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A STUDY ON THE EFFECTIVE EDGES IN THE CARTESIAN PRODUCT OF FUZZY GRAPHS

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Abstract

In this paper we discuss the number of effective edges in the Cartesian product of fuzzy graphs. Some basic theorems and results are obtained on effective edges. Based on the results we find the effective edge domination number for Cartesian product of complete fuzzy graphs.

Keywords: Fuzzy graph, complete fuzzy graph, Cartesian product, Domination number, Effective edge domination.
AMS classification: 05C72

1. INTRODUCTION

The concept of system modeling and analysis by means of linguistic variables was introduced by Zadeh [1]. He suitably chose the input and output variable as numerical by fuzzy sets. So, fuzzy sets or their membership functions provide an interface between the input and output numerical values. Fuzzy set approaches have several advantages over other intelligent modeling techniques such as neural networks, radical function networks, etc. The concept of fuzzy graph was first introduced by Kaufmann [2] from the fuzzy relation introduced by Zadeh [1]. Rosenfeld [10] introduced another elaborate definitions including the fuzzy vertex, the fuzzy edge, and the notion of fuzzy graphs and several fuzzy analogs of graph theoretic concepts such as paths, cycles and connectedness. The work on fuzzy graphs was also done by Mordeson and Peng Chang-Shyh [13], Debnath [15], and Yeh and Bang [16]. The concept of domination in graphs was introduced by in 1962 Ore [7]. The domination number and independent domination number are introduced by Haynes, Hedetniemi and Slater [9]. The concept of domination in the product of fuzzy graphs was introduced by Somasundaram [14]. We recall some basic definitions in fuzzy graphs and introduce some new definition and notations. For graph theoretic terminology we refer to Harary [11].

2. PRELIMINARIES

Definition 2.1
Fuzzy graph G(σ,μ) is pair of functions $V \to [0,1]$ and $\mu: V \times V \to [0,1]$ where for all $u, v \; in \; V$, we have $\mu(u,v) \le \sigma(u) \wedge \sigma(v)$.

Definition 2.2
The fuzzy graph $H(\tau,\rho)$ is called a fuzzy subgraph of G(σ,μ) if $\tau(u) \le \sigma(u)$ for all u in V and $\rho(u,v) \le \mu(u,v)$ for all $u, v \; in \; V$.

Definition 2.3
A fuzzy sub graph $H(\tau,\rho)$ is said to be a spanning sub graph of G (σ,μ) if $\tau(u) = \sigma(u)$ for all u in V. In this case the two graphs have the same fuzzy node set, they differ only in the arc weights.

Definition 2.4
Let G (σ,μ) be a fuzzy graph and τ be fuzzy subset of σ, that is, $\tau(u) \le \sigma(u)$ for all u in V. Then the fuzzy subgraph of G(σ,μ) induced by τ is the maximal fuzzy subgraph of G(σ,μ) that has the fuzzy node set τ. Evidently, this is just the fuzzy graph $H(\tau,\rho)$ where $\rho(u,v) = \tau(u) \wedge \tau(v) \wedge \mu(u,v)$ for all u, v in V.

Definition 2.5
The underlying crisp graph of a fuzzy graph G(σ,μ) is denoted by $G^* = (\sigma^*, \mu^*)$, where $\sigma^* = \{ u \in V \mid \sigma(u) > 0 \}$ and $\mu^* = \{ (u,v) \in V \times V \mid \mu(u,v) > 0 \}$.

Definition 2.6
A fuzzy graph G(σ,μ) is a strong fuzzy graph if $\mu(u,v) = \sigma(u) \wedge \sigma(v)$ for all $(u,v) \in \mu^*$ and is a complete fuzzy graph if $\mu(u,v) = \sigma(u) \wedge \sigma(v)$ for all $u, v \; in \; \sigma^*$. Two nodes u and v are said to be neighbours if $\mu(u,v) > 0$.

Definition 2.7
A fuzzy graph G = (σ, μ) is said to be Bipartite if the node set V can be partitioned into two non empty sets V_1 and V_2 such that $\mu(v_1, v_2) = 0$ if $v_1, v_2 \in V_1$ or $v_1, v_2 \in V_2$. Further if $\mu(v_1, v_2) > 0$ for all $v_1 \in V_1$ and $v_2 \in V_2$ then G is called complete bipartite graph and it is denoted by $K_{\sigma 1, \sigma 2}$ where σ_1 & σ_2 are respectively the restriction of σ to V_1 &V_2.

Definition 2.8
The complement of a fuzzy graph G(σ,μ) is a subgraph $\bar{G} = (\bar{\sigma}, \bar{\mu})$ where $\bar{\sigma} = \sigma$ and
$\bar{\mu}(u,v) = \sigma(u) \wedge \sigma(v) - \mu(u,v)$ for all $u, v \; in \; V$. A fuzzy graph is self complementary if $G = \bar{G}$.

Definition 2.9
The order p and size q of a fuzzy graph G(σ,μ) is defined as $p = \sum_{u \in V} \sigma(u)$ and $q = \sum_{(u,v) \in E} \mu(u,v)$.

Definition 2.10
The degree of the vertex u is defined as the sum of weight of arc incident at u, and is denoted by d(u).

Definition 2.11
An arc (u, v) of the fuzzy graph G (σ,μ) is called an effective edge if $\mu(u,v) = \sigma(u) \wedge \sigma(v)$ and effective edge neighborhood of u $\in V$ is $N_e(u) = \{v \in V: edge(u,v) \; is \; effective\}$.
$N_e[u] = N_e(u) \cup \{u\}$ is the closed neighbourhood of u. The minimum cardinality of effective neighbourhood $\delta_e(G) = \min\{|N_e(u)| \; u \in V(G)\}$.The Maximum cardinality of effective neighbourhood $\delta_e(G) = \max\{|N_e(u)| \; u \in V(G)\}$.

Definition 2.12
A path ρ in a fuzzy graph G (σ,μ) is a sequence of distinct nodes $v_0, v_1, v_2, \ldots v_n$ such that $\mu(v_{i-1}, v_i) > 0$ where $1 \le i \le n$ and n is called the length of P. The path ρ is called v_0-v_n path. Two vertices x,y in a fuzzy graph G(σ,μ) are said to be connected if there exists a x-y path in G(σ,μ). The strength of the path ρ is defined to be $\wedge_{i=i}^{n} \; \mu(v_{i-1}, v_i)$

Definition 2.13
If u and v are connected by means of length k, then $\mu^k(u,v) = \sup\{\mu(u,v_1) \wedge \mu(v_1,v_2) \dots \wedge \mu(v_{k-1},v_k) \mid u, v_1, v_2, \dots v$ in such path ρ }.

3. EFFECTIVE EDGES IN THE CARTESIAN PRODUCT OF FUZZY GRAPHS

Definition 3.1
Let G (σ,μ) be a fuzzy graph. Let u, v be two nodes of G (σ,μ). We say that u dominates v if edge (u, v) is an effective edge. A subset D of V is called a dominating set of G (σ,μ) if for every $v \in V - D$, there exists $u \in D$ such that u dominates v. A dominating set D is called a minimal dominating set if no proper subset of D is a dominating set. The minimum fuzzy cardinality taken over all dominating sets of a graph G is called the effective edge domination number and is denoted by $\gamma_E(G)$ and the corresponding dominating set is called the minimum effective edge dominating set. The number of elements in the minimum effective edge dominating set is denoted by n[$\gamma_E(G)$].

Definition 3.2
Let σ_i be a fuzzy subset of V_i and let μ_i be a fuzzy subset of X_i i = 1, 2. Define the fuzzy subsets $\sigma_1 X \sigma_2$ of V and $\mu_1 X \mu_2$ of X as follows:
$(\sigma_1 X \sigma_2)(u_1,u_2) = \min\{\sigma_1(u_1), \sigma_2(u_2)\}\ \forall (u_1,u_2) \in V$
$(\mu_1 X \mu_2)\big((u,u_2),(u,v_2)\big) = \min\{\sigma_1(u), \mu_2(u_2,v_2)\}$ $\forall u \in V_1 \ and\ \forall (u_2,v_2) \in X_2\ and$
$(\mu_1 X \mu_2)\big((u_1,w),(v_1,w)\big) = \min\{\sigma_2(w), \mu_1(u_1,v_1)\}$ $\forall w \in V_2\ and\ \forall (u_1,v_1) \in X_1$, where $V = V_1XV_2$.
Then the fuzzy graph G($\sigma_1 X \sigma_2$, $\mu_1 X \mu_2$) is said to be the Cartesian product of $G_1(\sigma_1,\mu_1)$ and $G_2(\sigma_2,\mu_2)$. We simply denote G($\sigma_1 X \sigma_2$, $\mu_1 X \mu_2$) by G_1XG_2 for our convenience.

Note 3.3

1. Throughout this paper we label the vertices of a fuzzy graph $G_1(\sigma_1,\mu_1)$ by $u_1, u_2, u_3 \dots \dots u_m$ and the vertices of a fuzzy graph $G_2(\sigma_2,\mu_2)$ by $v_1, v_2, v_3 \dots \dots v_n$. Then any vertex of G_1XG_2 be (u_i, v_j) i = 1, …, m; j = 1,…, n.
2. The number of edges in G_1XG_2 is $p_1q_2 + p_2q_1$ where p_1, p_2 are the number vertices in $G_1(\sigma_1,\mu_1)$ and $G_2(\sigma_2,\mu_2)$ respectively and q_1, q_2 are the the number of edges in $G_1(\sigma_1,\mu_1)$ and $G_2(\sigma_2,\mu_2)$ respectively.
3. If $G_1(\sigma_1,\mu_1)$ or $G_2(\sigma_2,\mu_2)$ are disconnected then so is G_1XG_2.

Definition 3.4
Let $G_1(\sigma_1,\mu_1)$ be a fuzzy graph with vertex set $u_1, u_2, u_3, \dots, u_m$ and $G_2(\sigma_2,\mu_2)$ be a fuzzy graph with vertex set $v_1, v_2, v_3, \dots, v_n$. The vertex u^* is called m-vertex if $\sigma(u^*) \le \sigma(u_i)\ \forall\ i = 1,\cdots,m$ and $\sigma(u^*) \le \sigma(v_j)\ \forall\ j = 1,\cdots, n$.

Note 3.5
The m-vertex u^* belongs either to $G_1(\sigma_1,\mu_1)$ or to $G_2(\sigma_2,\mu_2)$ or to both $G_1(\sigma_1,\mu_1)$ and $G_2(\sigma_2,\mu_2)$.

Theorem 3.4
Let $G_1(\sigma_1,\mu_1)$ and $G_2(\sigma_2,\mu_2)$ be two fuzzy graphs. Let $u^* \epsilon\ G_2(\sigma_2,\mu_2)$. If there exists an edge (u_i,u_{i+1}) in $G_1(\sigma_1,\mu_1)$ such that $\mu_1(u_i,u_{i+1}) \ge \sigma(u^*)$ then the edge connecting the points (u_i,u^*) and (u_{i+1},u^*) of G_1XG_2 is an effective edge.

Proof: By definition, $\mu[(u_i,u^*),(u_{i+1},u^*)] = \wedge[\sigma(u^*), \mu_1(u_i,u_{i+1})] = \sigma(u^*)$\{by hypothesis\}. Also by definition of u^*, $\sigma(u_i,u^*) = \sigma(u^*)$. Therefore $\mu[(u_i,u^*),(u_{i+1},u^*)] = \sigma(u^*) = \sigma(u_i,u^*)$. Hence the edge is effective.

Theorem 3.5
If a fuzzy graph $G_2(\sigma_2,\mu_2)$ has 'n' effective edges then G_1XG_2 has at least np_1 effective edges, where p_1 is the number vertices in $G_1(\sigma_1,\mu_1)$.

Proof: Let $x_1, x_2, \dots \dots, x_n$ be the n effective edges in $G_2(\sigma_2,\mu_2)$. Let $u_k \in G_1$. Let the end vertices of x_i be (v_i, v_{i+1}). Consider the edge connecting the points (u_k, v_i) and (u_k, v_{i+1}) of G_1XG_2 . $\mu[(u_k,v_i),(u_k,v_{i+1}) = \wedge[\sigma(u_k), \mu_2(v_i,v_{i+1})]$. Since (v_i,v_{i+1}) is an effective edge in $G_2(\sigma_2,\mu_2)$. $\mu_2(v_i,v_{i+1}) = \wedge[\sigma(v_i), \sigma(v_{i+1})]$.

Therefore

$$\mu[(u_k, v_i), (u_k, v_{i+1}) = \wedge [\sigma(u_k), \sigma(v_i), \sigma(v_{i+1})]. \quad \text{........(1)}$$

Case (i) If $\mu[(u_k, v_i), (u_k, v_{i+1})] = \sigma(u_k)$.Then $\sigma(u_k) \leq \sigma(v_i)$ and $\sigma(u_k) \leq \sigma(v_{i+1})$ by (1). Therefore, $\sigma(u_k, v_i) = \sigma(u_k) = \mu[(u_k, v_i), (u_k, v_{i+1})]$. Therefore, $\mu[(u_k, v_i), (u_k, v_{i+1})] = \sigma(u_k, v_i)$. Hence the edge is effective.

Case (ii) Suppose $\mu[(u_k, v_i), (u_k, v_{i+1})] = \sigma(v_i)$.Then by (1), $\sigma(v_i) \leq \sigma(u_k)$ and $\sigma(v_i) \leq \sigma(v_{i+1})$. Therefore, $\sigma(u_k, v_i) = \sigma(v_i)$, which implies $\mu[(u_k, v_i), (u_k, v_{i+1})] = \sigma(v_i) = \sigma(u_k, v_i)$. Hence, the edge is effective.

Case (iii) If $\mu[(u_k, v_i), (u_k, v_{i+1})] = \sigma(v_{i+1})$. Then by (1) $\sigma(v_{i+1}) \leq \sigma(v_i)$ and $\sigma(v_{i+1}) \leq \sigma(u_k)$. Therefore, $\sigma(u_k, v_{i+1}) = \sigma(v_{i+1}) = \mu[(u_k, v_i), (u_k, v_{i+1})]$. Hence the edge is effective. Therefore in all cases the edge connecting the points (u_k, v_i) and (u_k, v_{i+1}) of G_1XG_2 is effective. For each u_k *in* G_1 and edge x_i in G_2 there is an effective edge in G_1XG_2. Therefore the number of effective edges in G_1XG_2 is greater than or equal to np_1. □

Corollary 3.6

If $G_1(\sigma_1, \mu_1)$ has n effective edges then G_1XG_2 has atleast np_2 effective edges, where p_2 is the number vertices in $G_2(\sigma_2, \mu_2)$.

Notation 3.7

We denote the number of effective edges in $G_1(\sigma_1, \mu_1)$ by $n[E_{G_1}]$ and number of effective edges in $G_2(\sigma_2, \mu_2)$ by $n[E_{G_2}]$.

Corollary 3.8

$n[E_{G_1XG_2}] \geq p_1\, n[E_{G_2}] + p_2\, n[E_{G_1}]$.

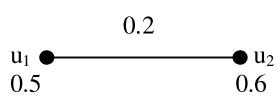
Corollary 3.9

If $G_1(\sigma_1, \mu_1)$ or $G_2(\sigma_2, \mu_2)$ has an effective edge then G_1XG_2 has an effective edge.

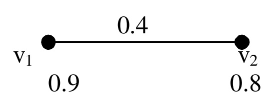
Remarks 3.10

1. There exists fuzzy graph G_1XG_2 which has no effective edge.

Examples:

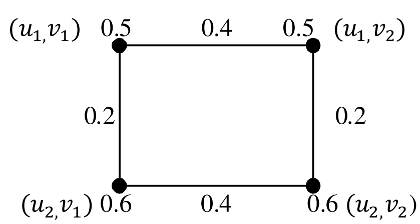


G_1

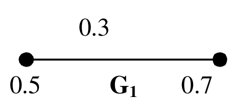


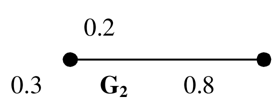
G_2

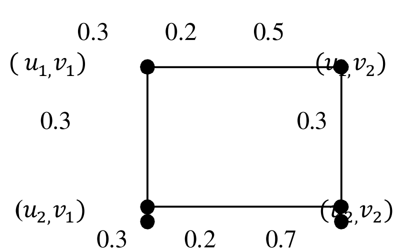
G_1XG_2



2. Consider the graph







G_1XG_2

Here G_1 and G_2 has no effective edge but G_1XG_2 has an effective edge.

Theorem 3.11
If $G_1(\sigma_1,\mu_1)$ and $G_2(\sigma_2,\mu_2)$ are complete fuzzy graphs then G_1XG_2 is a strong fuzzy graph.

Proof: Let $G_1(\sigma_1,\mu_1)$ have p_1 vertices and q_1 edges and let $G_2(\sigma_2,\mu_2)$ have p_2 vertices and q_2 edges. Therefore G_1XG_2 has $p_1\, q_2 + p_2 q_1$ edges. Since $G_1(\sigma_1,\mu_1)$ and $G_2(\sigma_2,\mu_2)$ are complete fuzzy graphs, we have $n[E_{G_1}] = q_1$ and $n[E_{G_2}] = q_2$. By corollary 3.8 $n[E_{G_1XG_2}] \geq p_1\, q_2 + p_2 q_1$. But G_1XG_2 has $p_1\, q_2 + p_2 q_1$ edges. Hence $n[E_{G_1XG_2}] = p_1\, q_2 + p_2 q_1$. Therefore, all the edges of G_1XG_2 are effective.

Definition 3.12 □
Let u_i, v_j be a vertex in G_1XG_2, define $N^c_{u_i,v_j} = \{u_k, v_k^{'} / u_k, v_k^{'}$ is not adjacent to $u_i, v_j\}$

Note 3.13
1. If a fuzzy graph $G_1(\sigma_1,\mu_1)$ has m vertices and $G_2(\sigma_2,\mu_2)$ has n vertices then $\left|N^c_{u_i,v_j}\right| \geq (m-1)(n-1)$.
2. If $G_1(\sigma_1,\mu_1)$ and $G_2(\sigma_2,\mu_2)$ are complete fuzzy graphs then $\left|N^c_{u_i,v_j}\right| = (m-1)(n-1)$.

Remark 3.14
1. Let $u_1, u_2, \ldots, u_m$ be the vertices in $G_1(\sigma_1,\mu_1)$ and $v_1, v_2, v_3, \ldots, v_n$ be the vertices in $G_2(\sigma_2,\mu_2)$. the vertices $u_1v_1, u_1v_2, \cdots, u_1v_n$ are called first row vertices of G_1XG_2.The vertices $u_2v_1, u_2v_2, \cdots, u_2v_n$ are called second row vertices of G_1XG_2 and so on.
2. We can view the vertices of G_1XG_2as follows
   $u_1v_1, u_1v_2, \cdots, u_1v_n$
   $u_2v_1, u_2v_2, \cdots, u_2v_n$
   $u_mv_1, u_mv_{2,} \cdots, u_mv_n$.
3. By the definition of G_1XG_2, there always exist vertices which are not adjacent in G_1XG_2. So G_1XG_2 can never be fuzzy complete.
4. If the two vertices x_i, x_k of G_1XG_2 are in the same row then we denote it by $x_i \mathbb{R} x_k$ and if they are in the same column then we denote it by $x_i \mathbb{C} x_k$.

Proposition 3.15
If $G_1(\sigma_1,\mu_1)$ and $G_2(\sigma_2,\mu_2)$ are complete fuzzy graphs and if $|V_1(G_1(\sigma_1,\mu_1)\ )| = m$, $|V_2(G_2(\sigma_2,\mu_2)\ )| = n$ then $n[\gamma_E(G_1XG_2)] \leq \wedge(m,n)$.

Proof: Assume m < n. Let $u_1, u_2, \ldots, u_m$ be the vertices in $G_1(\sigma_1,\mu_1)$ and $v_1, v_2, \ldots, v_n$ be the vertices in $G_2(\sigma_2,\mu_2)$. Let D = $\{u_1v_1, u_2v_2, \cdots, u_mv_m\}$. The vertex u_1v_1 dominates the first row vertices of G_1XG_2. The vertex u_2v_1 dominates the second row vertices of G_1XG_2. Likewise the vertex u_mv_m dominates the 'm'th row vertices of G_1XG_2.Therefore all the vertices of G_1XG_2 are dominated by D. Suppose n < m, then by the above argument , evidently $D_1 = \{u_1v_1, u_2v_2, \ldots, u_nv_n\}$ is a dominating set. Hence n$[\gamma_E(G_1XG_2)] \leq \wedge(m,n)$.

Corollary 3.16
If $G_1(\sigma_1,\mu_1)$ and $G_2(\sigma_2,\mu_2)$ are complete fuzzy graphs with $|V(G_1(\sigma_1,\mu_1)\ )| = m$,& $|V(G_2(\sigma_2,\mu_2)\ )| = n$ and if m < n then any 'm' distinct vertices of G_1XG_2 is a dominating set.

Theorem 3.17
Let $G_1(\sigma_1,\mu_1)$ and $G_2(\sigma_2,\mu_2)$ are complete fuzzy graphs and let $|V(G_1(\sigma_1,\mu_1)\ )| = m$, $|V(G_2(\sigma_2,\mu_2)\ )| = n$ then n$[\gamma_E(G_1XG_2)] = \wedge\{m,n\}$.

Proof: Without loss of generality assume m < n. By proposition 3.15, n$[\gamma_E(G)] \leq \wedge(m,n)$.
Suppose D = $\{x_1, x_2, \ldots, x_{m-1}\}$ is a dominating set. Since G_1XG_2 has 'm' rows of vertices, there exists a vertex $x \in V(G_1XG_2) - D$ such that $x\mathbb{R}x_i$| and $x\ \mathbb{C}x_i$| for all i = 1,2,...m-1. This implies that $x \in N^c(x_1) \cap N^c(x_2) \cap \ldots\ldots \cap N^c(x_{m-1})$.That is, there is no edge between x_i and x for all i = 1, 2,..., m-1. Hence D is not a dominating set. Therefore n $[\gamma_E(G_1XG_2)] = \wedge\{m,n\}$.

Result 3.18
Let $G_1(\sigma_1,\mu_1)$ and $G_2(\sigma_2,\mu_2)$ are complete fuzzy graphs with $|V(G_1(\sigma_1,\mu_1)\ )| = m$, $|V(G_2(\sigma_2,\mu_2)\ )| = n$. If m < n then $\gamma_E(G_1XG_2) = \min\{\sum_{i=1}^{m}\sigma(x_i)\ /x_i$ is a vertex of $G_1XG_2\}$.

Proof: Result follows from corollary 3.16

CONCLUSION

The concept of domination in graphs is very rich both in theoretical developments and applications. More than thirty domination parameters have been investigated by different authors. In this paper we have found the effective edge domination number for Cartesian product of complete fuzzy graphs. Also we have found the lower bound for the number of effective edges in the Cartesian product of fuzzy graphs.

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