
0.25	0.6652	0.6631	0.6652	1.5566	1.5564	1.5566	1.6398	1.6397	1.6398	1.7366	1.7365	1.7366
0.30	0.7163	0.7122	0.7163	1.7795	1.7792	1.7795	1.8839	1.8837	1.8839	2.0064	2.0063	2.0064
0.35	0.7544	0.7475	0.7544	1.9797	1.9791	1.9797	2.1058	2.1054	2.1058	2.2551	2.2548	2.2551
0.40	0.7830	0.7721	0.7829	2.1595	2.1584	2.1595	2.3076	2.3068	2.3076	2.4841	2.4837	2.4841
0.45	0.8043	0.7885	0.8042	2.3209	2.3192	2.3209	2.4910	2.4897	2.4910	2.6952	2.6944	2.6952
0.50	0.8202	0.7986	0.8202	2.4659	2.4632	2.4659	2.6577	2.6558	2.6577	2.8897	2.8884	2.8897
0.55	0.8321	0.8037	0.8321	2.5962	2.5922	2.5962	2.8092	2.8064	2.8092	3.0688	3.0670	3.0688
0.60	0.8410	0.8050	0.8410	2.7131	2.7075	2.7131	2.9470	2.9430	2.9470	3.2339	3.2313	3.2339
0.65	0.8476	0.8032	0.8476	2.8182	2.8105	2.8182	3.0723	3.0667	3.0723	3.3860	3.3824	3.3860
0.70	0.8526	0.7992	0.8526	2.9125	2.9023	2.9125	3.1861	3.1787	3.1861	3.5261	3.5213	3.5261
0.75	0.8563	0.7934	0.8563	2.9972	2.9841	2.9972	3.2896	3.2799	3.2896	3.6552	3.6488	3.6552
0.80	0.8590	0.7862	0.8590	3.0733	3.0567	3.0733	3.3837	3.3714	3.3837	3.7742	3.7660	3.7742
0.85	0.8611	0.7780	0.8611	3.1416	3.1210	3.1416	3.4692	3.4538	3.4692	3.8838	3.8734	3.8838
0.90	0.8627	0.7690	0.8627	3.2030	3.1779	3.2030	3.5470	3.5280	3.5470	3.9848	3.9718	3.9848
0.95	0.8638	0.7594	0.8638	3.2581	3.2278	3.2581	3.6176	3.5946	3.6176	4.0779	4.0620	4.0779
1.00	0.8647	0.7494	0.8647	3.3076	3.2716	3.3076	3.6819	3.6543	3.6819	4.1636	4.1444	4.1636

Table3: Comparison between Varinational Iteration Method & Diagonal Pade' method

t(time)	Glass		Iron		Copper		Silver	
sec	VIM RELATIVE ERROR %	Pade' RELATIVE ERROE %	VIM RELATIVE ERROR %	Pade' RELATIVE ERROE %	VIM RELATIVE ERROR %	Pade' RELATIVE ERROE %	VIM RELATIVE ERROR %	Pade' RELATIVE ERROE %
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.05	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.15	0.0000	0.4000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.20	0.0000	1.8000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.25	0.0000	4.2000	0.0000	0.4000	0.0000	0.2000	0.0000	0.2000
0.30	0.0000	8.2000	0.0000	0.6000	0.0000	0.4000	0.0000	0.2000
0.35	0.0000	13.8000	0.0000	1.2000	0.0000	0.8000	0.0000	0.6000
0.40	0.0020	21.1600	0.0000	2.2000	0.0000	1.6000	0.0000	0.8000
0.45	0.0020	31.4000	0.0000	3.4000	0.0000	2.6000	0.0000	1.6000
0.50	0.0000	43.2000	0.0000	5.4000	0.0000	3.8000	0.0000	2.6000
0.55	0.0000	56.8000	0.0000	8.0000	0.0000	5.6000	0.0000	3.6000
0.60	0.0000	72.0000	0.0000	12.4000	0.0000	8.0000	0.0000	5.2000
0.65	0.0000	88.8000	0.0000	15.4000	0.0000	11.2000	0.0000	7.2000
0.70	0.0000	106.8000	0.0000	20.4000	0.0000	14.8000	0.0000	9.6000
0.75	0.0000	125.8000	0.0000	26.2000	0.0000	19.4000	0.0000	12.8000
0.80	0.0000	145.6000	0.0000	33.2000	0.0000	24.6000	0.0000	16.4000
0.85	0.0000	166.2000	0.0000	41.2000	0.0000	30.8000	0.0000	20.8000
0.90	0.0000	187.4000	0.0000	50.2000	0.0000	38.0000	0.0000	26.0000
0.95	0.0000	208.8000	0.0000	60.6000	0.0000	46.0000	0.0000	31.8000
1.00	0.0000	230.6000	0.0000	72.0000	0.0000	55.2000	0.0000	38.4000

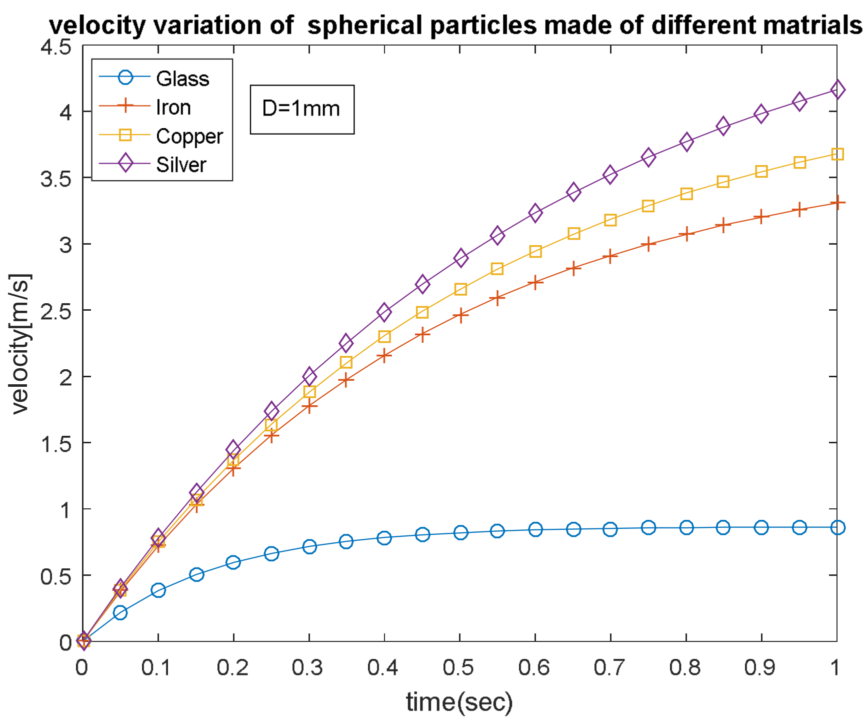

Fig. 1: The velocity results of vertically falling spherical particles in Newtonian fluid by R-K 4th order method.

The velocity results and relative errors% are tabulated in Table. 2 & 3. From Table 3, it is clear that the relative error % between VIM and R-K 4th order method is less as compare to Pade' approximant. So VIM gave the best result for current problem. Fig.1. indicates the velocity results of spherical particles made of different materials in falling procedure. It is obvious that the particles velocity is increased as density of particles increasing. It means the particles made of glass have lowest velocity due to its lower density as compare to other.

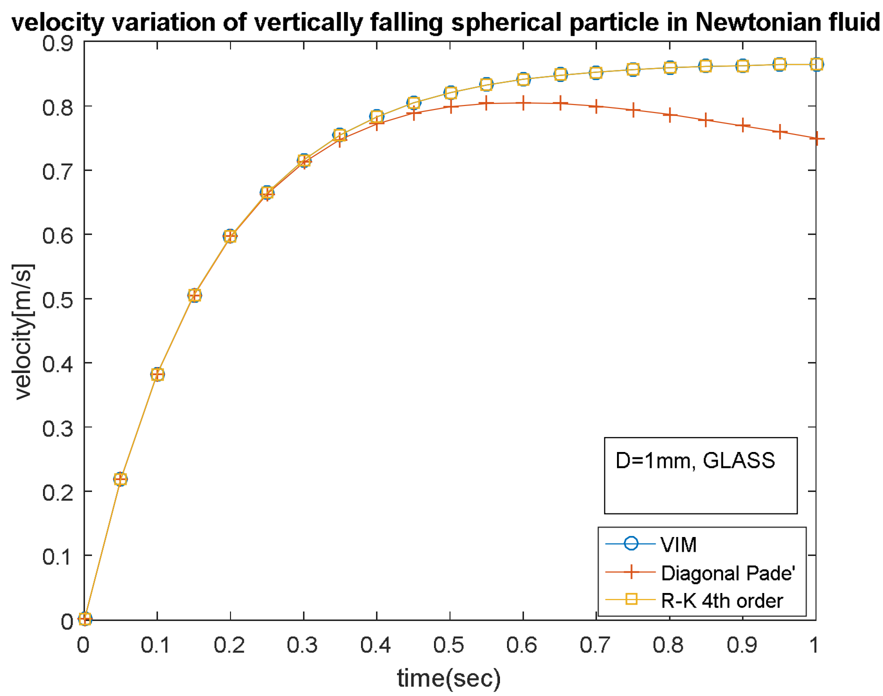

Fig. 2(a): The velocity results of vertically falling spherical particle (made of glass) in Newtonian fluid by different method.

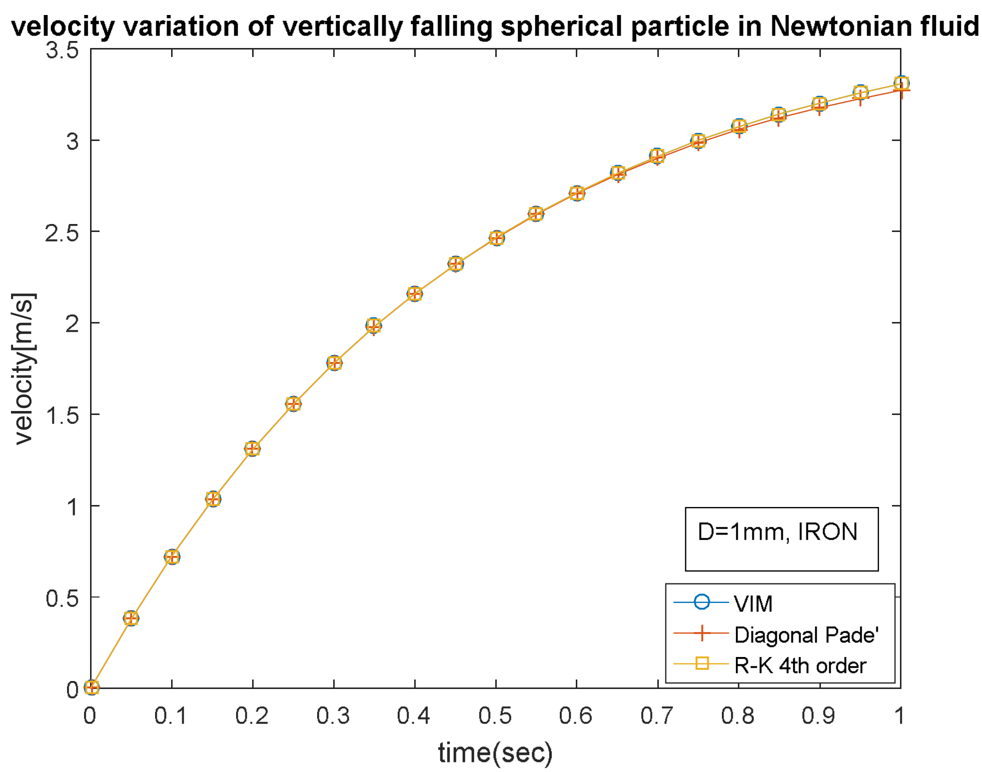

Fig. 2(b): The velocity results of vertically falling spherical particle (made of iron) in Newtonian fluid by different method.

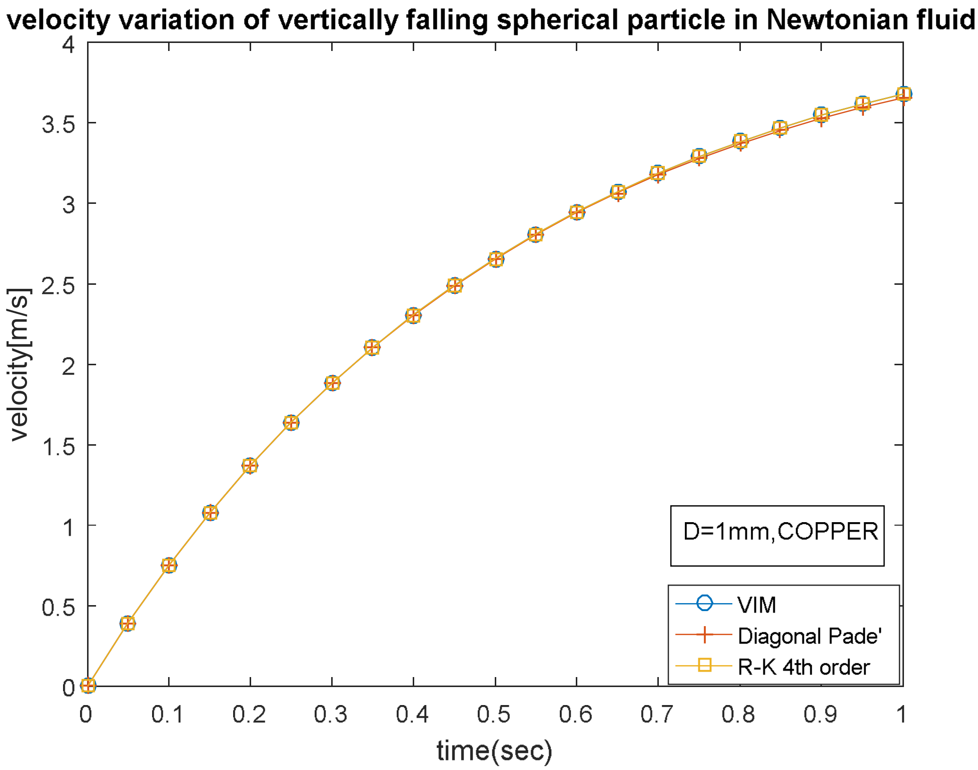

Fig. 2(c): The velocity results of vertically falling spherical particle (made of copper) in Newtonian fluid by different method.

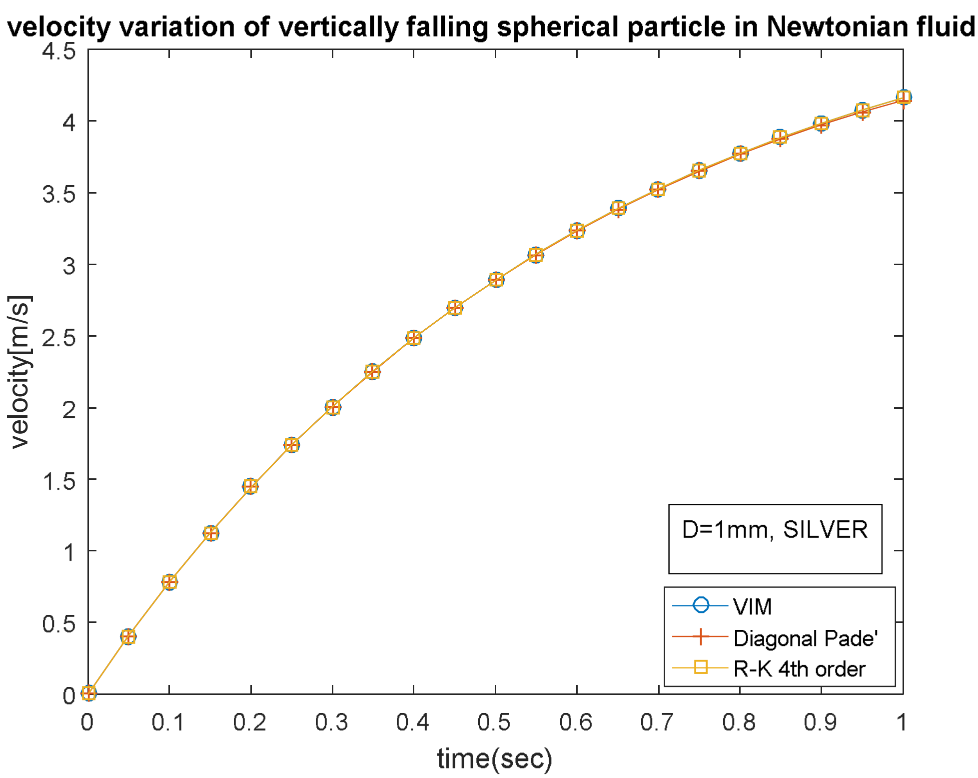

Fig. 2(d): The velocity results of vertically falling spherical particle (made of silver) in Newtonian fluid by different method.

Figures $2(a), (b), (c), (d)$ shows the velocity results of same size (D=1mm) particles made of glass, Iron, Copper and Silver which obtained by Varinational Iteration Method and compare the results by Diagonal Pade' & R-k 4^{th} order method in different time intervals as time taken (horizontally) and velocity (vertically). The velocity results clearly show that in short time both methods gave accurate results but VIM is most accurate method for solution of such problems.

Table4: Terminal Velocity [m/s] of spherical particles made of different materials

	Terminal Velocity[m/s]			
Diameter\Particles	Glass	Iron	Copper	Silver
D=1mm	0.8672m/s	3.7437m/s	4.3241m/s	5.1680m/s

In figs.3 & 4, indicates the terminal velocity and acceleration motion of vertically falling spherical particles in Newtonian fluid. It can be realized that the particles of lowest density has been taken less time to reaching terminal velocity as compare to other particles (i.e. acceleration motion of particles becomes zero).Fig. 4 indicates that for a constant diameter, initial acceleration time required to reach terminal velocity state increase by increasing the particle density. It also can be conclude that larger density particle reaches zero acceleration more slowly.

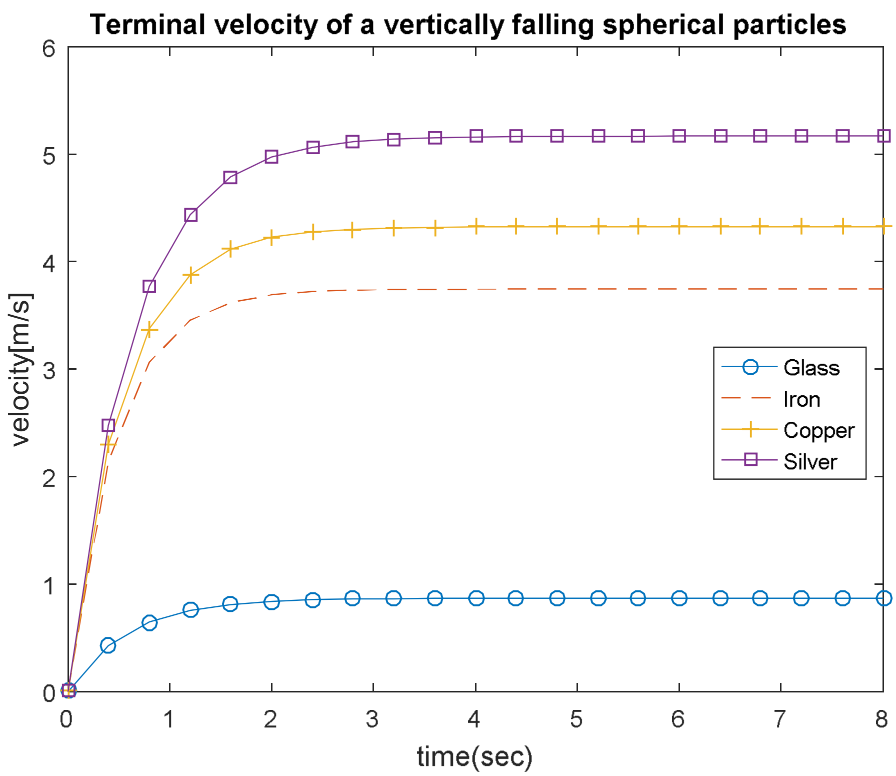

Fig. 3: The Terminal velocity results of vertically falling spherical particles (made of different materials) in Newtonian fluid.

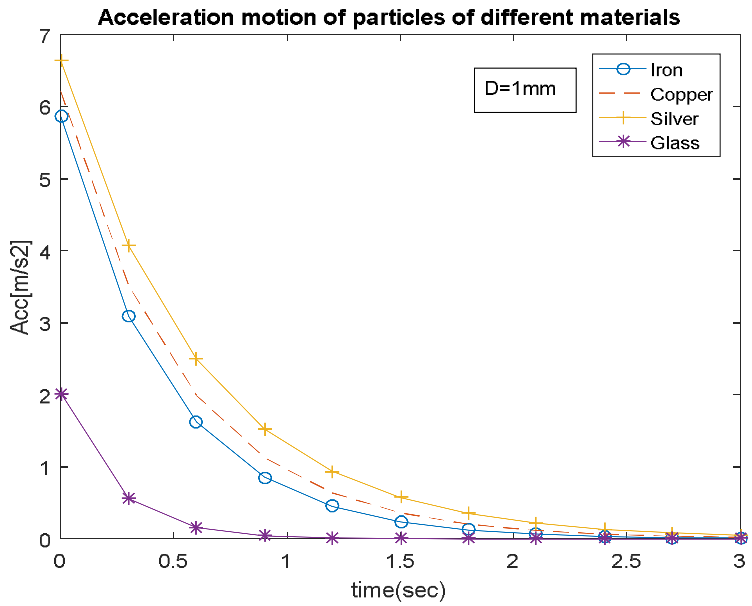

Fig. 4: The acceleration motion of vertically falling spherical particles in Newtonian fluid.

5. CONCLUSION

The achievement of this work is to apply VIM for velocity results of vertically falling spherical particles in Newtonian media. The current method is applied without using any linearization, discretization, restrictions or transformations. From above discussion, it is clear that the terminal velocity is increased as increasing density of particles and the acceleration period for lighter particles are shorter. It also shows that the effectiveness and simplicity of the current mathematical method. The VIM has a good agreement with R-k 4th order method and gives high degree of accuracy results as compare to Diagonal Pade' method. In addition, this method does not require many calculations to reach accurate results. Both methods give the accurate results in short time, but VIM is also suitable for long time. Also, the current method (VIM) can be used to develop the valid solution of other nonlinear differential equation of order one and more.

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Appendix – Matlab Code for R-K 4th order method

```
% GLASS(D = 1mm)
% VIM 1st iteration
symsts
formatlong
u = 0;
a = 1.6178 * 10^ − 6;
b = 9.4286 * 10^ − 6;
d = 8.1768 * 10^ − 6;
for i = 1: 1;
us = diff(u,s);
  d = simplify(int(−exp(b/a * (s − t)) * (us + (b/a) * u − (d/a)), s, 0, t));
  u = subs(u, s, t);
  u = simplify((u + d));
end
u(: )
ans = 40884/47143 − (40884 * exp(−(47143 * t)/8089))/47143
```

REFERENCES

[1] Chhabra R.P., Bubbles, (1993), Drops and Particles in Non-Newtonian Fluids, CRC Press, Boca Raton, FL.

[2] Clift R., Grace J.R., (1978), Weber M.E., Bubbles, Drops and Particles, Academic Press, New York.

[3] He J., (1999),Varinational Iteration Method – a Kind of Non-linear Analytical Technique: some example, International Journal of Non-Linear Mechanics, 34(4), pp. 699-708.

[4] He J., (2000), Varinational Iteration Method for Autonomous Ordinary Differential System, Applied Mathematics and Computation, 114(2/3), pp. 115-123.

[5] He J., (1997), Varinational Iteration Method for Delay Differential Equations, Communications in Nonlinear Science and Numerical Simulation, 2(4), pp. 235-236.

[6] He J., Xu-Hong Wu, (2007), Varinational Iteration Method: New Development and Applications, Journal of Computational and Applied Mathematics 207 n.1, pp. 3-17.

[7] Jalaal, Bararnia M. H., Domairry G., (2011), A Series Exact Solution for One-dimensional Non-linear Particles Equation of Motion, Powder Technology 207,pp. 461-464

[8] Kaur H., Garg B.P., (2017), Acceleration Motion of a Single Vertically Falling Non-Spherical Particle in Incompressible Newtonian Fluid by Different Methods, International Journal of Mathematics Trends and Technology, 52(7) ,pp. 452-457.

[9] Kaur H., Garg B.P., (2018), Effects of Density and Size on Terminal Velocity of a Vertically Falling Spherical Particles in Newtonian Fluid by Diagonal Pade' Approximant, International Journal of Statics and Applied Mathematics, 3(2), pp. 621-634

[10] Kaur H., Garg B.P., (2018), Radiation Effect on Terminal Velocity of a Vertically Falling Non-Spherical Particle by Diagonal Pade' [3/3] Approximants, International Journal of Statics and Applied Mathematics, 3(2), pp. 31-36.

[11] Liu Y., Gurram C.S., (2009), The use of He's Varinational Iteration Method for obtaining the Free Vibration of an Euler–Bernoulli Beam, Mathematical and Computer Modelling 50(11/12), pp. 1545-1552.

[12] Renaud M., Mauret E., Chhabra R.P., (*2004),* Power Law Fluid Flow over a Sphere: Average Shear Rate and Drag Coefficient, *Can. J*ournal of *Chemistry* and *Eng.82, pp. 1066*– 1070.

[13] Slota D., (2009), Exact Solution of the Heat Equation with Boundary Condition of the Fourth Kind by He's Varinational iteration method, Computers & Mathematics with Applications 58(11/12), pp. 2495-2503

[14] Tabuteauand H., Coussot P., Bruyn J. R.de, (2007) , Drag Force on a Sphere in Steady Motion Through a Yield Stress Fluid, The Society of Rheology Inc. J.Rheol.51, 125-137.

[15] Torabi M., Yaghoobi H., (2011), Novel Solution for Acceleration Motion of a Vertically Falling Spherical Particles by HPM-Pade' Approximant, Advanced Powder Technology, 22(5), pp. 674-677.

[16] Wang S.Q., He J., (2007), He's Varinational Iteration Method for Solving Integro-Differential Equations, Physics Letters A 367(3), pp. 188-191.

[17] Yaghoobi H., Torabi M., (2012), Novel Solution for Acceleration Motion of a Vertically Falling Non-Spherical Particles by VIM-Pade' Approximant, Powder Technology 215,pp. 206-209.