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SKOLEM DIFFERENCE LUCAS MEAN LABELLING FOR SOME SPECIAL GRAPHS

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Abstract

A graph G with p vertices and q edges is said to have Skolem difference Lucas mean labelling if it is possible to label the vertices $x \in v$ with distinct elements $f(x)$ from the set $\{1,2,\ldots,L_{p+q}\}$ in such a way that the edge $e = uv$ is labelled with $\left|\frac{f(u)-f(v)}{2}\right|$ if $|f(u)-f(v)|$ is even and $\frac{|f(u)-f(v|+1}{2}$ if $|f(u)-f(v)|$ is odd, then the resulting edge labels are distinct and are from $\{L_1, L_2, \ldots, L_q\}$. A graph that admits Skolem difference Lucas mean labelling is called a Skolem difference Lucas mean graph. In this paper, we prove for some graphs such as triangular snake graph TS_n, The triangular snake $C_2(P_n)$, Umbrella $U(m,n)$, $F_m \oplus K_{1,n}^+$, $F_n^{(t)}$, F_n^+ are Skolem difference Lucas mean graph.

Keywords: Skolem Mean Labelling, Skolem difference Mean Labelling, Skolem difference Lucas Mean Labelling

AMS subject classification: 05C78.

1. INTRODUCTION

By a graph, we mean a finite, undirected graph without loops and multiples edges, for terms not defined here we refer to Harary [4].

In a labelling of a particular type, the vertices are assigned values from a given set, the edges having a prescribed induced labelling must satisfy certain properties. An excellent reference on this subject is the survey by Gallian [3].

According to Beineke and Hegde [2] labelling of discrete structure is a frontier between graph theory and numbers.

The concept of Skolem mean labeling was introduced by Balaji, Ramesh and Subramanian [1], and the Skolem difference mean labeling introduced by Murugan and Subramanian [5]. We reproduce their definitions below for our ready reference.

Definition 1.1
A graph $G=(V,E)$ with p vertices and q edges is said to be a Skolem mean graph if there exists a function f from the vertex set of $G\ to\ \{1,2,3,\dots,p\}$ such that the induced map f^* from the edge set of $G\ to\ \{2,3,4,\dots,p\}$ defined by $f^*(e=uv)=\left|\frac{f(u)-f(v)}{2}\right|$ if $|f(u)-f(v)|$ is even and $\frac{|f(u)-f(v|+1}{2}$ if $|f(u)-f(v)|$ is odd. Then the resulting edge labels are distinct and are from {2,3,4,…,p}. A graph that admits Skolem mean labelling is called a Skolem mean graph.

Definition 1.2
A graph $G=(V,E)$ with p vertices and q edges is said to be a Skolem difference mean graph if there exists a function f from the vertex set of $G\ to\ \{1,2,3,\dots,p+q\}$ such that the induced map f^* from the edge set of $G\ to\ \{1,2,3,\dots,q\}$ defined by $f^*(e=uv)=\left|\frac{f(u)-f(v)}{2}\right|$ if $|f(u)-f(v)|$ is even and $\frac{|f(u)-f(v|+1}{2}$ if $|f(u)-f(v)|$ is odd. The resulting edge labels are then distinct and are from {1,2,3,…,q}. A graph that admits Skolem difference mean labelling is called a Skolem difference mean graph.
These definitions motivate us to define the Skolem difference Lucas mean labelling.

Definition 1.3
A graph G with p vertices and q edges is said to have Skolem difference Lucas mean labelling if it is possible to label the vertices $x\in v$ with distinct elements $f(x)$ from the set $\{1,2,\dots,L_{p+q}\}$ in such a way that the edge $e=uv$ is labelled with $\left|\frac{f(u)-f(v)}{2}\right|$ if $|f(u)-f(v)|$ is even and $\frac{|f(u)-f(v)|+1}{2}$ if $|f(u)-f(v)|$ is odd, then the resulting edge labels are distinct and are from $\{L_1,L_2,\dots,L_q\}$. A graph that admits Skolem difference Lucas mean labelling is called a Skolem difference Lucas mean graph.

2. RESULTS

Theorem 2.1
The triangular snake graph TS_n is Skolem difference Lucas mean graph.
Proof:
Let G be the graph TS_n .
Let $V(G)=\{v_i,w_j : 1\le i\le n+1\ and\ 1\le j\le n\}$
$E(G)=\{v_iv_{i+1} : 1\le i\le n\}\cup\{v_jw_j :\ 1\le j\le n\}\cup\{v_jw_{(j-1)} : 2\le j\le n+1\}$
Then $|V(G)|=2n+1$ and $|E(G)|=3n$
Let $f\colon V(G)\to\{1,2,\dots,L_{5n+1}\}$ be defined as follows
$f(v_1)=3$
$f(w_1)=1$
$f(v_2)=9$
$f(v_{i+1})=2L_{3i}+f(v_i),\quad 2\le i\le n$
$f(w_i)=2L_{4+3(i-2)}+f(v_i),\quad 2\le i\le n$
$f^+(E)=\{f(v_iv_{i+1}) : 1\le i\le n\}\cup\{f(v_jw_j) :\ 1\le j\le n\}\cup\{f(v_jw_{j-1}) : 2\le j\le n+1\}$
$=\ \{f(v_1v_2),f(v_2v_3),\dots,f(v_nv_{n+1})\}\cup\{f(v_1w_1),f(v_2w_2),\dots,f(v_nw_n)\}$
$\cup\{f(v_2w_1),f(v_3w_2),\dots,f(v_{n+1}w_n)\}$

$$=\left\{\left|\frac{f(v_1)-f(v_2)}{2}\right|,\left|\frac{f(v_2)-f(v_3)}{2}\right|,\dots,\left|\frac{f(v_n)-f(v_{n+1})}{2}\right|\right\}$$
$$\cup\left\{\left|\frac{f(v_1)-f(w_1)}{2}\right|,\left|\frac{f(v_2)-f(w_2)}{2}\right|,\dots,\left|\frac{f(v_n)-f(w_n)}{2}\right|\right\}$$
$$\cup\left\{\left|\frac{f(v_2)-f(w_1)}{2}\right|,\left|\frac{f(v_3)-f(w_2)}{2}\right|,\dots,\left|\frac{f(v_{n+1})-f(w_n)}{2}\right|\right\}$$

$$= \left\{\left|\frac{3-9}{2}\right|, \left|\frac{f(v_2)-2L_6-f(v_2)}{2}\right|, \dots, \left|\frac{f(v_n)-2L_{3n}-f(v_n)}{2}\right|\right\}$$
$$\cup \left\{\left|\frac{3-1}{2}\right|, \left|\frac{f(v_2)-2L_4-f(v_2)}{2}\right|, \dots, \left|\frac{f(v_n)-2L_{3n-2}-f(v_n)}{2}\right|\right\}$$
$$\cup \left\{\left|\frac{9-1}{2}\right|, \left|\frac{2L_6+f(v_2)-2L_4-f(v_2)}{2}\right|, \dots, \left|\frac{2L_{3n}+f(v_n)-2L_{3n-2}-f(v_n)}{2}\right|\right\}$$
$= \{L_2, L_6, \dots, L_{3n}\} \cup \{L_1, L_4, \dots, L_{3n-2}\} \cup \{L_3, L_5, \dots, L_{3n-1}\}$
$= \{L_1, L_2, L_3, L_4, L_5, L_6, \dots, L_{3n-2}, L_{3n-1}, L_{3n}\}$
$f^+(E) = \{L_1, L_2, \dots, L_{3n}\}$
Thus, the induced edge labels are distinct and are $L_1, L_2, \dots, L_{3n}$.
Hence, the triangular snake graph TS_n is Skolem difference Lucas mean graph.

Theorem 2.2
The triangular snake $C_2(P_n)$ is Skolem difference Lucas mean graph.

Proof:
Let G be the graph $C_2(P_n)$.
Let $V(G) = \{u_i : 1 \le i \le n+1\} \cup \{v_i, w_i : 1 \le i \le n\}$
$E(G) = \{u_i v_i : 1 \le i \le n\} \cup \{u_i w_i : 1 \le i \le n\} \cup \{u_{i+1} v_i : 1 \le i \le n\} \cup \{u_{i+1} w_i : 1 \le i \le n\} \cup \{u_i u_{i+1} : 1 \le i \le n\}$
Then $|V(G)| = 3n+1$ and $|E(G)| = 5n$
Let $f: V(G) \to \{1, 2, \dots, L_{8n+1}\}$ be defined as follows:
$f(u_1) = 1$
$f(u_i) = 2L_{5i-7} + f(u_{i-1}), \quad 2 \le i \le n+1$
$f(v_i) = 2L_{5i-4} + f(u_i), \quad 1 \le i \le n$
$f(v_i) = 2L_{5i} + f(u_i), \quad 1 \le i \le n$
$f^+(E) = \{f(u_i v_i) : 1 \le i \le n\} \cup \{f(u_i w_i) : 1 \le i \le n\} \cup \{f(u_{i+1} v_i) : 1 \le i \le n\} \cup \{f(u_{i+1} w_i) : 1 \le i \le n\}$
$\cup \{f(u_i u_{i+1}) : 1 \le i \le n\}$
$= \{f(u_1 v_1), f(u_2 v_2), \dots, f(u_n v_n)\} \cup \{f(u_1 w_1), f(u_2 w_2), \dots, f(u_n w_n)\} \cup \{f(u_2 v_1), f(u_3 v_2), \dots, f(u_{n+1} v_n)\}$
$\cup \{f(u_2 w_1), f(u_3 w_2), \dots, f(u_{n+1} w_n)\} \cup \{f(u_1 u_2), f(u_2 u_3), \dots, f(u_n u_{n+1})\}$
$$= \left\{\left|\frac{f(u_1)-f(v_1)}{2}\right|, \left|\frac{f(u_2)-f(v_2)}{2}\right|, \dots, \left|\frac{f(u_n)-f(v_n)}{2}\right|\right\}$$
$$\cup \left\{\left|\frac{f(u_1)-f(w_1)}{2}\right|, \left|\frac{f(u_2)-f(w_2))}{2}\right|, \dots, \left|\frac{f(u_n)-f(w_n)}{2}\right|\right\}$$
$$\cup \left\{\left|\frac{f(u_2)-f(v_1)}{2}\right|, \left|\frac{f(u_3)-f(v_2)}{2}\right|, \dots, \left|\frac{f(u_{n+1})-f(v_n)}{2}\right|\right\}$$
$$\cup \left\{\left|\frac{f(u_2)-f(w_1)}{2}\right|, \left|\frac{f(u_3)-f(w_2)}{2}\right|, \dots, \left|\frac{f(u_{n+1})-f(w_n)}{2}\right|\right\}$$
$$\cup \left\{\left|\frac{f(u_1)-f(u_2)}{2}\right|, \left|\frac{f(u_2)-f(u_3)}{2}\right|, \dots, \left|\frac{f(u_n)-f(u_{n+1})}{2}\right|\right\}$$
$$= \left\{\left|\frac{f(u_1)-2L_1-f(u_1)}{2}\right|, \left|\frac{f(u_2)-2L_6-f(u_2)}{2}\right|, \dots, \left|\frac{f(u_n)-2L_{5n-4}-f(u_n)}{2}\right|\right\}$$
$$\cup \left\{\begin{matrix}\left|\frac{f(u_1)-2L_5-f(u_1)}{2}\right|, \left|\frac{f(u_2)-2L_{10}-f(u_2))}{2}\right|, \dots, \\ \left|\frac{f(u_n)-2L_{5n}-f(u_n)}{2}\right|\end{matrix}\right\}$$

$$\cup \left\{ \begin{array}{c} \left|\frac{2L_3 + f(u_1) - 2L_1 - f(u_1)}{2}\right|, \left|\frac{2L_8 + f(u_2) - 2L_6 - f(u_2)}{2}\right|, \dots, \\ \left|\frac{2L_{5n-2} + f(u_n) - 2L_{5n-4} - f(u_n)}{2}\right| \end{array} \right\}$$

$$\cup \left\{ \begin{array}{c} \left|\frac{2L_3 + f(u_1) - 2L_5 - f(u_1)}{2}\right|, \left|\frac{2L_8 + f(u_2) - 2L_{10} - f(u_2)}{2}\right|, \dots, \\ \left|\frac{2L_{5n-2} + f(u_n) - 2L_{5n} - f(u_n)}{2}\right| \end{array} \right\}$$

$$\cup \left\{ \begin{array}{c} \left|\frac{f(u_1) - 2L_3 - f(u_1)}{2}\right|, \left|\frac{f(u_2) - 2L_8 - f(u_2)}{2}\right|, \dots, \\ \left|\frac{f(u_n) - 2L_{5n-2} - f(u_n)}{2}\right| \end{array} \right\}$$

$= \{L_1, L_6, \dots, L_{5n-4}\} \cup \{L_5, L_{10}, \dots, L_{5n}\} \cup \{L_2, L_7, \dots, L_{5n-3}\} \cup \{L_4, L_9, \dots, L_{5n-1}\} \cup$
$\{L_3, L_8, \dots, L_{5n-2}\}$
$f^+(E) = \{L_1, L_2, L_3, L_4, L_5, L_6, L_7, L_8, L_9, L_{10}, \dots, L_{5n-2}, L_{5n-1}, L_{5n}\}$
Thus, the induced edge labels are distinct and are $L_1, L_2, \dots, L_{5n}$.
Hence, the triangular snake graph $C_2(P_n)$ is Skolem difference Lucas mean graph.

Theorem 2.3
The graph Umbrella $U(m,n)$ is Skolem difference Lucas mean graph.

Proof:
Let G be the graph Umbrella $U(m,n)$.
Let $V(G) = \{u, u_i, v_j : 1 \le i \le m \ and \ 1 \le j \le n-1\}$.

$$E(G) = \{uu_i : 1 \le i \le m\} \cup \{u_i u_{i+1} : 1 \le i \le m-1\} \cup \{v_j v_{j+1} : 1 \le j \le n-2\} \cup \begin{cases} v_1 u_{\frac{m}{2}} & if \ \ n \ is \ even \\ v_1 u_{\frac{m+1}{2}} & if \ \ n \ is \ odd \end{cases}$$

Then $|V(G)| = m + n$ and $|E(G)| = 2m + n - 2$.
Let $f: V(G) \to \{1, 2, \dots, L_{3m+2n-2}\}$ be defined as follows:
$f(u) = 1$
$f(u_i) = 2L_{2i-1} + f(u), \quad 1 \le i \le m$

$$f(v_1) = \begin{cases} 2L_{2m} + f(u_{\frac{m}{2}}) & if \ \ n \ is \ even \\ 2L_{2m} + f(u_{\frac{m+1}{2}}) & if \ \ n \ is \ odd \end{cases}$$

$f(v_j) = 2L_{2m+j-1} + f(v_{j-1}), \quad 2 \le j \le n-1$

$$f^+(E) = \{f(uu_i) : 1 \le i \le m\} \cup \{f(u_i u_{i+1}) : 1 \le i \le m-1\} \cup \{f(v_j v_{j+1}) : 1 \le j \le n-2\} \cup \begin{cases} f(v_1 u_{\frac{m}{2}}) & if \ \ n \ is \ even \\ f(v_1 u_{\frac{m+1}{2}}) & if \ \ n \ is \ odd \end{cases}$$

$$= \{f(uu_1), f(uu_2), \dots, f(uu_m)\} \cup \{ f(u_1u_2), f(u_2u_3), \dots, f(u_{m-1}u_m)\} \cup \{ f(v_1v_2), f(v_2v_3), \dots, f(v_{n-2}v_{n-1})\} \cup \begin{cases} f(v_1 u_{\frac{m}{2}}) & if \ \ n \ is \ even \\ f(v_1 u_{\frac{m+1}{2}}) & if \ \ n \ is \ odd \end{cases}$$

$$= \left\{\left|\frac{f(u) - f(u_1)}{2}\right|, \left|\frac{f(u) - f(u_2)}{2}\right|, \dots, \left|\frac{f(u) - f(u_m)}{2}\right|\right\}$$

$$\cup \left\{\left|\frac{f(u_1) - f(u_2)}{2}\right|, \left|\frac{f(u_2) - f(u_3)}{2}\right|, \dots, \left|\frac{f(u_{m-1}) - f(u_m)}{2}\right|\right\}$$

$$\cup \left\{\left|\frac{f(v_1) - f(v_2)}{2}\right|, \left|\frac{f(v_2) - f(v_3)}{2}\right|, \dots, \left|\frac{f(v_{n-2}) - f(v_{n-1})}{2}\right|\right\}$$

$$\cup \begin{cases} f(v_1 u_{\frac{m}{2}}) & if \ \ n \ is \ even \\ f(v_1 u_{\frac{m+1}{2}}) & if \ \ n \ is \ odd \end{cases}$$

$$= \left\{\left|\frac{f(u)-2L_1 - f(u)}{2}\right|, \left|\frac{f(u)-2L_3 - f(u)}{2}\right|, \dots, \left|\frac{f(u)-2L_{2m-1} - f(u)}{2}\right|\right\}$$
$$\cup \left\{ \begin{array}{c} \left|\frac{2L_1 + f(u)-2L_3 - f(u)}{2}\right|, \left|\frac{2L_3 + f(u)-2L_5 - f(u)}{2}\right|, \dots, \\ \left|\frac{2L_{2m-3} + f(u)-2L_{2m-1} - f(u)}{2}\right| \end{array} \right\}$$
$$\cup \left\{ \begin{array}{c} \left|\frac{f(v_1) - 2L_{2m+1} - f(v_1)}{2}\right|, \left|\frac{f(v_2) - 2L_{2m+2} - f(v_2)}{2}\right|, \dots, \\ \left|\frac{f(v_{n-2}) - 2L_{2m+n-2} - f(v_{n-2})}{2}\right| \end{array} \right\}$$
$$\cup \begin{cases} 2L_{2m} + f\left(u_{\frac{m}{2}}\right) - f(u_{\frac{m}{2}}) & if \quad n \text{ is even} \\ 2L_{2m} + f\left(u_{\frac{m+1}{2}}\right) - f(u_{\frac{m+1}{2}}) & if \quad n \text{ is odd} \end{cases}$$
$$= \{ L_1, L_3, \dots, L_{2m-1}\} \cup \{L_2, L_4, \dots, L_{2m-2}\} \cup \{L_{2m+1}, L_{2m+2}, \dots, L_{2m+n-2}\} \cup \{L_{2m}\}$$
$$= \{L_1, L_2, L_3, L_4, \dots, L_{2m-2}, L_{2m-1}, L_{2m}, L_{2m+1}, L_{2m+2}, \dots, L_{2m+n-2}\}$$
$$f^+(E) = \{L_1, L_2, \dots, L_{2m+n-2}\}$$

Thus, the induced edge labels are distinct and are $L_1, L_2, \dots, L_{2m+n-2}$.
Hence, the graph Umbrella $U(m,n)$ is Skolem difference Lucas mean graph.

Theorem 2.4
The graph $F_m \oplus K_{1,n}^+$ is Skolem difference Lucas mean graph for $m \geq 2, n \geq 1$.

Proof:
Let G be the graph $F_m \oplus K_{1,n}^+$.
Let $V(G) = \{u_0, u_i : 1 \leq i \leq m\} \cup \{v_i, w_i : 1 \leq i \leq n\}$
$E(G) = \{u_i u_{i+1} : 1 \leq i \leq m-1\} \cup \{u_0 u_i : 1 \leq i \leq m\} \cup \{u_0 v_i : 1 \leq i \leq n\} \cup \{v_i w_i : 1 \leq i \leq n\}$
Then $|V(G)| = 2m + n + 1$ and $|E(G)| = 2m + 2n - 1$
Let $f: V(G) \to \{1, 2, \dots, L_{4m+3n}\}$ be defined as follows
$f(u_0) = 1$
$f(u_i) = 2L_{2i-1} + f(u_0), \quad 1 \leq i \leq m$
$f(v_i) = 2L_{2m+2i-2} + f(u_0), \quad 1 \leq i \leq n$
$f(w_i) = 2L_{2m+2i-1} + f(v_i) - 1, \quad 1 \leq i \leq n$
$f(w_1) = 2L_{2m+n-1} + 1$
$f(w_j) = 2L_{2m+n+j-2} + f(w_{j-1}), \quad 2 \leq j \leq n-1$

$$f^+(E) = \{f(u_i u_{i+1}) : 1 \leq i \leq m-1\} \cup \{f(u_0 u_i) : 1 \leq i \leq m\} \cup \{f(u_0 v_i) : 1 \leq i \leq n\} \cup \{f(v_i w_i) : 1 \leq i \leq n\}$$
$$= \{ f(u_1 u_2), f(u_2 u_3), \dots, f(u_{m-1} u_m)\} \cup \{ f(u_0 u_1), f(u_0 u_2), \dots, f(u_0 u_m)\}$$
$$\cup \{f(u_0 v_1), f(u_0 v_2), \dots, f(u_0 v_n)\} \cup \{f(v_1 w_1), f(v_2 w_2), \dots, f(v_n w_n)\}$$
$$= \left\{\left|\frac{f(u_1) - f(u_2)}{2}\right|, \left|\frac{f(u_2) - f(u_3)}{2}\right|, \dots, \left|\frac{f(u_{m-1}) - f(u_m)}{2}\right|\right\}$$
$$\cup \left\{\left|\frac{f(u_0) - f(u_1)}{2}\right|, \left|\frac{f(u_0) - f(u_2)}{2}\right|, \dots, \left|\frac{f(u_0) - f(u_m)}{2}\right|\right\}$$
$$\cup \left\{\left|\frac{f(u_0) - f(v_1)}{2}\right|, \left|\frac{f(u_0) - f(v_2)}{2}\right|, \dots, \left|\frac{f(u_0) - f(v_n)}{2}\right|\right\}$$
$$\cup \left\{\left|\frac{f(v_1) - f(w_1)}{2}\right|, \left|\frac{f(v_2) - f(w_2)}{2}\right|, \dots, \left|\frac{f(v_n) - f(w_n)}{2}\right|\right\}$$
$$= \left\{ \begin{array}{c} \left|\frac{2L_1 + f(u_0) - 2L_3 - f(u_0)}{2}\right|, \left|\frac{2L_3 + f(u_0) - 2L_5 - f(u_0)}{2}\right|, \dots, \\ \left|\frac{2L_{2m-3} + f(u_0) - 2L_{2m-1} - f(u_0)}{2}\right| \end{array} \right\} \cup$$
$$\left\{\left|\frac{f(u_0) - 2L_1 - f(u_0)}{2}\right|, \left|\frac{f(u_0) - 2L_3 - f(u_0)}{2}\right|, \dots, \left|\frac{f(u_0) - 2L_{2m-1} - f(u_0)}{2}\right|\right\} \cup \left\{ \begin{array}{c} \left|\frac{f(u_0) - 2L_{2m} - f(u_0)}{2}\right|, \left|\frac{f(u_0) - 2L_{2m+2} - f(u_0)}{2}\right|, \dots, \\ \left|\frac{f(u_0) - 2L_{2m+2n-2} - f(u_0)}{2}\right| \end{array} \right\} \cup$$
$$\{\frac{|f(v_1) - 2L_{2m+1} - f(v_1) + 1| + 1}{2}, \frac{|f(v_2) - 2L_{2m+3} - f(v_2) + 1| + 1}{2}, \dots,$$

$\frac{|f(v_n) - 2L_{2m+2n-1} - f(v_n) + 1| + 1}{2}\}$

$= \cup \{L_2, L_4, \dots, L_{2m-2}\} \cup \{L_1, L_3, \dots, L_{2m-1}\} \cup \{L_{2m}, L_{2m+2}, \dots, L_{2m+n-2}\}$
$\cup \{L_{2m+1}, L_{2m+3}, L_{2m+n+1}, \dots, L_{2m+2n-1}\}$

$= \{L_1, L_2, L_3, L_4, \dots, L_{2m-2}, L_{2m-1}, L_{2m}, L_{2m+1}, L_{2m+2}, \dots, L_{2m+n-2}, L_{2m+n-1}\}$

$f^+(E) = \{L_1, L_2, \dots, L_{2m+n-1}\}$

Thus, the induced edge labels are distinct and are $L_1, L_2, \dots, L_{2m+n-1}$.

Hence, the graph $F_m \oplus K_{1,n}^+$ is Skolem difference Lucas mean graph for $m \geq 2$, $n \geq 1$.

Theorem 2.5

The graph $F_n^{(t)}$ is Skolem difference Lucas mean graph for all $n \geq 2$.

Proof:

Let G be the graph $F_n^{(t)}$.

Let $V(G) = \{u_0, u_{ij} : 1 \leq i \leq t \text{ and } 1 \leq j \leq n\}$

$E(G) = \{u_0 u_{ij} : 1 \leq i \leq t \text{ and } 1 \leq j \leq n\} \cup \{u_{ij} u_{i(j+1)} : 1 \leq i \leq t \text{ and } 1 \leq j \leq n-1\}$

Then $|V(G)| = nt + 1$ and $|E(G)| = 2nt - t$

Let $f: V(G) \to \{1, 2, \dots, L_{3nt-t+1}\}$ be defined as follows

$f(u_0) = 1$

$f(u_{1j}) = 2L_{2j-1} + f(u_0), \quad 1 \leq j \leq n$

$f(u_{ij}) = 2L_{2n(i-1)+2(j-1)-(i-2)} + f(u_0), \quad 2 \leq i \leq t \text{ and } 1 \leq j \leq n$

$f^+(E) = \{f(u_0 u_{ij}) : 1 \leq i \leq t \text{ and } 1 \leq j \leq n\} \cup \{f(u_{ij} u_{i(j+1)}) : 1 \leq i \leq t \text{ and } 1 \leq j \leq n-1\}$

$= \left\{ \begin{matrix} f(u_0 u_{11}), f(u_0 u_{12}), \dots, f(u_0 u_{1n}), f(u_0 u_{21}), f(u_0 u_{22}), \dots, f(u_0 u_{2n}), \\ \dots, f(u_0 u_{t1}), f(u_0 u_{t2}), \dots, f(u_0 u_{tn}) \end{matrix} \right\}$
$\cup \{f(u_{11} u_{12}), f(u_{12} u_{13}), \dots, f(u_{1(n-1)} u_{1n}), f(u_{21} u_{22}), f(u_{22} u_{23}),$
$\dots, f(u_{2(n-1)} u_{2n}), \dots, f(u_{t1} u_{t2}), f(u_{t2} u_{t3}), \dots, f(u_{t(n-1)} u_{tn})\}$

$= \{\left|\frac{f(u_0) - f(u_{11})}{2}\right|, \left|\frac{f(u_0) - f(u_{12})}{2}\right|, \dots, \left|\frac{f(u_0) - f(u_{1n})}{2}\right|, \left|\frac{f(u_0) - f(u_{21})}{2}\right|,$
$\left|\frac{f(u_0) - f(u_{22})}{2}\right|, \dots, \left|\frac{f(u_0) - f(u_{2n})}{2}\right|, \dots, \left|\frac{f(u_0) - f(u_{t1})}{2}\right|, \left|\frac{f(u_0) - f(u_{t2})}{2}\right|$
$, \dots, \left|\frac{f(u_0) - f(u_{tn})}{2}\right| \{\left|\frac{f(u_{11}) - f(u_{12})}{2}\right|, \left|\frac{f(u_{12}) - f(u_{13})}{2}\right|, \dots,$
$\left|\frac{f(u_{1(n-1)}) - f(u_{1n})}{2}\right|, \left|\frac{f(u_{21}) - f(u_{22})}{2}\right|, \left|\frac{f(u_{22}) - f(u_{23})}{2}\right|, \dots,$
$\left|\frac{f(u_{2(n-1)}) - f(u_{2n})}{2}\right|, , \dots, \left|\frac{f(u_{t1}) - f(u_{t2})}{2}\right|, \left|\frac{f(u_{t2}) - f(u_{t3})}{2}\right|,$
$\dots, \left|\frac{f(u_{t(n-1)}) - f(u_{tn})}{2}\right|\}$

$= \{\left|\frac{f(u_0) - 2L_1 - f(u_0)}{2}\right|, \left|\frac{f(u_0) - 2L_3 - f(u_0)}{2}\right|, \dots, \left|\frac{f(u_0) - 2L_{2n-1} - f(u_0)}{2}\right|,$
$\left|\frac{f(u_0) - 2L_{2n} - f(u_0)}{2}\right|, \left|\frac{f(u_0) - 2L_{2n+2} - f(u_0)}{2}\right|, \dots, \left|\frac{f(u_0) - 2L_{4n-2} - f(u_0)}{2}\right|,$
$\dots, \left|\frac{f(u_0) - 2L_{2nt-2n-t+2} - f(u_0)}{2}\right|, \left|\frac{f(u_0) - 2L_{2nt-2n-t+4} - f(u_0)}{2}\right|,$
$\dots, \left|\frac{f(u_0) - 2L_{2nt-t} - f(u_0)}{2}\right|\} \cup \{\left|\frac{2L_1 + f(u_0) - 2L_3 - f(u_0)}{2}\right|, \left|\frac{2L_3 + f(u_0) - 2L_5 - f(u_0)}{2}\right|, \dots,$
$\left|\frac{2L_{2n-3} + f(u_0) - 2L_{2n-1} - f(u_0)}{2}\right|, \left|\frac{2L_{2n} + f(u_0) - 2L_{2n+2} - f(u_0)}{2}\right|$
$, \left|\frac{2L_{2n+2} + f(u_0) - 2L_{2n+4} - f(u_0)}{2}\right|, \dots, \left|\frac{2L_{4n-4} + f(u_0) - 2L_{4n-2} - f(u_0)}{2}\right|,$
$\dots, \left|\frac{2L_{2nt-2n-t+2} + f(u_0) - 2L_{2nt-2n-t+4} - f(u_0)}{2}\right|,$

$\left|\frac{2L_{2nt-2n-t+4}+f(u_0)-2L_{2nt-2n-t+6}-f(u_0)}{2}\right|,$
$\ldots,\left|\frac{2L_{2nt-t-2}+f(u_0)-2L_{2nt-t}-f(u_0)}{2}\right|\}$
$=\{L_1,L_3,\ldots,L_{2n-1},L_{2n},L_{2n+2},\ldots,L_{4n-2},\ldots,L_{2nt-2n-t+2},L_{2nt-2n-t+4},\ldots,L_{2nt-t}\}$
$\cup\{L_2,L_4,\ldots,L_{2n-2},L_{2n+1},L_{2n+3},\ldots,L_{4n-3},\ldots,L_{2nt-2n-t+3},L_{2nt-2n-t+5},\ldots,L_{2nt-t-1}\}$
$=\{L_1,L_2,L_3,L_4,L_5,L_6,\ldots,L_{2n-1},L_{2n},L_{2n+1},L_{2n+2},L_{2n+3},\ldots,L_{4n-3},$
$L_{4n-2},\ldots,L_{2nt-2n-t+5},L_{2nt-2n-t+4},L_{2nt-2n-t+3},L_{2nt-2n-t+2},\ldots,L_{2nt-t-1},L_{2nt-t}\}$
$f^+(E)=\{L_1,L_2,\ldots,L_{2nt-t}\}$
Thus, the induced edge labels are distinct and are $L_1,L_2,\ldots,L_{2nt-t}$.
Hence, the graph $F_n^{(t)}$ is Skolem difference Lucas mean graph for all $n\geq 2$.

Theorem 2.6
The graph F_n^+ is Skolem difference Lucas mean graph for all $n\geq 2$.

Proof:
Let G be the graph F_n^+.
Let $V(G)=\{u,u_{11},v_i,v_{i1}: 1\leq i\leq n\}$.
$E(G)=\{uv_i: 1\leq i\leq n\}\cup\{v_iv_{i+1}: 1\leq i\leq n-1\}\cup\{uu_{11}\}\cup\{v_iv_{i1}: 1\leq i\leq n\}$.
Then $|V(G)|=2n+2$ and $|E(G)|=3n$.
Let $f: V(G)\to\{1,2,\ldots,L_{5n+2}\}$ be defined as follows:
$f(u) = 1$
$f(v_i) = 2L_{2i-1}+f(u),\quad 1\leq i\leq n$
$f(u_{11}) = L_{2n}+f(u)$
$f(v_{i1}) = L_{2n+i}+f(v_i),\quad 1\leq i\leq n$
$f^+(E)=\{f(uv_i): 1\leq i\leq n\}\cup\{f(v_iv_{i+1}): 1\leq i\leq n-1\}\cup\{f(uu_{11})\}$
$\{f(v_iv_{i1}): 1\leq i\leq n\}$
$=\{f(uv_1),f(uv_2),\ldots,f(uv_n)\}\cup\{f(v_1v_2),f(v_2v_3),\ldots,f(v_{n-1}v_n)\}\cup$
$\{f(v_1v_2),f(v_2v_3),\ldots,f(v_{n-1}v_n)\}\cup\{f(uu_{11})\}\cup\{f(v_1v_{11}),f(v_2v_{21}),\ldots,f(v_nv_{n1})\}$
$=\left\{\left|\frac{f(u)-f(v_1)}{2}\right|,\left|\frac{f(u)-f(v_2)}{2}\right|,\ldots,\left|\frac{f(u)-f(v_n)}{2}\right|\right\}$
$\cup\left\{\left|\frac{f(v_1)-f(v_2)}{2}\right|,\left|\frac{f(v_2)-f(v_3)}{2}\right|,\ldots,\left|\frac{f(v_{n-1})-f(v_n)}{2}\right|\right\}\cup\left\{\left|\frac{f(u)-f(u_{11})}{2}\right|\right\}$
$\cup\left\{\left|\frac{f(v_1)-f(v_{11})}{2}\right|,\left|\frac{f(v_2)-f(v_{21})}{2}\right|,\ldots,\left|\frac{f(v_n)-f(v_{n1})}{2}\right|\right\}$
$=\left\{\left|\frac{f(u)-2L_1-f(u)}{2}\right|,\left|\frac{f(u)-2L_3-f(u)}{2}\right|,\ldots,\left|\frac{f(u)-2L_{2n-1}-f(u)}{2}\right|\right\}$
$\cup\left\{\left|\frac{2L_1+f(u)-2L_3-f(u)}{2}\right|,\left|\frac{2L_3+f(u)-2L_5-f(u)}{2}\right|,\ldots,\left|\frac{2L_{2n-3}+f(u)-2L_{2n-1}-f(u)}{2}\right|\right\}\cup\left\{\left|\frac{f(u)-2L_{2n}-f(u)}{2}\right|\right\}$
$\cup\left\{\left|\frac{f(v_1)-2L_{2n+1}-f(v_{11})}{2}\right|,\left|\frac{f(v_2)-2L_{2n+2}-f(v_{21})}{2}\right|,\ldots,\right.$
$\left.\left|\frac{f(v_n)-2L_{3n}-f(v_{n1})}{2}\right|\right\}$
$=\{L_1,L_3,\ldots,L_{2n-1}\}\cup\{L_2,L_4,\ldots,L_{2n-2}\}\cup\{L_{2n}\}\cup\{L_{2n+1},L_{2n+2},\ldots,L_{3n}\}$
$=\{L_1,L_2,L_3,L_4,\ldots,L_{2n-2},L_{2n-1},L_{2n},L_{2n+1},L_{2n+2},\ldots,L_{3n}\}$
$f^+(E)=\{L_1,L_2,\ldots,L_{3n}\}$.
Thus, the induced edge labels are distinct and are $L_1,L_2,\ldots,L_{3n}$.
Hence, the graph F_n^+ is Skolem difference Lucas mean graph for all $n\geq 2$.

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