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STOCHASTIC ANALYSIS OF POTATO CHIPS PLANT MODEL UNDER CLASSICAL AND BAYESIAN SET UPS

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Abstract

This paper presents the stochastic analysis of a real existing industrial system model of potato chips under Classical and Bayesian set ups. The system consists of four different subsystems viz. Destoning and Peeling (D), Slicing (S), Coloring (C) and Frying &Salting (F). Out of four subsystems, one subsystem (Destoning and Peeling) has its cold standby unit. All these subsystems are connected in series configuration. Life time distributions and repair time distributions of each sub system are assumed to be independent Weibull with different scale parameters but common shape parameter. A single repairman is always available with the system .The repair discipline is first come first served (FCFS). Maximum Likelihood Estimation of the parameters representing the reliability characteristics is also done. A Bayesian approach is also adopted to evaluate the reliability characteristics .Finally, a Monte Carlo simulation study is carried out to judge the performances of the ML and Bayes estimator.

Keywords: Regenerative point, Mean time to System Failure (MTSF), Busy period, Net Expected profit, Fisher Information Matrix.

Mathematics Subject Classification 2010: 60H10; 62F10; 62F15

1. INTRODUCTION

A lot of research work on reliability modeling of maintained systems has been carried out by several researchers in the field of reliability theory by considering static environmental condition. Gupta and Bhardwaj [6] analyzed the performance measures of a two-unit warm standby system model with repair, inspection and post-repair. Goel et al. [4] studied a two-unit warm standby system with fault detection and inspection. Chaudhary et al. [2] analyzed a two non-identical unit parallel system model with single or double phase(s) of repair. It is worth mentioning here that all the above studies are not based on real existing system models. However, some researchers like Gupta and Kumar [5] carried out the analysis of Reliability characteristics of a distillery plant. Chaudhary et al. [1] analyzed reliability characteristic of bread making system. Gupta and Kishan [7] developed a model pertaining to power, inverter and generator and obtained various reliability measures. Gupta and Shivakar [8] analyzed a cloth weaving system model using regenerative point technique.

We also note that all the above studies were mainly concerned to obtain various reliability characteristics such as mean time to system failure (MTSF), point wise and steady state availabilities etc. by using different life time and repair time distributions of units and not to estimate the parameter(s) involved in the life time/repair time distribution of the system/unit.

In this paper we analyze a real existing system model of a potato chips plant assuming the failure and repair time distributions of each sub system as independent Weibull with common and known shape parameter but different scale parameters and also find the maximum likelihood estimators of the parameters representing various reliability characteristics. Since the lifetime experiments are very time consuming and as such the environmental conditions throughout the experiment may not be same. Therefore, it seems reasonable to treat the failure time parameters representing various system reliability characteristics as random variables instead of fixed constants. Keeping this in view, a Bayesian approach is also adopted to evaluate the various measures of system effectiveness by taking different priors and the comparative analysis is also carried out to access the performances of the MLE and Bayesian estimators.

The probability density function (p.d.f) of Weibull distribution with shape parameter p and scale parameter α is given by

$$f(t) = \alpha p t^{p-1} \exp(-\alpha t^p);\ \alpha, p > 0, t \geq 0 \qquad (1)$$

The reliability/survival function and hazard (failure /repair) rate for Weibull distribution are respectively given by

$$R(t) = \exp(-\alpha t^p)$$

and

$$h(t) = \alpha p t^{p-1}$$

It is important to note that for p=1, the Weibull distribution given in (1), reduces to exponential distribution and for p=2, it reduces to the Rayleigh distribution.

2. SYSTEM DESCRIPTION

The potato chips system consists of four main subsystems- Destoning & Peeling, Slicing, Coloring and Salting & Frying. All are arranged in series network. Out of these four subsystems, one subsystem namely- Destoning machine has its cold standby unit. The system stops functioning if any one of the subsystems stops functioning. Destoning & Peeling machine, Coloring machine and Frying & Salting machine becomes as good as new after repair while after the repair of Slicing machine, it first goes for inspection with known probabilities to decide whether the repair is perfect or not. If the repair of Slicing machine is found to be perfect then it becomes operational, otherwise it is sent for post repair. The service discipline of the repairman is First Come First Served (FCFS). A single repair facility is used to repair each subsystem and inspection & post repair of slicing machine. The failure and repair time distributions of each subsystem are taken as independent having the Weibull density with common shape parameter 'p' but different scale parameters α_d, α_s, α_c, α_f and β_d, β_s, β_c, β_f respectively as follows:

$$f_d(t) = \alpha_d p t^{p-1} \exp(-\alpha_d t^p), t \geq 0, \alpha_d, p > 0$$

$$f_s(t) = \alpha_s p t^{p-1} \exp(-\alpha_s t^p), t \geq 0, \alpha_s, p > 0$$

$$f_c(t) = \alpha_c p t^{p-1} \exp(-\alpha_c t^p), t \geq 0, \alpha_c, p > 0$$

$$f_f(t) = \alpha_f p t^{p-1} \exp(-\alpha_f t^p), t \geq 0, \alpha_f, p > 0$$

and

$$g_d(t) = \beta_d p t^{p-1} \exp(-\beta_d t^p), t \geq 0, \beta_d, p > 0$$

$$g_s(t) = \beta_s p t^{p-1} \exp(-\beta_s t^p), t \geq 0, \beta_s, p > 0$$

$$g_c(t) = \beta_c p t^{p-1} \exp(-\beta_c t^p), t \geq 0, \beta_c, p > 0$$

$$g_f(t) = \beta_f p t^{p-1} \exp(-\beta_f t^p), t \geq 0, \beta_f, p > 0$$

The inspection and post repair time distributions of slicing machine are taken to be independent having the Weibull density with common shape parameter 'p' but different scale parameters ν_s and λ_s as follows:

$$o(t) = \nu_s p t^{p-1} \exp(-\nu_s t^p);\ \nu_s,\ p > 0,\ t \geq 0$$
$$m(t) = \lambda_s p t^{p-1} \exp(-\lambda_s t^p);\ \lambda_s,\ p > 0,\ t \geq 0$$

2.1 Roll of Subsytems

• **Destoning & Peeling (D) –** Destoning & Peeling machine pushes the potatoes up to a conveyer belt to the automatic peeling machine. After they have been peeled, the potatoes are washed with cold Water.

• **Slicing (S)** – The main work of slicing machine is to cut the potatoes in to paper thin slices.

• **Slicing (S)** – The main work of slicing machine is to cut the potatoes in to paper thin slices.

• **Coloring (C) –** After the slices of potatoes, the potatoes are chemically treated to enhance their color.

• **Frying & Salting (F) –** Frying and salting machine is used to remove the excess water as they flow into 40-75ft troughs filled with oil. As the slices tumble, salt is sprinkled to each of chips.

3. NOTATIONS AND STATES OF THE SYSTEM

E : Set of regenerative states.

α_d, α_s, α_c, α_f : Scale parameters of failure time distribution for destoning & peeling, slicing, coloring, frying & salting machine respectively.

β_d, β_s, β_c, β_f : Scale parameters of repair time distribution for destoning & peeling, slicing, coloring, frying & salting machine respectively.

P : Shape parameter of failure/repair time distribution of each subsystem.

$h_d(t)$: Failure rate of destoning & peeling machine.
$= \alpha_d p t^{p-1}$, α_d, p, t > 0

$h_s(t)$: Failure rate of slicing machine.
$= \alpha_s p t^{p-1}$, α_s, p, t > 0

$h_c(t)$: Failure rate of coloring machine.
$= \alpha_c p t^{p-1}$, α_c, p, t > 0

$h_f(t)$: Failure rate of frying & salting machine.
$= \alpha_f p t^{p-1}$, α_f, p, t > 0

$J_d(t)$: Repairs rate of destoning & peeling machine.
$= \beta_d p t^{p-1}$, β_d, p, t > 0

$J_s(t)$: Repair rate of slicing machine.
$= \beta_s p t^{p-1}$, β_s, p, t > 0

$J_c(t)$: Repair rate of coloring machine.
$= \beta_c p t^{p-1}$, β_c, p, t > 0

$J_f(t)$: Repair rate of frying & salting machine.
$= \beta_f p t^{p-1}$, β_f, p, t > 0

ν_s, λ_s : Scale parameter of inspection/post repair time distribution of slicing machine.

$l_s(t)$: Inspection rate of slicing machine having the form
$= \nu_s p t^{p-1};\ \nu_s$, p, t > 0

$x_s(t)$: Post repair rate of slicing machine having the form
$= \lambda_s p t^{p-1};\ \lambda_s$, p, t > 0

a/b : Probabilities that the repair of slicing machine is perfect or imperfect such i.e. a+b=1

$q_{ij}(\cdot), Q_{ij}(\cdot)$: p.d.f. (probability density function) and c.d.f. (cumulative density function) of one step or direct transition time from $S_i \epsilon E$ to $S_j \epsilon E$.

p_{ij} : Steady state transition probability from state S_i to S_j such that
$p_{ij} = \lim_{t\to\infty} Q_{ij}(t)$.

$p_{ij}^{(k)}$: Steady state transition probability from state S_i to S_j via S_k such that

$$p_{ij}^{(k)} = \lim_{t\to\infty} Q_{ij}^{(k)}(t).$$

ψ_i : Mean sojourn time in regenerative state S_i i.e. $=\int_0^\infty P[T_i > t]dt$

θ_1 : Mean repair time of Destoning and Peeling machine.

$*$: Symbol for Laplace transform of a function i.e. $q_{ij}^*=\int_0^\infty e^{-st} q_{ij}(t)dt.$

· : Regenerative point.

× : Non regenerative point.

Symbols for the States of the System

D_o, D_s, D_g, D_r, D_{wr} : Destoning & Peeling machine is operative, standby, good, under repair, waiting for repair.

S_o, S_g, S_r, S_{wr}, S_I, S_{Pr} : Slicing machine is operative, good, under repair, waiting for repair, under inspection after repair/and post repair.

C_o, C_g, C_r, C_{wr} : Coloring machine is operative, good, under repair, waiting fo repair.

F_o, F_g, F_r, F_{wr} : Frying and Salting machine is operative, good, under repair, waiting for repair.

With these symbols and assumptions stated above, the transition diagram of the system model along with transition rates between different states is shown in Fig.1. Here S_5, S_6, S_7, S_8 states are non-regenerative whereas the other states are regenerative. Also note that states S_0 and S_1 are up states whereas states S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9 and S_{10} are failed states.

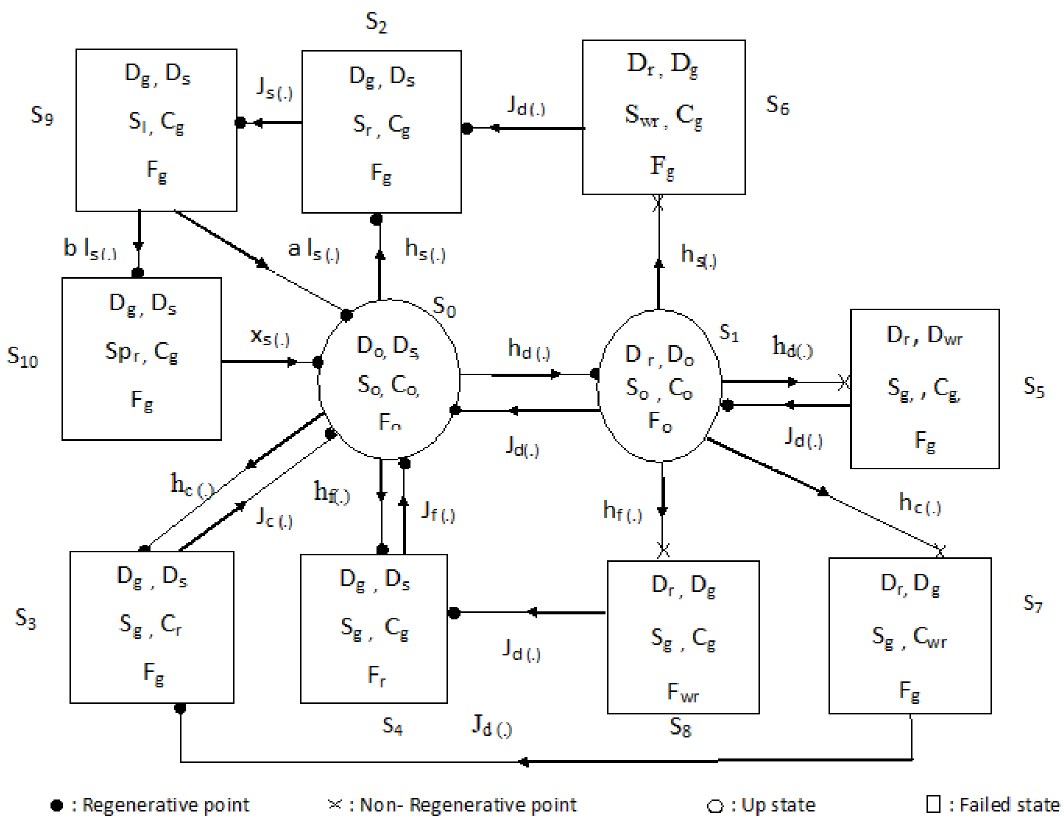


Fig. 1: Transition Diagram

4. TRANSITION PROBABILITIES AND SOJOURN TIMES

The elements p_{ij} of transition probability matrix (t.p.m.) of the embedded Markov chain is as follows:

$$P = \begin{bmatrix} p_{00} & p_{01} & p_{02} & p_{03} & p_{04} & p_{09} & p_{010} \\ p_{10} & p_{11}^{(5)} & p_{12}^{(6)} & p_{13}^{(7)} & p_{14}^{(8)} & p_{19} & p_{110} \\ p_{20} & p_{21} & p_{22} & p_{23} & p_{24} & p_{29} & p_{210} \\ p_{30} & p_{31} & p_{32} & p_{33} & p_{34} & p_{39} & p_{310} \\ p_{40} & p_{41} & p_{42} & p_{43} & p_{44} & p_{49} & p_{410} \\ p_{90} & p_{91} & p_{92} & p_{93} & p_{94} & p_{99} & p_{910} \\ p_{100} & p_{101} & p_{102} & p_{103} & p_{104} & p_{109} & p_{1010} \end{bmatrix}$$

The steady state transition probabilities can be obtained by using the results,

$$p_{ij} = \lim_{t \to \infty} Q_{ij}(t) \text{ and } p_{ij}^{(k)} = \lim_{t \to \infty} Q_{ij}^{(k)}(t)$$

$$p_{01} = \int \alpha_d p t^{p-1} e^{-\alpha_d t^p} e^{-\alpha_f t^p} e^{-\alpha_c t^p} e^{-\alpha_s t^p} dt = \frac{\alpha_d}{\alpha_d + \alpha_f + \alpha_c + \alpha_s}$$

Note that the limits of integration are from 0 to ∞ whenever not mentioned.

Similarly,

$$p_{02} = \frac{\alpha_s}{\alpha_d + \alpha_f + \alpha_c + \alpha_s}; p_{03} = \frac{\alpha_c}{\alpha_d + \alpha_f + \alpha_c + \alpha_s}; p_{04} = \frac{\alpha_f}{\alpha_d + \alpha_f + \alpha_c + \alpha_s}$$

$$p_{11}^{(5)} = \frac{\alpha_d}{\alpha_d + \alpha_f + \alpha_c + \alpha_s + \beta_d}; p_{14}^{(8)} = \frac{\alpha_f}{\alpha_d + \alpha_f + \alpha_c + \alpha_s + \beta_d}; p_{13}^{(7)} = \frac{\alpha_c}{\alpha_d + \alpha_f + \alpha_c + \alpha_s + \beta_d}$$

$$p_{12}^{(6)} = \frac{\alpha_s}{\alpha_d + \alpha_f + \alpha_c + \alpha_s + \beta_d}; p_{10} = \frac{\beta_d}{\alpha_d + \alpha_f + \alpha_c + \alpha_s + \beta_d}$$

$$p_{9,10} = b^* \left[\frac{\nu_s}{\nu_s + \nu_s} \right]; p_{90} = a^* \frac{\nu_s}{\nu_s + \nu_s}$$

It can be easily verified that

$$p_{01} + p_{02} + p_{03} + p_{04} = 1$$

$$p_{10} + p_{11}^{(5)} + p_{12}^{(6)} + p_{13}^{(7)} + p_{14}^{(8)} = 1 \qquad (2\text{-}5)$$

$$p_{90} + p_{9,10} = 1$$

$$p_{29} = p_{30} = p_{40} = 1$$

Mean Sojourn Times

If T_i be the sojourn time in state S_i, then mean sojourn time in state S_i is given by,

$$\psi_0 = \int P(T_i > t)\, dt$$

Therefore, the mean sojourn times for various states are as follows:

$$\psi_0=\int e^{-\alpha_d t^p}e^{-\alpha_f t^p}e^{-\alpha_c t^p}e^{-\alpha_s t^p}dt=\int e^{-(\alpha_d+\alpha_f+\alpha_c+\alpha_s)t^p}dt=\frac{\Gamma(1+\frac{1}{p})}{(\alpha_d+\alpha_f+\alpha_c+\alpha_s)^{1/p}}$$

$$\psi_1=\int e^{-\alpha_d t^p}e^{-\alpha_f t^p}e^{-\alpha_c t^p}e^{-\alpha_s t^p}e^{-\beta_d t^p}dt=\int e^{-(\alpha_d+\alpha_f+\alpha_c+\alpha_s+\beta_d)t^p}dt=\frac{\Gamma(1+\frac{1}{p})}{(\alpha_d+\alpha_f+\alpha_c+\alpha_s+\beta_d)^{1/p}}$$

Similarly,

$$\psi_2=\frac{\Gamma(1+\frac{1}{p})}{(\beta_s)^{1/p}},\ \psi_3=\frac{\Gamma(1+\frac{1}{p})}{(\beta_c)^{1/p}},\ \psi_4=\frac{\Gamma(1+\frac{1}{p})}{(\beta_f)^{1/p}},\ \psi_9=\frac{\Gamma(1+\frac{1}{p})}{(\nu_s+\nu_s)^{1/p}},\ \psi_{10}=\frac{\Gamma(1+\frac{1}{p})}{(\lambda_s)^{1/p}}$$

(6-12)

5. ANALYSIS OF CHARACTERISTICS

5.1 Reliability and Mean Time to System Failure (MTSF)

Let the random variable 'T' be the time to system failure (TSF) when the system starts from $S_i \in E$, then the reliability of the system is given by

$$R_i(t) = P\,[T_i > t]$$

To determine the reliability of the system, we regard the failed states of the system as absorbing states. By simple probabilistic arguments, we have the following recursive relations among $R_i(t)$'s.

$$R_0(t)=Z_0(t)+q_{01}(t)\copyright R_1(t)$$

$$R_1(t)=Z_1(t)+q_{10}(t)\copyright R_0(t)$$

(13-14)

Taking Laplace transform of Equations (13-14) and solving for $R_0^*(s)$ (omitting the argument's' for brevity), we get

$$R_0^*(s)=\frac{N_1(s)}{D_1(s)}=\frac{Z_0^*+q_{01}^*Z_1^*}{1-q_{01}^*q_{10}^*} \quad (15)$$

where

$Z_0^*(s)$ and $Z_1^*(s)$ are the Laplace transforms of $Z_0(t)$ and $Z_1(t)$ given by

$$Z_0(t)=e^{-(\alpha_d+\alpha_f+\alpha_c+\alpha_s)t^p} \quad \text{and} \quad Z_1(t)=e^{-(\alpha_d+\alpha_f+\alpha_c+\alpha_s+\beta_d)t^p}$$

Taking the inverse Laplace transform (ILT) of Equation (15), one can get the reliability of the system when system starts from the state S_0.

The mean time to system failure (MTSF) can be obtained by using the well known formula-

$$\text{MTSF}=E(T_0)=\lim_{s\to 0}R_0^*(s)=\frac{N_1(s)}{D_1(s)}=\frac{N_1(0)}{D_1(0)}=\frac{N_1}{D_1} \quad (16)$$

Now using the results $q_{ij}^*(0)=p_{ij}$ and $Z_i^*(0)=\psi_i$, we get

$$N_1=\psi_0+p_{01}\psi_1=\frac{\Gamma(1+\frac{1}{p})}{(\alpha_d+\alpha_f+\alpha_c+\alpha_s)^{1/p}}+\frac{\alpha_d}{\alpha_d+\alpha_f+\alpha_c+\alpha_s}*\frac{\Gamma(1+\frac{1}{p})}{(\alpha_d+\alpha_f+\alpha_c+\alpha_s+\beta_d)^{1/p}} \quad (17)$$

$$D_1=1-p_{01}p_{10}=1-\frac{\alpha_d}{\alpha_d+\alpha_f+\alpha_c+\alpha_s}*\frac{\beta_d}{\alpha_d+\alpha_f+\alpha_c+\alpha_s+\beta_d} \quad (18)$$

5.2 Availability Analysis

Let us define $A_i(t)$ as the probability that the system is up at time t when initially it starts from state $S_i \in E$.

By simple probabilistic arguments, we have the following recursive relations among $A_i(t)$'s:

$$A_0(t) = Z_0(t) + q_{01}(t) © A_1(t) + q_{02}(t) © A_2(t) + q_{03}(t) © A_3(t) + q_{04}(t) © A_4(t)$$

$$A_1(t) = Z_1(t) + q_{10}(t) © A_0(t) + q_{11}^{(5)}(t) © A_1(t) + q_{12}^{(6)}(t) © A_2(t) + q_{13}^{(7)}(t) © A_3(t) + q_{14}^{(8)}(t) © A_4(t)$$

$$A_2(t) = q_{29}(t) © A_9(t)$$

$$A_3(t) = q_{30}(t) © A_0(t)$$

$$A_4(t) = q_{40}(t) © A_0(t)$$

$$A_9(t) = q_{9,10}(t) © A_{10}(t) + q_{90}(t) © A_0(t)$$

$$A_{10}(t) = q_{10,0}(t) © A_0(t)$$

(19-25)

Where $Z_0(t) = e^{-(\alpha_d+\alpha_s+\alpha_c+\alpha_f)t^p}$ and $Z_1(t) = e^{-(\alpha_d+\alpha_s+\alpha_c+\alpha_f+\beta_d)t^p}$

Taking Laplace transforms of relations (19-25) and simplifying for $A_0^*(s)$ (omitting the argument's' for brevity), we get

$$A_0^*(s) = \frac{N_2(s)}{D_2(s)}$$

$$= \frac{Z_0^*(1-q_{11}^{(5)*}) + q_{01}^* Z_1^*}{[(1-q_{11}^{(5)*})(1-q_{02}^* Y - q_{03}^* q_{30}^* - q_{04}^* q_{40}^*)] - q_{01}^*[q_{10}^* + Y q_{12}^{(6)*} + q_{30}^* q_{13}^{(7)*} + q_{40}^* q_{14}^{(8)*}]}$$

$$\text{where, } Y = \{q_{29}^* q_{9,10}^* q_{10,0}^* + q_{29}^* q_{90}^*\}$$

(26)

Taking inverse Laplace Transform of Equation (26), we get the availability of the system when it starts from state S_0.

In the long run, the steady state availability of the system when it starts from state S_0 is given by

$$A_0 = \lim_{t \to \infty} A_0(t)$$

$$= \lim_{s \to 0} s A_0^*(s) = \frac{N_2}{D_2}$$

(27)

where,

$$N_2 = \psi_0[1 - p_{11}^{(5)}] + \psi_1 p_{01}$$

and

$$D_2 = (1-p_{11}^{(5)})\psi_0 + p_{01}\theta_1 + (1-p_{11}^{(5)})(p_{02}\psi_2 + p_{03}\psi_3 + p_{04}\psi_4 + p_{02}p_{29}\psi_9 + p_{02}p_{29}p_{9,10}\psi_{10})$$
$$+ p_{01}(p_{12}^{(6)}\psi_2 + p_{13}^{(7)}\psi_3 + p_{14}^{(8)}\psi_4 + p_{29}p_{12}^{(6)}\psi_9 + p_{29}p_{90}p_{12}^{(6)}\psi_{10})$$

(28)

where,

$\theta_1 = \dfrac{\Gamma\left(1+\frac{1}{p}\right)}{(\beta_d)^{1/p}}$ = Mean repair time of Destoning and Peeling machine.

The expected up time of the system during (0, t) is given by

$$\mu_{up}(t)=\int_0^t A_0(u)du \text{, so that, } \mu^*_{up}(s) = \frac{A^*_0(s)}{s} \tag{29}$$

5.3 Busy Period Analysis

(i)The expected busy period of the repairman in repair of Destoning and Peeling machine (D)

Let us define $B^D_i(t)$ as the probability that the repairman is busy in the repair of a failed Destoning and Peeling machine (D) at epoch t when the system starts from state $S_i \in E$. By simple probabilistic arguments, we have the following recursive relations among $B^D_i(t)$'s:

$$B^D_0(t) = q_{01}(t) © B^D_1(t) + q_{02}(t) © B^D_2(t) + q_{03}(t) © B^D_3(t) + q_{04}(t) © B^D_4(t)$$

$$B^D_1(t)=Z_1(t)+q_{10}(t)© B^D_0(t)+q^{(5)}_{11}(t)© B^D_1(t)+q^{(6)}_{12}(t)© B^D_2(t) + q^{(7)}_{13}(t) © B^D_3(t)+q^{(8)}_{14}(t)© B^D_4(t)$$

$$B^D_2(t) = q_{29}(t) © B^D_9(t)$$

$$B^D_3(t) = q_{30}(t) © B^D_0(t)$$

$$B^D_4(t) = q_{40}(t) © B^D_0(t)$$

$$B^D_9(t) = q_{9,10}(t) © B^D_{10}(t) + q_{90}(t) © B^D_0(t)$$

$$B^D_{10}(t) = q_{10,0}(t) © B^D_0(t)$$

(30-36)

where,

$$Z_1(t) = e^{-(\alpha_d+\alpha_s+\alpha_c+\alpha_f+\beta_d)t^p}.$$

Taking the Laplace transform of Equations (30-36) and solving for $B^{D*}_0(s)$
(omitting the argument's' for brevity), we get

$$B^{D*}_0(s)=\frac{N_3(s)}{D_2(s)}=\frac{Z^*_1q^*_{01}}{[(1-q^{(5)*}_{11})(1-q^*_{02}Y-q^*_{03}q^*_{30}-q^*_{04}q^*_{40})]-q^*_{01}[q^*_{10}+Yq^{(6)*}_{12}+q^*_{30}q^{(7)*}_{13}+q^*_{40}q^{(8)*}_{14}]} \tag{37}$$

where, $Y=\{q^*_{29}q^*_{9,10}q^*_{10,0}+q^*_{29}q^*_{90}\}$

Taking the inverse Laplace Transform of Equation (37), we get the probability that the repairman is busy in the repair of a failed Destoning and Peeling machine at epoch t, given that the system starts from the state S_0. In the long run, the fraction of time for which the system is under repair, starting from the state S_0, is given by

$$B^D_0 = \lim_{t\to\infty} B^D_0(t)= \lim_{s\to0} s\, B^{D*}_0(s) = \frac{N_3}{D_2} \tag{38}$$

where,

$$N_3 = p_{01}\psi_1 \tag{39}$$

and D_2 is the same as given in (28).

The expected busy period of the repairman in the repair of Destoning and Peeling machine (D) is

$$\mu^D_b(t)=\int_0^t B^D_0(u)du \quad \text{so that} \quad \mu^{D*}_b(s) = \frac{B^{D*}_0(s)}{s} \tag{40}$$

(ii) The expected busy period of the repairman in repair of Slicing machine (S)

Let us define $B_i^S(t)$ as the probability that the repairman is busy in the repair of a failed Slicing machine(S) at epoch t when the system starts from state $S_i \in E$. By simple probabilistic arguments, we have following recursive relations among $B_i^S(t)$'s:

$$B_0^S(t) = q_{01}(t) \copyright B_1^S(t) + q_{02}(t) \copyright B_2^S(t) + q_{03}(t) \copyright B_3^S(t) + q_{04}(t) \copyright B_4^S(t)$$

$$B_1^S(t) = q_{10}(t) \copyright B_0^S(t) + q_{11}^{(5)}(t) \copyright B_1^S(t) + q_{12}^{(6)}(t) \copyright B_2^S(t) + q_{13}^{(7)}(t) \copyright B_3^S(t) + q_{14}^{(8)}(t) \copyright B_4^S(t)$$

$$B_2^S(t) = Z_2(t) + q_{29}(t) \copyright B_9^S(t)$$

$$B_3^S(t) = q_{30}(t) \copyright B_0^S(t)$$

$$B_4^S(t) = q_{40}(t) \copyright B_0^S(t)$$

$$B_9^S(t) = q_{9,10}(t) \copyright B_{10}^S(t) + q_{90}(t) \copyright B_0^S(t)$$

$$B_{10}^S(t) = q_{10,0}(t) \copyright B_0^S(t)$$

(41-47)

where, $Z_2(t) = e^{-\beta_s t^p}$

Taking the Laplace transform of Equations (41-47) and solving for $B_0^{S*}(s)$
(omitting the argument's' for brevity), we get

$$B_0^{S*}(s) = \frac{N_4(s)}{D_2(s)} = \frac{Z_2^*[q_{02}^*(1-q_{11}^{(5)*}) + q_{01}^* q_{12}^{(6)*}]}{[(1-q_{11}^{(5)*})(1-q_{02}^* Y - q_{03}^* q_{30}^* - q_{04}^* q_{40}^*)] - q_{01}^*[q_{10}^* + Y q_{12}^{(6)*} + q_{30}^* q_{13}^{(7)*} + q_{40}^* q_{14}^{(8)*}]} \quad (48)$$

where, $Y = \{q_{29}^* q_{9,10}^* q_{10,0}^* + q_{29}^* q_{90}^*\}$

Taking the inverse Laplace Transform of equation (48), we get the probability that the repairman is busy in the repair of a failed Slicing machine at epoch t given that the system starts from the state S_0. In the long run, the fraction of time for which the system is under repair, starting from state S_0, is given by

$$B_0^S = \lim_{t\to\infty} B_0^S(t) = \lim_{s\to 0} s\, B_0^{S*}(s) = \frac{N_4}{D_2}$$

(49)

where

$$N_4 = [(1-p_{11}^{(5)})p_{02} + p_{01}p_{12}^{(6)}]\psi_2 \quad (50)$$

and D_2 is as given in (28).

The expected busy period of the repairman in the repair of Slicing machine (M) is

$$\mu_b^S(t) = \int_0^t B_0^S(u)\,du \qquad \text{so that} \qquad \mu_b^{S*}(s) = \frac{B_0^{S*}(s)}{s}$$

(51)

(iii) The expected busy period of the repairman in inspection of Slicing machine (S)

Let us define $B_i^{SI}(t)$ as the probability that the repairman is busy in the inspection after the repair of Slicing machine(S) at epoch t when the system starts from the state $S_i \in E$. By simple probabilistic arguments, we have the following recursive relations in $B_i^{SI}(t)$'s:

$$B_0^{SI}(t) = q_{01}(t) © B_1^{SI}(t) + q_{02}(t) © B_2^{SI}(t) + q_{03}(t) © B_3^{SI}(t) + q_{04}(t) © B_4^{SI}(t)$$
$$B_1^{SI}(t) = q_{10}(t) © B_0^{SI}(t) + q_{11}^{(5)}(t) © B_1^{SI}(t) + q_{12}^{(6)}(t) © B_2^{SI}(t) + q_{13}^{(7)}(t) © B_3^{SI}(t) + q_{14}^{(8)}(t) © B_4^{SI}(t)$$
$$B_2^{SI}(t) = q_{29}(t) © B_9^{SI}(t)$$
$$B_3^{SI}(t) = q_{30}(t) © B_0^{SI}(t)$$
$$B_4^{SI}(t) = q_{40}(t) © B_0^{SI}(t)$$
$$B_9^{SI}(t) = Z_9(t) + q_{9,10}(t) © B_{10}^{SI}(t) + q_{90}(t) © B_0^{SI}(t)$$
$$B_{10}^{SI}(t) = q_{10,0}(t) © B_0^{SI}(t)$$

(52-58)

where,

$$Z_9(t) = e^{-(\nu_s+\nu_s)t^p}$$

Taking the Laplace transform of Equations (52-58) and solving for $B_0^{SI*}(s)$ (omitting the argument's' for brevity), we get

$$B_0^{SI*}(s) = \frac{N_5(s)}{D_2(s)} = \frac{Z_9^* q_{29}^* [q_{02}^*(1-q_{11}^{(5)*}) + q_{01}^* q_{12}^{(6)*}]}{[(1-q_{11}^{(5)*})(1-q_{02}^* Y - q_{03}^* q_{30}^* - q_{04}^* q_{40}^*)] - q_{01}^*[q_{10}^* + Y q_{12}^{(6)*} + q_{30}^* q_{13}^{(7)*} + q_{40}^* q_{14}^{(8)*}]} \quad (59)$$

where, $Y = \{q_{29}^* q_{9,10}^* q_{10,0}^* + q_{29}^* q_{90}^*\}$

(59)

Taking the inverse Laplace Transform of (59), we get the probability that the repairman is busy in the inspection of the Slicing machine(S) at epoch t after its repair, given that the system starts from the state S_0.

In the long run, the fraction of time for which the system is under inspection, starting from state S_0, is given by

$$B_0^{SI} = \lim_{t \to \infty} B_0^{SI}(t) = \lim_{s \to 0} s\, B_0^{SI*}(s) = \frac{N_5}{D_2} \quad (60)$$

where,

$$N_5 = [(1-p_{11}^{(5)})p_{02} + p_{01}p_{12}^{(6)}]p_{29}\psi_9 \quad (61)$$

and D_2 is given by (28).

The expected busy period of the repairman in the inspection of Slicing Machine (S) is

$$\mu_b^{SI}(t) = \int_0^t B_0^{SI}(u)\,du \quad \text{so that} \quad \mu_b^{SI*}(s) = \frac{B_0^{SI*}(s)}{s} \quad (62)$$

(iv) The expected busy period of the repairman in post repair of Slicing machine (S)

Let us define $B_i^{SPr}(t)$ as the probability that the repairman is busy in the post repair after the inspection of the repaired Slicing Machine (S) at epoch t when the system starts from the state $S_i \in E$. By simple probabilistic arguments, we have the following recursive relations in $B_i^{SPr}(t)$'s:

$$B_0^{SPr}(t) = q_{01}(t) © B_1^{SPr}(t) + q_{02}(t) © B_2^{SPr}(t) + q_{03}(t) © B_3^{SPr}(t) + q_{04}(t) © B_4^{SPr}(t)$$
$$B_1^{SPr}(t) = q_{10}(t) © B_0^{SPr}(t) + q_{11}^{(5)}(t) © B_1^{SPr}(t) + q_{12}^{(6)}(t) © B_2^{SPr}(t) + q_{13}^{(7)}(t) © B_3^{SPr}(t) + q_{14}^{(8)}(t) © B_4^{SPr}(t)$$
$$B_2^{SPr}(t) = q_{29}(t) © B_9^{SPr}(t)$$
$$B_3^{SPr}(t) = q_{30}(t) © B_0^{SPr}(t)$$
$$B_4^{SPr}(t) = q_{40}(t) © B_0^{SPr}(t)$$
$$B_9^{SPr}(t) = q_{9,10}(t) © B_{10}^{SPr}(t) + q_{90}(t) © B_0^{SPr}(t)$$
$$B_{10}^{SPr}(t) = Z_{10}(t) + q_{10,0}(t) © B_0^{SPr}(t)$$

(63-69)

where,

$$Z_{10}(t)=e^{-\lambda_s t^p}$$

Taking the Laplace transform of (63-69) and solving for $B_0^{SPr*}(s)$
(omitting the argument's' for brevity), we get

$$B_0^{SPr*}(s)=\frac{N_6(s)}{D_2(s)}=\frac{Z_{10}^{*}q_{29}^{*}q_{9,10}^{*}[q_{02}^{*}(1-q_{11}^{(5)*})+q_{01}^{*}q_{12}^{(6)*}]}{[(1-q_{11}^{(5)*})(1-q_{02}^{*}Y-q_{03}^{*}q_{30}^{*}-q_{04}^{*}q_{40}^{*})]-q_{01}^{*}[q_{10}^{*}+Yq_{12}^{(6)*}+q_{30}^{*}q_{13}^{(7)*}+q_{40}^{*}q_{14}^{(8)*}]} \quad (70)$$

where, $Y=\{q_{29}^{*}q_{9,10}^{*}q_{10,0}^{*}+q_{29}^{*}q_{90}^{*}\}$

Taking the inverse Laplace Transform of (70), we get the probability that the repairman is busy in the post repair after the inspection of the repaired Slicing Machine (S) at epoch t given that the system starts from the state S_0. In the long run, the fraction of time for which the system is under post repair, starting from state S_0, is given by

$$B_0^{SPr} = \lim_{t\to\infty} B_0^{SPr}(t) = \lim_{s\to 0} s\,B_0^{SPr*}(s) = \frac{N_6}{D_2} \quad (71)$$

where,

$$N_6 =[(1-p_{11}^{(5)})p_{02}+p_{01}p_{12}^{(6)}]p_{29}p_{9,10}\psi_{10} \quad (72)$$

and D_2 given by (28).

The expected busy period of the repairman in post repair of Slicing Machine (S) is

$$\mu_b^{SPr}(t)=\int_0^t B_0^{SPr}(u)\,du \quad \text{so that} \quad \mu_b^{SPr*}(s) = \frac{B_0^{SPr*}(s)}{s} \quad (73)$$

(v) The expected busy period of the repairman in the repair of the Coloring machine (C)

Let us define $B_i^C(t)$ as the probability that the repairman is busy in the repair of a failed Coloring machine (C) at epoch t when the system starts from the state $S_i \in E$. By simple probabilistic arguments, we have the following recursive relations among $B_i^C(t)$'s:

$$B_0^C(t) = q_{01}(t) © B_1^C(t) + q_{02}(t) © B_2^C(t) + q_{03}(t) © B_3^C(t) + q_{04}(t) © B_4^C(t)$$

$$B_1^C(t) = q_{10}(t) © B_0^C(t)+ q_{11}^{(5)}(t)© B_1^C(t)+ q_{12}^{(6)}(t)© B_2^C(t)+ q_{13}^{(7)}(t)© B_3^C(t)+ q_{14}^{(8)}(t) © B_4^C(t)$$

$$B_2^C(t) = q_{29}(t) © B_9^C(t)$$

$$B_3^C(t) = Z_3(t) + q_{30}(t) © B_0^C(t)$$

$$B_4^C(t) = q_{40}(t) © B_0^C(t)$$

$$B_9^C(t) = q_{9,10}(t) © B_{10}^C(t) + q_{90}(t) © B_0^C(t)$$

$$B_{10}^C(t) =q_{10,0}(t) © B_0^C(t)$$

(74-80)

where, $Z_3(t)=e^{-\beta_c t^p}$.

Taking the Laplace transform of (74-80) and solving for $B_0^{C*}(s)$
(omitting the argument's' for brevity), we get

$$B_0^{C*}(s)=\frac{N_7(s)}{D_2(s)}=\frac{Z_3^{*}[q_{03}^{*}(1-q_{11}^{(5)*})+q_{01}^{*}q_{13}^{(7)*}]}{[(1-q_{11}^{(5)*})(1-q_{02}^{*}Y-q_{03}^{*}q_{30}^{*}-q_{04}^{*}q_{40}^{*})]-q_{01}^{*}[q_{10}^{*}+Yq_{12}^{(6)*}+q_{30}^{*}q_{13}^{(7)*}+q_{40}^{*}q_{14}^{(8)*}]} \quad (81)$$

where, $Y=\{q_{29}^{*}q_{9,10}^{*}q_{10,0}^{*}+q_{29}^{*}q_{90}^{*}\}$

Taking the inverse Laplace Transform of (81), we get the probability that the repairman is busy in the repair of a failed Coloring machine (C) at epoch t given that the system starts from the state S_0. In the long run, the fraction of time for which the system is under repair, starting from the state S_0, is given by

$$B_0^C = \lim_{t\to\infty} B_0^C(t) = \lim_{s\to 0} s\, B_0^{C*}(s) = \frac{N_7}{D_2} \quad (82)$$

where,

$$N_7 = [(1-p_{11}^{(5)})p_{03} + p_{01}p_{13}^{(7)}]\psi_3 \quad (83)$$

and D_2 is given by (28).

The expected busy period of the repairman in the repair of Coloring machine (C) is

$$\mu_b^C(t) = \int_0^t B_0^C(u)\,du \text{ so that } \mu_b^{C*}(s) = \frac{B_0^{C*}(s)}{s} \quad (84)$$

(vi)The expected busy period of the repairman in repair of Frying & Salting machine (F)

Let us define $B_i^F(t)$ as the probability that the repairman is busy in the repair of a failed Frying and Salting machine (F) at epoch t when the system starts from the state $S_i \in E$. By simple probabilistic arguments, we have the following recursive relations among $B_i^F(t)$'s:

$$B_0^F(t) = q_{01}(t) © B_1^F(t) + q_{02}(t) © B_2^F(t) + q_{03}(t) © B_3^F(t) + q_{04}(t) © B_4^F(t)$$
$$B_1^F(t) = q_{10}(t) © B_0^F(t) + q_{11}^{(5)}(t) © B_1^F(t) + q_{12}^{(6)}(t) © B_2^F(t) + q_{13}^{(7)}(t) © B_3^F(t) + q_{14}^{(8)}(t) © B_4^F(t)$$
$$B_2^F(t) = q_{29}(t) © B_9^F(t)$$
$$B_3^F(t) = q_{30}(t) © B_0^F(t)$$
$$B_4^F(t) = Z_4(t) + q_{40}(t) © B_0^F(t)$$
$$B_9^F(t) = q_{9,10}(t) © B_{10}^F(t) + q_{90}(t) © B_0^F(t)$$
$$B_{10}^F(t) = q_{10,0}(t) © B_0^F(t)$$

(85-91)

where,

$$Z_4(t) = e^{-\beta_f t^p}$$

Taking the Laplace transform of (85-91) and solving for $B_0^{F*}(s)$
(omitting the argument's' for brevity), we get

$$B_0^{F*}(s) = \frac{N_8(s)}{D_2(s)} = \frac{Z_4^*[q_{04}^*(1-q_{11}^{(5)*}) + q_{01}^* q_{14}^{(8)*}]}{[(1-q_{11}^{(5)*})(1-q_{02}^* Y - q_{03}^* q_{30}^* - q_{04}^* q_{40}^*)] - q_{01}^*[q_{10}^* + Y q_{12}^{(6)*} + q_{30}^* q_{13}^{(7)*} + q_{40}^* q_{14}^{(8)*}]} \quad (92)$$

$$\text{where, } Y = \{q_{29}^* q_{9,10}^* q_{10,0}^* + q_{29}^* q_{90}^*\}$$

Taking the inverse Laplace Transform of (92), we get the probability that the repairman is busy in the repair of a failed Frying and Salting Machine at epoch t given that the system starts from the state S_0. In the long run, fraction of time for which the system is under repair, starting from the state S_0, is given by

$$B_0^F = \lim_{t\to\infty} B_0^F(t) = \lim_{s\to 0} s\, B_0^{F*}(s) = \frac{N_8}{D_2} \quad (93)$$

where,

$$N_8 = [(1-p_{11}^{(5)})p_{04} + p_{01}p_{14}^{(8)}]\psi_4 \quad (94)$$

and D_2 is the same as given in (28).

The expected busy period of the repairman in the repair of the Frying and Salting machine (F) is

$$\mu_b^F(t)=\int_0^t B_0^F(u)\,du \quad \text{so that} \quad \mu_b^{F*}(s)=\frac{B_0^{F*}(s)}{s} \tag{95}$$

5.4 Profit Function Analysis

Let us define

K_0 = revenue (in Rs.) per-unit up time of the system.

K_1 = cost (in Rs.) per unit time when the repairman is busy in the repair of the Destoning and Peeling machine

K_2 = cost (in Rs.) per unit time when the repairman is busy in the repair of the Slicing machine.

K_3 = cost (in Rs.) per unit time when the repairman is busy in the repair of the Coloring machine.

K_4 = cost (in Rs.) per unit time when the repairman is busy in the repair of the Frying & Salting.

K_5 = cost (in Rs.) per unit time when the repairman is busy in the inspection of the Slicing machine.

K_6 = cost (in Rs.) per unit time when the repairman is busy in the post repair of the Slicing machine after inspection.

Then, the expected total profit incurred in time interval (0, t) is

P (t) = Expected total revenue in (0, t) – Expected total cost of repair in (0, t) - Expected total cost of inspection in (0, t) - Expected total cost of post repair in (0, t)

$$= K_0\mu_{up}(t) - K_1\mu_b^D(t)-K_2\mu_b^S(t)-K_3\mu_b^C(t)-K_4\mu_b^F(t)-K_5\mu_b^{SI}(t)-K_6\mu_b^{SPr}(t) \tag{96}$$

The expected total profit per-unit time in steady-state is given by

$$P = K_0A_0-K_1B_0^D-K_2B_0^S-K_3B_0^C-K_4B_0^F-K_5B_0^{SI}-K_6B_0^{SPr} \tag{97}$$

where $A_0, B_0^D, B_0^S, B_0^C, B_0^F B_0^{SI}, B_0^{SPr}$ are given in (27), (38), (49), (82),(93), (60) and (71), respectively.

6. ESTIMATION OF PARAMETERS, MTSF AND PROFIT FUNCTION

6.1 Classical Estimation

6.1.1 ML Estimation

Suppose that the failure, repair, inspection and post repair times of units of system are independently distributed as Weibull with failure rates h_d (.), h_s (.),h_c (.), h_f (.),repair rates J_d(.), J_s(.),J_c(.),J_f(.),inspection rate l_s(.) and post repair rate x_s(.) as defined in Section 3. Let

$$\underset{\sim}{X}_1=(x_{11},x_{12},\ldots\ldots,x_{1n_1}),\ \underset{\sim}{X}_2=(x_{21},x_{22},\ldots\ldots,x_{2n_2}),\ \underset{\sim}{X}_3=(x_{31},x_{32},\ldots\ldots,x_{3n_3}),\ \underset{\sim}{X}_4=(x_{41},x_{42},\ldots\ldots,x_{4n_4}),$$

$$\underset{\sim}{X}_5=(x_{51},x_{52},\ldots\ldots,x_{5n_5}),\ \underset{\sim}{X}_6=(x_{61},x_{62},\ldots\ldots,x_{6n_6}),\ \underset{\sim}{X}_7=(x_{71},x_{72},\ldots\ldots,x_{7n_7}),$$

$$\underset{\sim}{X}_8=(x_{81},x_{82},\ldots\ldots,x_{8n_8}),\ \underset{\sim}{X}_9=(x_{91},x_{92},\ldots\ldots,x_{9n_9}),\ \underset{\sim}{X}_{10}=(x_{101},x_{102},\ldots\ldots,x_{10n_{10}})$$

be ten independent random samples of size n_i (i=1,2,3,4,5,6,7,8,9,10) drawn from Weibull distribution with failure rates h_d (.), h_s (.),h_c (.), h_f (.),repair rates J_d(.), J_s(.),J_c(.),J_f(.),inspection rate l_s(.) and post repair rate x_s(.) respectively.

The likelihood function of the combined sample is

$$\begin{aligned} L= L\Big(&\underset{\sim}{X}_1,\underset{\sim}{X}_2,\underset{\sim}{X}_3,\underset{\sim}{X}_4,\underset{\sim}{X}_5,\underset{\sim}{X}_6,\underset{\sim}{X}_7,\underset{\sim}{X}_8,\underset{\sim}{X}_9,\underset{\sim}{X}_{10}\Big|\alpha_d,\ \alpha_s,\ \alpha_c,\ \alpha_f,\ \beta_d,\beta_s,\ \beta_c,\ \beta_f,\nu_s,\lambda_s\Big) \\ &= \alpha_d^{\,n_1}\alpha_s^{\,n_2}\alpha_c^{\,n_3}\alpha_f^{\,n_4}\beta_d^{\,n_5}\beta_s^{\,n_6}\beta_c^{\,n_7}\beta_f^{\,n_8}\nu_s^{\,n_9}\lambda_s^{\,n_{10}}\,p^{n_1+n_2+n_3+n_4+n_5+n_6+n_7+n_8+n_9+n_{10}} \\ &\quad \times Z_1 Z_2 Z_3 Z_4 Z_5 Z_6 Z_7 Z_8 Z_9 Z_{10}\ e^{-(\alpha_d W_1+\alpha_s W_2+\alpha_c W_3+\alpha_f W_4+\beta_d W_5+\beta_s W_6+\beta_c W_7+\beta_f W_8+\nu_s W_9+\lambda_s W_{10s})} \end{aligned} \tag{98}$$

where

$$W_i = \sum_{j=1}^{n_i} x_{ij}^{\,p} \text{ and } Z_i = \prod_{j=1}^{n_i} x_{ij}^{\,p-1} \;; \quad i=1,2,3,4,5,6,7,8,9,10.$$

By using the usual maximization likelihood approach, the M.L. estimates (say $\hat{\alpha}_d, \hat{\alpha}_s, \hat{\alpha}_c, \hat{\alpha}_f, \hat{\beta}_d, \hat{\beta}_s, \hat{\beta}_c, \hat{\beta}_f, \hat{\nu}_s, \hat{\lambda}_s$) of the parameters ($\alpha_d, \alpha_s, \alpha_c, \alpha_f, \beta_d, \beta_s, \beta_c, \beta_f, \nu_s, \lambda_s$) are

$\hat{\alpha}_d = n_1/W_1$, $\hat{\alpha}_s = n_2/W_2$, $\hat{\alpha}_c = n_3/W_3$, $\hat{\alpha}_f = n_4/W_4$, $\hat{\beta}_d = n_5/W_5$,
$\hat{\beta}_s = n_6/W_6$, $\hat{\beta}_c = n_7/W_7$, $\hat{\beta}_f = n_8/W_8$, $\hat{\nu}_s = n_9/W_9$, $\hat{\lambda}_s = n_{10}/W_{10}$

Now, using the invariance property of ML estimates, the MLEs of the MTSF and profit function, say, $\hat{M}$ and $\hat{P}$ can be obtained. The asymptotic distribution of

$$\left(\hat{\alpha}_d-\alpha_d, \hat{\alpha}_s-\alpha_s, \hat{\alpha}_c-\alpha_c, \hat{\alpha}_f-\alpha_f, \hat{\beta}_d-\beta_d, \hat{\beta}_s-\beta_s, \hat{\beta}_c-\beta_c, \hat{\beta}_f-\beta_f, \hat{\nu}_s-\nu_s, \hat{\lambda}_s-\lambda_s\right)' \sim N_{10}(0, I^{-1}),$$

where I denotes the Fisher information matrix with diagonal elements.

$$I_{11}=\frac{n_1}{\alpha_d^2},\ I_{22}=\frac{n_2}{\alpha_s^2},\ I_{33}=\frac{n_3}{\alpha_c^2},\ I_{44}=\frac{n_4}{\alpha_f^2},\ I_{55}=\frac{n_5}{\beta_d^2},\ I_{66}=\frac{n_6}{\beta_s^2},\ I_{77}=\frac{n_7}{\beta_c^2},\ I_{88}=\frac{n_8}{\beta_f^2},\ I_{99}=\frac{n_9}{\nu_s^{\,2}},\ I_{1010}=\frac{n_{10}}{\lambda_s^{\,2}}$$

and the non diagonal elements are all zero.

Also, the asymptotic distribution of $\left(\hat{M}-M\right)$ is $N\left(0, A'I^{-1}A\right)$ and that of $\left(\hat{P}-P\right)$ is $N\left(0, B'I^{-1}B\right)$, where

$$A'=\left(\frac{\partial M}{\partial\alpha_d},\frac{\partial M}{\partial\alpha_s},\frac{\partial M}{\partial\alpha_c},\frac{\partial M}{\partial\alpha_f},\frac{\partial M}{\partial\beta_d},\frac{\partial M}{\partial\beta_s},\frac{\partial M}{\partial\beta_c},\frac{\partial M}{\partial\beta_f},\frac{\partial M}{\partial\nu_s},\frac{\partial M}{\partial\lambda_s}\right),$$

$$B'=\left(\frac{\partial P}{\partial\alpha_d},\frac{\partial P}{\partial\alpha_s},\frac{\partial P}{\partial\alpha_c},\frac{\partial P}{\partial\alpha_f},\frac{\partial P}{\partial\beta_d},\frac{\partial P}{\partial\beta_s},\frac{\partial P}{\partial\beta_c},\frac{\partial P}{\partial\beta_f},\frac{\partial P}{\partial\nu_s},\frac{\partial P}{\partial\lambda_s}\right)$$

6.1.2 Bayesian Estimation

In the Bayesian method of estimation parameters are taken as random variables. Suppose the parameters involved in the model are random variables having independent Gamma prior distributions as

$\alpha_d \sim \text{Gamma}(a_1, b_1)$

$\alpha_s \sim \text{Gamma}(a_2, b_2)$

$\alpha_c \sim \text{Gamma}(a_3, b_3)$

$\alpha_f \sim \text{Gamma}(a_4, b_4)$

$\beta_d \sim \text{Gamma}(a_5, b_5)$

$\beta_s \sim \text{Gamma}(a_6, b_6)$

$\beta_c \sim \text{Gamma}(a_7, b_7)$

$\beta_f \sim \text{Gamma}(a_8, b_8)$

$\nu_s \sim \text{Gamma}(a_9, b_9)$

$\lambda_s \sim \text{Gamma}(a_{10}, b_{10})$ (99-108)

Now, using the likelihood function in (98) and taking the prior distributions (99-108), the posterior distributions of these parameters, given the data, are obtained as follows:

$$\alpha_d \mid \underset{\sim}{X}_1 \sim Gamma(n_1+a_1, b_1+W_1)$$
$$\alpha_s \mid \underset{\sim}{X}_2 \sim Gamma(n_2+a_2, b_2+W_2)$$
$$\alpha_c \mid \underset{\sim}{X}_3 \sim Gamma(n_3+a_3, b_3+W_3)$$
$$\alpha_f \mid \underset{\sim}{X}_4 \sim Gamma(n_4+a_4, b_4+W_4)$$
$$\beta_d \mid \underset{\sim}{X}_5 \sim Gamma(n_5+a_5, b_5+W_5)$$
$$\beta_s \mid \underset{\sim}{X}_6 \sim Gamma(n_6+a_6, b_6+W_6)$$
$$\beta_c \mid \underset{\sim}{X}_7 \sim Gamma(n_7+a_7, b_7+W_7)$$
$$\beta_f \mid \underset{\sim}{X}_8 \sim Gamma(n_8+a_8, b_8+W_8)$$
$$\nu_s \mid \underset{\sim}{X}_9 \sim Gamma(n_9+a_9, b_9+W_9)$$
$$\lambda_s \mid \underset{\sim}{X}_{10} \sim Gamma(n_{10}+a_{10}, b_{10}+W_{10}) \qquad (109\text{-}118)$$

Now, under the squared error loss function, the Bayes estimates of the parameters are the means of the posterior distributions (109-118). For obtaining the Bayes estimates and width of the highest posterior density (HPD) intervals of the parameters, we generated observations from the above posteriors distributions. For obtaining Bayesian estimation and width of HPD intervals of MTSF and Profit function, we substituted the above draws directly into (16) and (97). Finally, the sample means of the respective draws are taken as the Bayes estimates of the parameter and reliability characteristics. For obtaining the width of HPD intervals, 'boa' package of R-software was used.
The highest posterior density (HPD) intervals of the parameters are obtained using the concept of Chen and Shao [3].

7. SIMULATION STUDY

A simulation study is carried out to examine the behavior of the estimates of parameters and reliability characteristics. .

Samples of sizes $n_1 = n_2 = n_3 = n_4 = n_5 = n_6 = n_7 = n_8 = n_9 = n_{10} = 120$ were drawn from the ten considered distributions by assuming various values of the parameters as given in Tables 1-6. All calculations were performed on R.2.14.2.

For a more concrete study of the system behavior, we also plot curves for MTSF and Profit function with respect to the failure rate α_d for different values of repair rate β_d=0.5,1.5,2.5 while the other parameters are kept fixed as

p=1.0, α_s=0.9, α_c=0.2, α_f=.6, β_s=1.5, β_c= 0.5, β_f=1.4, ν_s= 0.4, λ_s=1.5

K_0=3000, K_1=1200, K_2=400, K_3=200, K_4=100, K_5=2500 and K_6=1500.

8. CONCLUDING REMARKS

From the Figs. 2-4, it is observed that Mean time to system failure (MTSF) decreases as failure rate α_1 increases while it increases as repair rate β_2 increases. Same trends for profit function are also observed from Figs 5-7.

From the Tables 1-6, it is also observed that for fixed β_2 and varying α_1, Bayes estimates of MTSF and profit function perform well as compared to their MLEs as they have lesser posterior standard error (PSE) than that of MLEs. Also the width of the HPD intervals is more conservative as compared to the width of the confidence intervals, so here we conclude that Bayes estimates perform well as compared to their MLEs.

Table1: The values of MTSF for fixed β_d =.5 and varying α_d

α_d	TRUE. MTSF	ML. MTSF	SE	[LOWER LIMIT, UPPER LIMIT] OF C.I	WIDTH_C.I.	GAMMA -BAYES. MTSF	PSE	[LOWER LIMIT, UPPER LIMIT] OF HPD	WIDTH_ HPD INTERVAL
0.1	0.586797	0.63277	0.035	0.5641777 0.7013777	0.137	0.508	0.027	0.4582073 0.5633709	0.105
0.2	0.5829	0.632768	0.034	0.5661288 0.6994088	0.133	0.508	0.027	0.4578306 0.5631876	0.105
0.3	0.57731	0.631566	0.033	0.5668863 0.6962463	0.129	0.478	0.023	0.4543530 0.5453532	0.091
0.4	0.57034	0.623941	0.032	0.5612211 0.6866611	0.125	0.411	0.017	0.4542650 0.5199373	0.065
0.5	0.56239	0.609848	0.031	0.5490885 0.6710885	0.122	0.386	0.015	0.4308935 0.4889530	0.058
0.6	0.553745	0.5944	0.030	0.535639 0.653639	0.118	0.373	0.015	0.4121326 0.4661301	0.054
0.7	0.54462	0.5808	0.029	0.5240492 0.6380492	0.114	0.311	0.017	0.4112600 0.4639130	0.052
0.8	0.5352	0.57000	0.028	0.5151232 0.6251232	0.110	0.303	0.018	0.4010023 0.4500068	0.049
0.9	0.5256	0.5615	0.028	0.5066911 0.6166911	0.110	0.297	0.02	0.3900560 0.4360564	0.046
1.0	0.5159	0.5551	0.027	0.5022058 0.6082058	0.106	0.295	0.021	0.3700560 0.4150567	0.045

Table 2: The values of MTSF for fixed β_d =1.5 and varying α_d

α_d	TRUE. MTSF	ML. MTSF	SE	[LOWER LIMIT, UPPER LIMIT] OF C.I	WIDTH _C.I.	GAMMA -BAYES. MTSF	PSE	[LOWER LIMIT, UPPER LIMIT] OF HPD	WIDTH_ HPD INTERVAL
0.1	0.58721	0.632777	0.035	0.5641777 0.7013777	0.137	0.164	0.005	0.1548411 0.1727726	0.018
0.2	0.58441	0.632770	0.034	0.5661307 0.6994107	0.133	0.164	0.005	0.1544836 0.1725691	0.018
0.3	0.58015	0.631819	0.033	0.5651794 0.6984594	0.133	0.162	0.005	0.1543062 0.1723479	0.018
0.4	0.57471	0.62558	0.033	0.5609075 0.6902675	0.129	0.156	0.004	0.1536105 0.1696105	0.016

0.5	0.5683	0.6136	0.032	0.5508968 0.6763368	0.125	0.154	0.005	0.1532001 0.1682357	0.015
0.6	0.56122	0.6000	0.031	0.539333 0.660853	0.122	0.153	0.004	0.1520496 0.1670417	0.014
0.7	0.5535	0.5879	0.030	0.529117 0.646717	0.118	0.150	0.004	0.1515592 0.1655592	0.014
0.8	0.5454	0.5779	0.030	0.5191737 0.6367737	0.118	0.149	0.004	0.1510085 0.1646234	0.013
0.9	0.5370	0.5701	0.029	0.5133444 0.6270244	0.114	0.130	0.004	0.1503497 0.1613497	0.011
1.0	0.5284	0.5641	0.028	0.509303 0.619063	0.110	0.127	0.004	0.1492637 0.1582637	0.009

Table 3: The values of MTSF for fixed β_d =2.5 and varying α_d

α_d	TRUE. MTSF	ML. MTSF	SE	[LOWER LIMIT, UPPER LIMIT] OF C.I	WIDTH_C.I.	GAMMA-BAYES. MTSF	PSE	[LOWER LIMIT, UPPER LIMIT] OF HPD	WIDTH_ HPD INTERVAL
0.1	0.58744	0.63277	0.035	0.5641777 0.7013777	0.137	0.164	0.005	0.1549011 0.1729011	0.018
0.2	0.58524	0.63277	0.034	0.5661308 0.6994108	0.133	0.164	0.005	0.1544890 0.1724890	0.018
0.3	0.5818	0.6318	0.034	0.565188 0.698468	0.133	0.162	0.005	0.1543543 0.1723543	0.018
0.4	0.5773	0.6256	0.033	0.5609651 0.6903251	0.129	0.156	0.004	0.1537713 0.1707713	0.017
0.5	0.5720	0.6137	0.033	0.5490725 0.6784325	0.129	0.153	0.004	0.1532344 0.1692344	0.016
0.6	0.5660	0.600	0.032	0.5375809 0.6630209	0.125	0.152	0.004	0.1521287 0.1671287	0.015
0.7	0.5594	0.58817	0.031	0.5274191 0.6489391	0.122	0.150	0.004	0.1517029 0.1657038	0.014
0.8	0.5523	0.5782	0.031	0.517514 0.639034	0.122	0.148	0.004	0.1512378 0.1652378	0.014
0.9	0.5449	0.5705	0.03	0.5117112 0.6293112	0.118	0.147	0.004	0.1523920 0.1633920	0.011
1.0	0.5372	0.5645	0.029	0.5076885 0.6213685	0.114	0.140	0.004	0.1522908 0.1612908	0.009

Table 4: The values of PROFIT for fixed β_d =.5 and varying α_d

α_d	TRUE. PROFIT	ML. PROFIT	SE	[LOWER LIMIT, UPPER LIMIT] OF C.I	WIDTH _C.I.	GAMMA -BAYES. PROFIT	PSE	[LOWER LIMIT, UPPER LIMIT] OF HPD	WIDTH_ HPD INTERV AL
0.1	213.7174	320.7097	50.272	222.1766 419.2428	197.066	222.33	40.6	144.2740 303.0659	158.792
0.2	193.3077	319.015	47.28	221.3462 406.6842	185.338	213.711	39.512	137.9324 292.2892	154.357
0.3	175.2433	301.0575	44.651	213.5416 388.5735	175.032	144.992	30.891	84.64397 205.28177	120.638
0.4	159.2118	268.1629	42.32	185.2157 351.1101	165.894	87.776	23.484	42.00488 134.27635	92.271
0.5	144.9457	236.9921	40.233	158.1354 315.8487	157.713	73.109	21.457	30.98141 115.34771	84.366
0.6	132.2163	213.4774	38.351	138.3094 288.6454	150.336	72.037	20.009	27.7681 111.2971	83.529
0.7	120.8277	196.8815	36.641	125.0652 268.6979	143.633	70.76	19.482	23.99323 106.25723	82.264
0.8	110.6121	185.2721	35.079	116.5173 254.0270	137.51	67.573	17.193	19.57317 98.52017	78.947
0.9	101.4254	177.0665	33.644	111.1243 243.0088	131.884	64.405	16.868	16.76429 92.16729	75.403
1.0	93.14342	171.1767	32.32	107.8295 234.5239	126.694	61.354	13.402	12.86457 87.27757	74.413

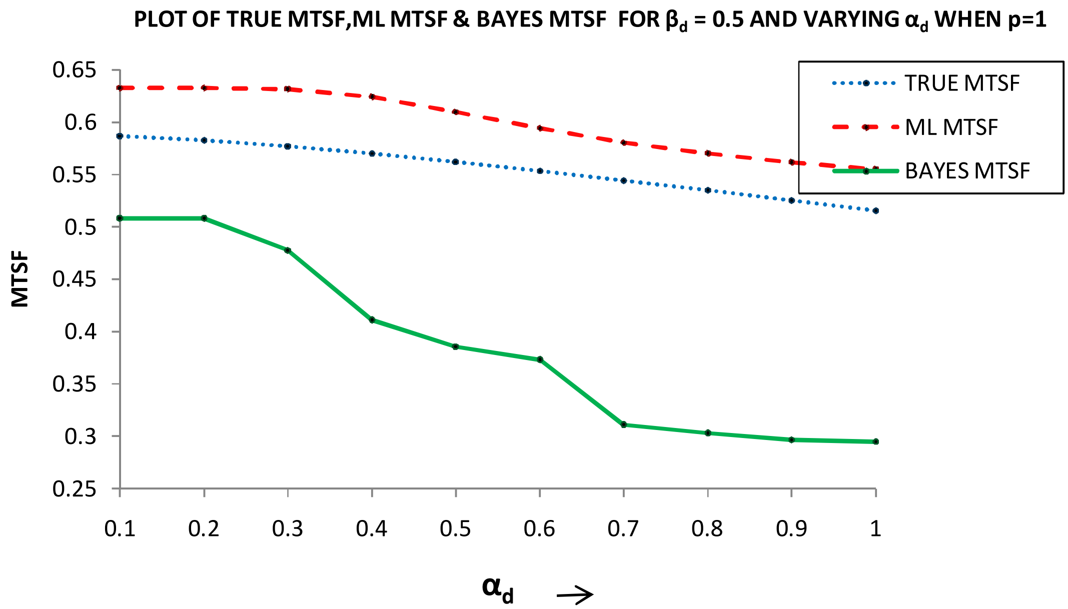
Table 5: The values of Profit for fixed β_d =1.5 and varying α_d

α_d	TRUE. PROFIT	ML. PROFIT	SE	[LOWER LIMIT, UPPER LIMIT] OF C.I	WIDTH_ C.I.	GAMMA-BAYES. PROFIT	PSE	[LOWER LIMIT, UPPER LIMIT] OF HPD	WIDTH_ HPD INTERV AL
0.1	214.4173	321.9047	51.378	222.1790 419.2690	197.090	223.678	40.90	144.3756 304.2726	159.897
0.2	194.8977	320..021	47.95	221.57862 408.0092	186.663	214.119	39.989	138.2920 293.581`	155.289
0.3	175.5783	303.0678	45.678	214.3853 391.3976	177.012	146.338	31.899	86.25087 208.70687	122.456
0.4	161.2118	270.2863	43.567	187.1520 353.541	166.389	88.103	25.678	43.03044 136.79744	93.767
0.5	146.4987	237.9921	41.567	160.0489 317.1429	157.094	74.109	21.957	31.00278 117.02078	86.018
0.6	134.6783	215.7744	38.567	142.2673 293.9313	151.664	73.037	20.789	29.02139 116.30839	87.287
0.7	122.209	197.9941	36.989	126.0987 270.2167	144.118	71.76	19.987	24.38278 107.54778	83.165
0.8	114.5478	185.2744	37.156	117.1256 254.9386	137.813	68.573	18.287	21.27384 100.30784	79.034

0.9	103.8764	179.733	35.249	113.1567 246.2747	133.118	64.789	17.478	19.28104 96.39004	77.109
1.0	95.4672	173.1278	33.409	109.4586 237.6626	128.204	62.678	14.402	15.16781 90.38881	75.221

Table 6: The values of PROFIT for fixed β_d =2.5 and varying α_d

α_d	TRUE. PROFIT	ML. PROFIT	SE	[LOWER LIMIT, UPPER LIMIT] OF C.I	WIDTH _C.I.	GAMMA -BAYES. PROFIT	PSE	[LOWER LIMIT, UPPER LIMIT] OF HPD	WIDTH_ HPD INTERVAL
0.1	214.989	323.4098	51.990	223.3726 421.8556	198.483	223.450	41.90	145.2019 305.6109	160.409
0.2	195.387	321.173	48.934	222.1835 409.3025	187.119	217.578	40.02	139.07780 297.0148	157.937
0.3	177.281	304.134	46.598	215.0947 394.6837	179.589	149.002	33.389	88.56289 217.69089	129.128
0.4	162.114	273.2657	45.134	189.1429 358.2519	169.109	90.220	26.908	44.42046 141.76546	97.345
0.5	148.567	240.1260	42.423	161..0389 319.5169	158.478	77.567	23.506	33.27897 122.37197	89.093
0.6	135.112	217.3387	39.903	144.7210 300.410	155.689	74.045	21.309	30.00965 118.35465	88.345
0.7	123.406	200.0012	37.108	128.0927 275.2107	147.118	73.290	20.005	25.10326 109.30126	84.198
0.8	117.345	186.1674	39.249	118.2768 258.4858	140.209	69.590	19.012	22.57292 101.65092	79.078
0.9	106.467	181.244	39.118	114.6629 249.6649	135.002	65.630	18.408	20.84298 98.07098	77.228
1.0	97.789	176.2345	34.012	110.1830 239.292	129.109	63.678	16.334	18.18272 94.28072	76.098


PLOT OF TRUE MTSF,ML MTSF & BAYES MTSF FOR βd = 0.5 AND VARYING αd WHEN p=1
TRUE MTSF
ML MTSF
BAYES MTSF
MTSF
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.1
0.2
0.3
0.4
0.5
0.6
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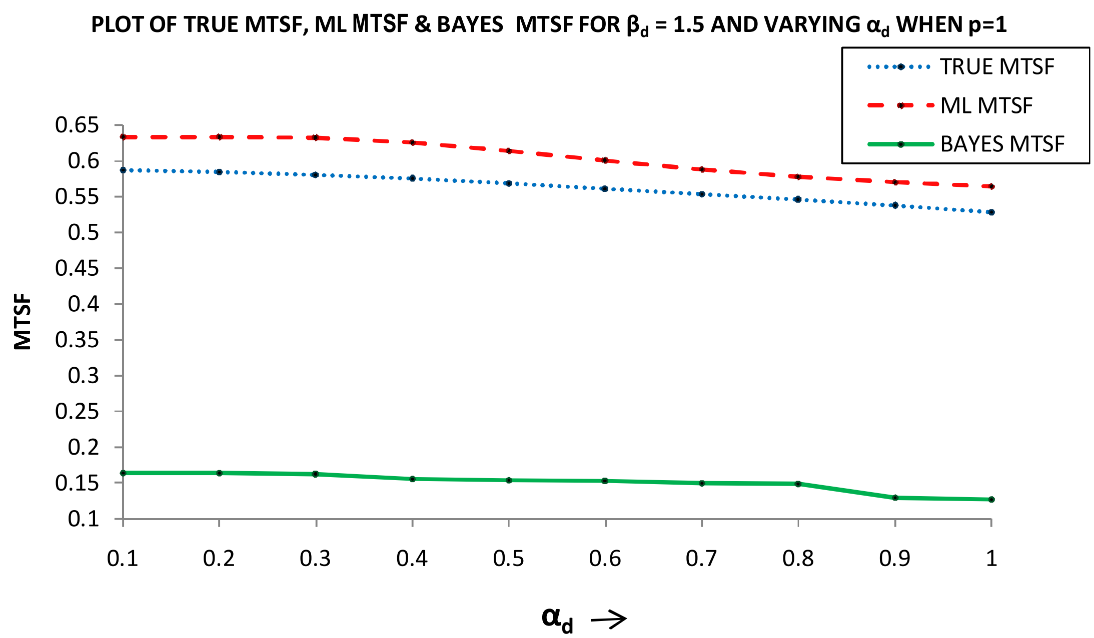
0.8
0.9
1
αd →

Fig. 2:


PLOT OF TRUE MTSF, ML MTSF & BAYES MTSF FOR βd = 1.5 AND VARYING αd WHEN p=1
TRUE MTSF
ML MTSF
BAYES MTSF
MTSF
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
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0.5
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0.7
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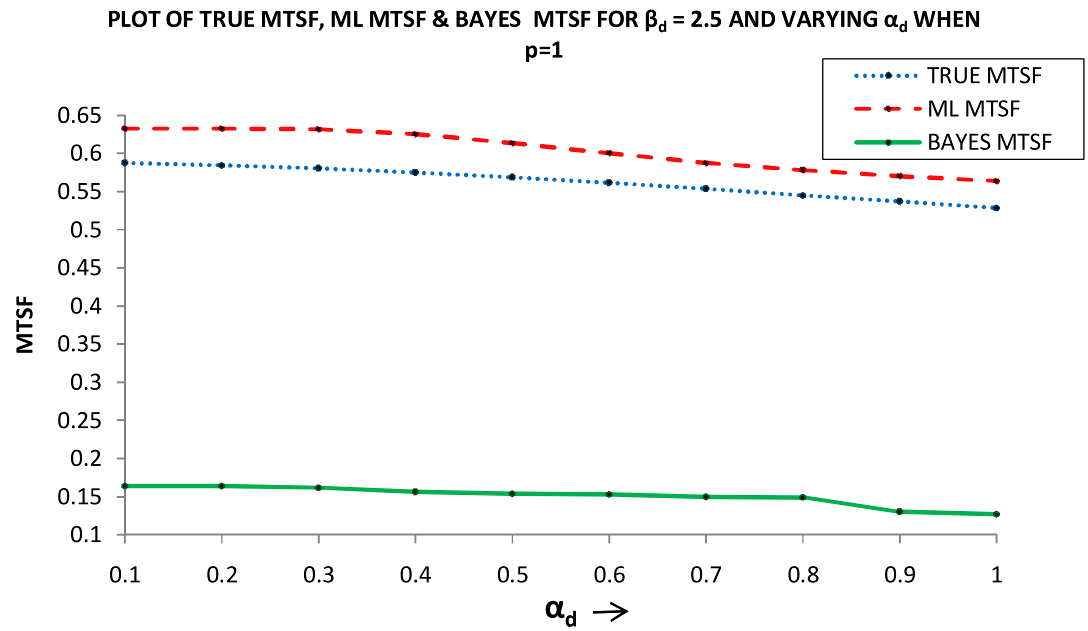
0.9
1
αd →

Fig. 3:


PLOT OF TRUE MTSF, ML MTSF & BAYES MTSF FOR βd = 2.5 AND VARYING αd WHEN p=1
TRUE MTSF
ML MTSF
BAYES MTSF
MTSF
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
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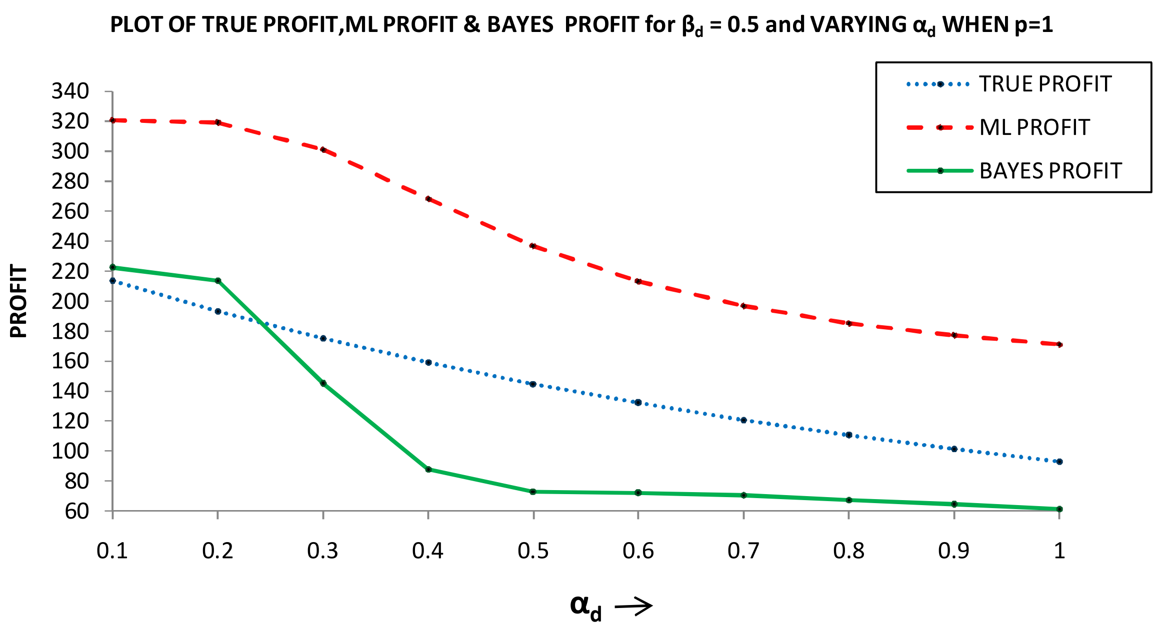
0.9
1
αd →

Fig. 4:


PLOT OF TRUE PROFIT,ML PROFIT & BAYES PROFIT for βd = 0.5 and VARYING αd WHEN p=1
TRUE PROFIT
ML PROFIT
BAYES PROFIT
PROFIT
340
320
300
280
260
240
220
200
180
160
140
120
100
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0.9
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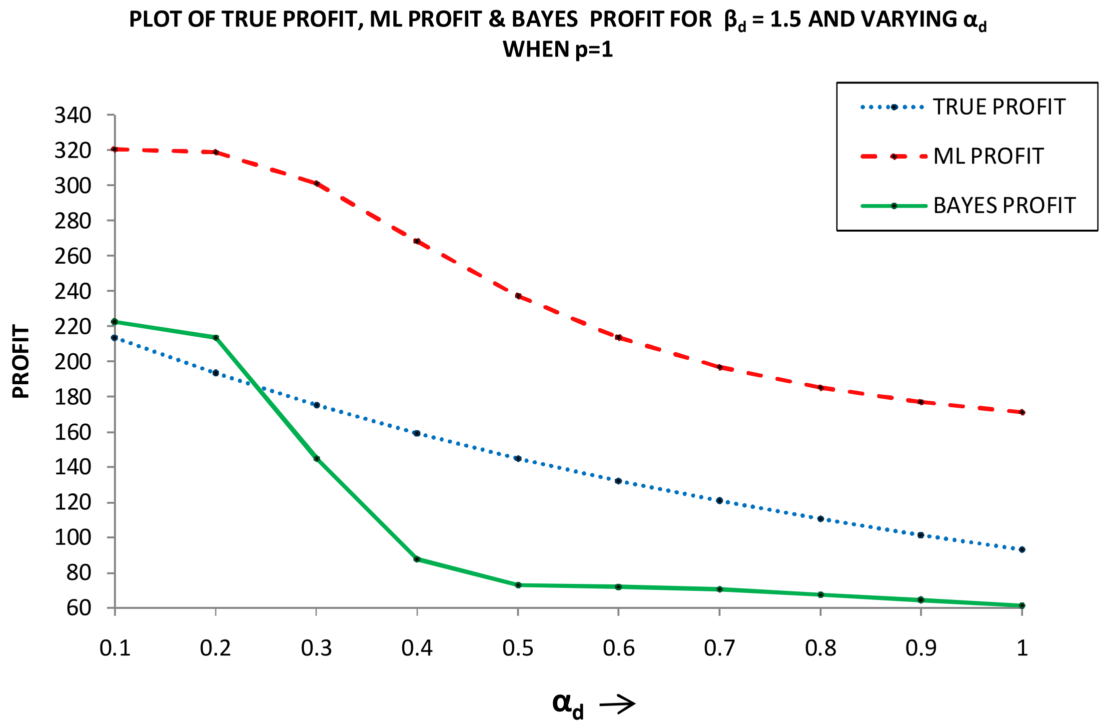
αd →

Fig. 5:


PLOT OF TRUE PROFIT, ML PROFIT & BAYES PROFIT FOR β_d = 1.5 AND VARYING α_d WHEN p=1
TRUE PROFIT
ML PROFIT
BAYES PROFIT
PROFIT
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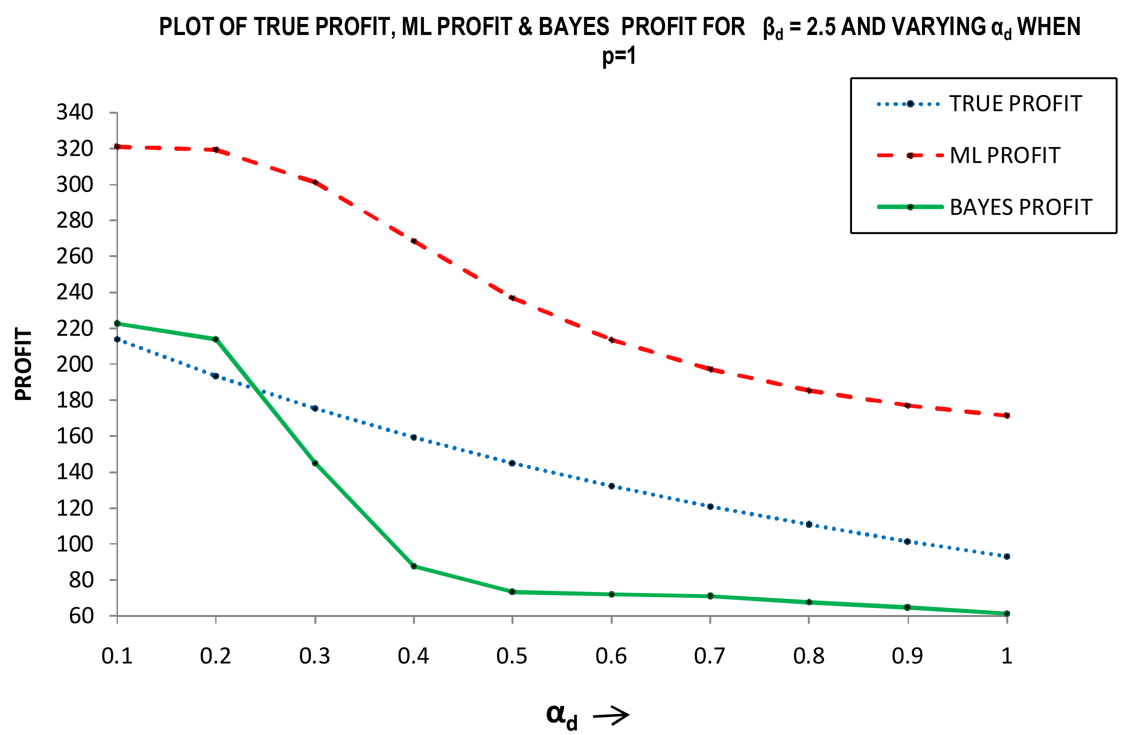
0.9
1
$\alpha_d \rightarrow$

Fig. 6:


PLOT OF TRUE PROFIT, ML PROFIT & BAYES PROFIT FOR β_d = 2.5 AND VARYING α_d WHEN p=1
TRUE PROFIT
ML PROFIT
BAYES PROFIT
PROFIT
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0.4
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0.6
0.7
0.8
0.9
1
$\alpha_d \rightarrow$

Fig. 7:

ACKNOWLEDGEMENT

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