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MODELLING AGE SPECIFIC FERTILITY RATE IN INDIA THROUGH FERTILITY CURVES

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Abstract

One of the important demographic features of any population is the fertility rate which has a direct relationship with both the social and the biological environment.Demographic factors like age at marriage, present family size, gender preference (Mahadevan [27], Bhasin [3], Asari and John [2], Chachra and Bhasin [18], Bhasin and Nag [19]) and socio economic factors like education, occupation, religion, contraceptive practice, etc. (Bhatia [20], Asari and John [2]) are the determinants of desired family size and all these are considered as the cause of the variation of fertility. So far researchers have proposed a variety of reproductivity measures and mathematical models to describe the reproductivity pattern of human population (Islam and Ali [21], Peristera and Kostaki [22] and Nasir et al. [23]). The objective of this study is to examine the current pattern of Age Specific Fertility Rate(ASFR) and to study the trend in fertile age groups by fitting non linear models to the ASFR data for all the states in India obtained from the sample registration system [1], of India. Cross Validity Prediction Power (CVPP),Shrinkage, and R^2 are used to identify the best model for the states. Model identification for Forward Age Specific Fertility Rate (FASFR) and Backward Age Specific Fertility Rate (BASFR) along with validity measures are also presented in this paper.

Keywords: Age Specific Fertility rate (ASFR), Forward Age Specific Fertility (FASFR) Rate, Backward Age Specific Fertility Rate (BASFR), Non linear Model, Cross Validity Prediction Power (CVPP), Shrinkage.

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1. INTRODUCTION

One of the important demographic features of any population is the fertility rate which has the direct relationship with both the social and the biological environments. Fertility profile in human population has two

phases namely estimation of standard measures of fertility and fertility modelling. Demographic factors like age at marriage, present family size, gender preference and also socio economic factors like education, occupation, religion, contraceptive practice, etc. (Bhatia [20], Asari and John [2]) are the determinants of desired family size and considered as the cause of the variation of fertility. Some of the other factors like place of residence, family type and mass media indirectly affect the fertility. The decision of a married couple about when they wish to have their next child also affects the birth interval and this also indirectly affects the fertility rate. Old age security motivates higher fertility (Vlassof and Vlassof [6], Goody et.al. [5]) and also the parent's desire for more children is directly correlated with their fear of losing them (Mysore Population Study[24], Preston [25]). Desired family size depends on the relative utilities of economic benefits and the cost of upbringing children.The second phase of the fertility profile deals with fertility modelling that is primarily concerned with fertility curves. In the literature a variety of reproductivity measures and mathematical models have been proposed which have described the reproductivity pattern of human population (Islam and Ali [21], Peristera and Kostaki [22] and Nasir et al. [23]).

Some of these mathematical models have been shown to provide excellent fits to age specific fertility rate distributions of human population. Modelling fertility of a population is imperative among demographers in recent years.Building and validating mathematical models is vital for the population projection and estimation. It is used to find out structural relationships and their dynamic behaviour among the various elements in a demographic process. Various types of models have been proposed for modelling the five year age specific fertility curves of many populations. Different models which are so far applied to the data in various countries include the Beta and Gamma functions (Hoem et al. [16]) , Pearson type I and type IIIcurves (Hoem et al. [16]), Hadwiger distribution function (Hadwiger [17], Gilje,E. [8], Yntema, L. [9]), Cubic Splines (Hoem, J.M. and Rennermalm, B. [10], Gilks, W.R. [13], Gompertz curve (Wunsch, G. [15], Murphy, E.M. and Nagnur, D.N. [11] and Farid, S.M [12]), and the polynomial models (Brass, W. [14]).

India is the second most populous country with 1.36 billion population. The total fertility ratefor India has fallen from 4.97 during the period 1975-1980to 2.3 for the period 2015-2020. In the year 2025-2030 it is expected to go down to 2.1 with the population growth at the rateof at 1.2% it is predicted to have more than 1.53 billion people by the end of 2030.

The objective of this exploratory study is to examine the current pattern of Age Specific Fertility Rate(ASFR) and to study the trend in fertile age groups by fitting non linear models to the ASFR data for all the states in India obtained from sample registration system[1], of India.Validation measures like R^2,shrinkage and cross validity prediction power (CVPP) are used for the justification of the models. These models are also fitted for the Forward Age Specific Fertility (FASFR) and the Backward Age Specific Fertility Rate (BASFR).

2. METHODS

A brief description of the models used in this study are listed below:

Model 1:Polynomial models

Age specific fertility rate can be fitted by polynomial models with respect to different ages in year. The p^{th} degree polynomial model considered is given by $y^A = b_0 + \sum_{j=1}^{p} b_j z^j + \in$ where z is mid age group in years. $y^A$is age specific fertility rates , b_0 is the constant, $b_j isthe$coefficient of z_j(j=1,2,3,...,p) and $\in$is the stochastic error term of the model. The general form of forward cumulative age specific fertility model is given by $y^F = b_0 + \sum_{j=i}^{p} b_j z^j + \varepsilon$ where z is mid age group in years. $y^F$is the forward cumulative age specific fertility rate , b_0 is the constant, $b_j$is the coefficient of z_j (j=1,2,3,...,p) and $\in$is the chance error term of the model. The general form of backward cumulative age specific fertility model is given by $y^B = b_0 + \sum_{j=i}^{p} b_j z^j + \varepsilon$ where z is mid age group in years.,$y^B$is the backward cumulative age specific fertility rate , b_0 is the constant, $b_j$is the coefficient of z_j (j=1,2,3,...,p) and $\in$is the disturbance error term of the model. If j=3 we get a cubic model and quardratic model is obtained when j=2.

Model 2:Compoundmodel

The general form of the exponential model is represented by $y = b_0 = \exp(b_1 * z)$.

Model 3: "S"model

The general form ofthe " S" model is represented by $Y_t = \exp(\beta_0 + \frac{\beta_1}{t})$.

Model 4: Power model

The general form of the power model is represented as $y_t = \beta_0 {\beta_1}^t$

Model Validation:

F – test:

The F test is used to verify the overall measure of the significance of the model as well as the significance of R^2. F test is given by $F = \frac{\frac{R^2}{(m-1)}}{\frac{(1-R^2)}{(n-m)}}$ with $(m-1, n-m)$degrees of freedom, where m is the number of parameters of the fitted model, n is the number of cases and $R^2$is the coefficient of determination of the model.

To check the population model which is more stable, a measure of effectiveness the cross validity prediction power(CVPP), ${\rho^2}_{cv}$is used with the value of ${\rho^2}_{cv} = 1 - \frac{(n-1)(n-2)(n+1)}{n(n-p-1)(n-p-2)}(1-R^2)$ (Herzerg, 1969), where n is the number of cases, p is the number of explanatory variables in the model and the cross validated R is the correlation between the observed and the predicted values of the dependent variable.

Shrinkage of the fitted model:

The shrinkage of the model is given by shrinkage=$\left|{\rho^2}_{cv} - R^2\right|$ where${\rho^2}_{cv}$ is the cross validity prediction power and R^2 is the coefficient of determination of the model.

3. RESULTS AND DISCUSSIONS

The age specific fertility rate in India for the years 2011 to 2015 obtained from sample registration system [1] of India is given in Table 1.

Table 1: Age specific fertility rate in India

Age group	2011	2012	2013	2014	2015
15-19	30.7	31.5	28.1	27.3	11.1
20-24	196.7	191.9	194.3	174.9	173.8
25-29	153.4	154.6	149.7	143.7	150.3
30-34	69.8	64.5	63.9	76.6	77.6
35-39	26.4	23.9	22	26.4	26.2
40-44	8.7	8.2	7.4	10.5	10.9
45-49	2.8	2.2	2	3.6	3.6

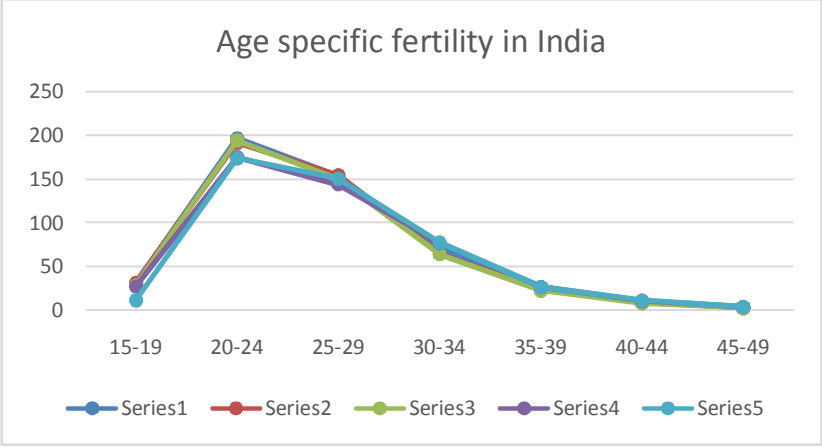

Figure1: Age Specific fertility Rate.

Theabove figure 1 shows the trend of age specific fertility rate and it observed that there is a decline in the ASFR for all age groups from 2011 to 2015. For each year the ASFR is highest in the age group 20-24 and then declines gradually to older age cohorts in all years and also there is a shift in increase in age specific fertility rate in the age group 30-34. This indicates a shift in the fertile age group.

The Forward Age Specific Fertility rate in India for the years 2011 to 2015 obtained from sample registration system [1] of India is given in Table 2.

Table 2: Forward Age specific fertility rate in India

		FASFR			
Age group	2011	2012	2013	2014	2015
15-19	30.7	31.5	28.1	27.3	11.1
20-24	227.4	223.4	222.4	202.2	184.9
25-29	380.8	378	371.7	345.9	335.2
30-34	450.6	442.5	435.6	422.5	412.8
35-39	477	466.4	457.6	448.9	439
40-44	485.7	474.6	465	459.4	449.9
45-49	488.5	476.8	467	463	453.5

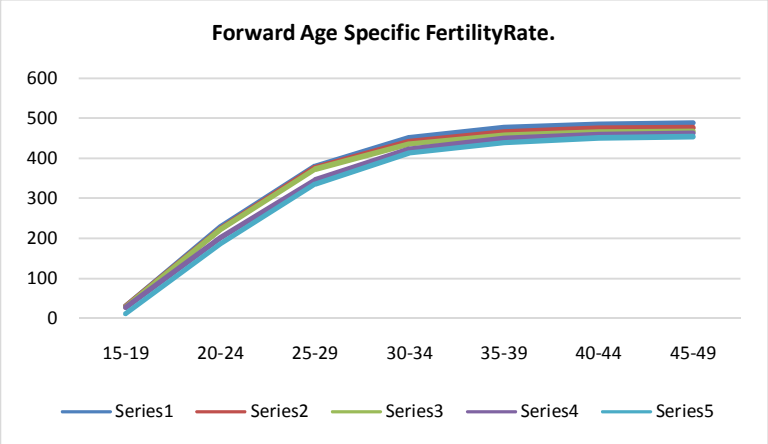

Figure 2: Growth of Forward Age specific fertility rate in India

Thefig. 2 shows the trend of forward age specific fertility rate and it observed that there is a decline in the FASFR for all age groups from 2011 to 2015. FASFR is highest in the age group 20-24 and then declines gradually to older age cohorts in all years and also there is a shift in increase in age specific fertility rate in the age group 30-34. This indicates a shift in the fertile age group.

The Backward Age Specific Fertility rate in India for the years 2011 to 2015 obtained from sample registration system [1] of India is given in Table 3.

Table 3: Backward Age specific fertility rate in India

		BASFR			
Age group	2011	2012	2013	2014	2015
15-19	488.5	476.8	467	463	453.5
20-24	457.8	445.3	438.9	435.7	442.4
25-29	261.1	253.4	244.6	260.8	268.6
30-34	107.7	98.8	95.3	117.1	118.3
35-39	37.9	34.3	31.4	40.5	40.7
40-44	11.5	10.4	9.4	14.1	14.5
45-49	2.8	2.2	2	3.6	3.6

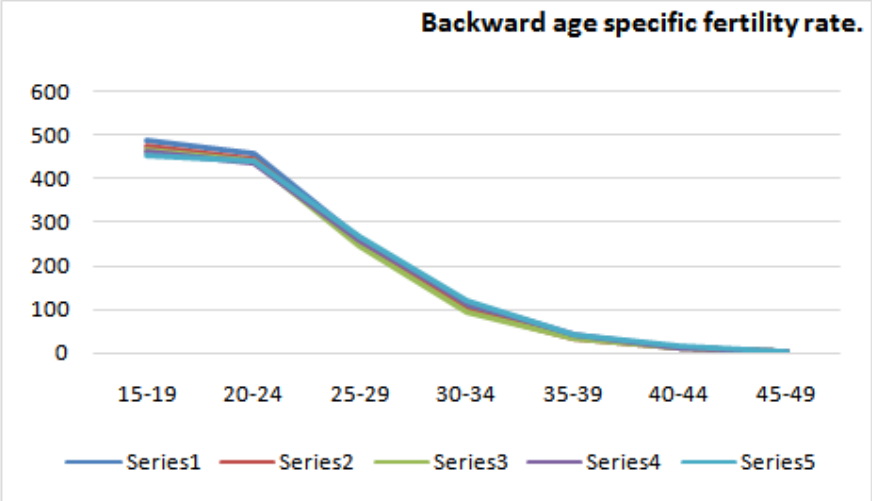

Figure 3: Distribution of Backward Age Specific Fertility rate in India

The Fig. 3 shows the trend of backward age specific fertility rate and it observed that there is a decline in the BASFRonly in the age group 15-19 for all the years. For each age group BASFR declines and then slightly increases in the year 2015.

To examine the fertility pattern in India and its states, linear and non linear models are fitted to the available data by taking the midvalue of age groups as the independent variable and the ASFR/FASFR/BASFRas the dependent variable. Tables 4, 56 give the R^2 and p value for the year 2011 obtained by fitting Quadratic, compound, cubic, power and "S" models to the ASFR, FASFR and BASFR data for all the states of India respectively.

Table 4: R^2 and p value for the fitted models(ASFR)

State name	Quadratic model		Compound model		Cubic model		Power model		"S" model	
	R	p value	R	p value	R	p value	R	p value	R	p value
India	0.499	0.251	0.587	0.045	0.895	0.056	0.447	0.101	0.306	0.198
Andhra Pradesh	0.424	0.331	0.803	0.006	0.748	0.197	0.675	0.023	0.532	0.063
Assam	0.608	0.153	0.624	0.034	0.945	0.022	0.484	0.083	0.343	0.167
Bihar	0.568	0.187	0.399	0.128	0.937	0.026	0.266	0.236	0.149	0.393
Chhattisgarh	0.494	0.256	0.656	0.027	0.871	0.076	0.515	0.07	0.371	0.147
Delhi	0.493	0.257	0.467	0.091	0.934	0.028	0.327	0.18	0.199	0.315
Gujarat	0.477	0.274	0.548	0.057	0.908	0.046	0.413	0.119	0.276	0.226
Haryana	0.394	0.367	0.569	0.05	0.805	0.37	0.43	0.11	0.295	0.207
Himachal Pradesh	0.416	0.341	0.556	0.054	0.86	0.085	0.426	0.112	0.29	0.212
Jammu &Kashmir	0.706	0.087	0.066	0.578	0.972	0.008	0.014	0.8	0.001	0.954
Jharkhand	0.761	0.178	0.512	0.07	0.956	0.015	0.373	0.145	0.239	0.266
Karnataka	0.44	0.314	0.772	0.009	0.798	0.144	0.649	0.029	0.506	0.073
Kerala	0.503	0.247	0.637	0.031	0.93	0.031	0.494	0.078	0.35	0.162
Madhya Pradesh	0.442	0.311	0.566	0.051	0.845	0.098	0.431	0.109	0.293	0.21
Maharashtra	0.433	0.322	0.739	0.013	0.809	0.133	0.608	0.039	0.462	0.093
Odisha	0.537	0.215	0.606	0.039	0.932	0.029	0.465	0.091	0.323	0.183

Punjab	0.443	0.31	0.523	0.066	0.897	0.054	0.383	0.138	0.248	0.256
Rajasthan	0.493	0.257	0.533	0.063	0.899	0.053	0.395	0.131	0.259	0.244
Tamil Nadu	0.447	0.305	0.678	0.023	0.877	0.07	0.542	0.059	0.397	0.129
Uttar Pradesh	0.573	0.182	0.346	0.165	0.96	0.013	0.22	0.289	0.112	0.496
West Bengal	0.574	0.181	0.841	0.004	0.839	0.104	0.729	0.015	0.591	0.043

Table 4 shows that inthequadratic models all the states are not significant . In the Power model Andhra Pradesh, Karnataka,Maharashtra and West Bengal are the significant states.Inthe cubic model Assam, Odisha, Kerala,Gujarat, Punjab,Jharkhand, Jammu&Kashmir,Rajasthan and Uttar Pradesh are the significant states. In the compound model the states like Chhattisgarh, Karnataka, Maharashtra, West Bengal, Andhra Pradesh, Haryana, Kerala, Odisha, Assam, Himachal Pradesh, Mathya Pradesh and Tamil Nadu are significant besides the whole country.West Bengal is the only one state which is significant in the S model.

Table 5: R^2 and p value for the fitted models:(FASFR)

State name	Quadratic model		Compound model		Cubic model		Power model		"S" model	
Fasfr(2011)	R	Sig	R	Sig	R	Sig	R	Sig	R	Sig
India	0.989	**0**	0.586	0.045	0.999	**0**	0.715	0.017	0.832	0.004
Andhra Pradesh	0.946	**0.003**	0.513	0.07	0.998	**0**	0.643	0.03	0.768	0.01
Assam	0.995	**0**	0.651	0.028	0.998	**0**	0.776	0.009	0.883	0.002
Bihar	0.997	**0**	0.619	0.036	0.998	**0**	0.745	0.012	0.857	0.003
Chhattisgarh	0.985	**0**	0.576	0.048	0.999	**0**	0.705	0.018	0.824	0.005
Delhi	0.99	**0**	0.554	0.055	0.996	**0**	0.684	0.022	0.805	0.006
Gujarat	0.987	**0**	0.569	0.05	0.997	**0**	0.699	0.019	0.819	0.005
Haryana	0.971	**0.001**	0.516	0.069	0.999	**0**	0.645	0.03	0.77	0.009
Himachal Pradesh	0.977	**0.001**	0.525	0.065	0.998	**0**	0.655	0.028	0.779	0.009
Jammu & Kashmir	0.989	**0**	0.645	0.03	0.992	**0.001**	0.771	0.009	0.879	0.002
Jharkhand	0.995	**0**	0.636	0.032	0.997	**0**	0.762	0.01	0.872	0.002
Karnataka	0.961	**0.002**	0.53	0.063	0.999	**0**	0.66	0.026	0.784	0.008
Kerala	0.987	**0**	0.575	0.048	0.996	**0**	0.706	0.018	0.825	0.005
Madhya Pradesh	0.982	**0**	0.555	0.055	0.999	**0**	0.683	0.022	0.804	0.006
Maharashtra	0.967	**0.001**	0.533	0.062	0.999	**0**	0.663	0.026	0.786	0.008
Odisha	0.991	**0**	0.603	0.04	0.998	**0**	0.731	0.014	0.847	0.003
Punjab	0.982	**0**	0.537	0.061	0.996	**0**	0.667	0.025	0.79	0.007
Rajasthan	0.99	**0**	0.584	0.046	0.999	**0**	0.712	0.017	0.83	0.004
Tamil Nadu	0.977	**0.001**	0.546	0.058	0.997	**0**	0.677	0.023	0.799	0.007
Uttar Pradesh	0.996	**0**	0.618	0.036	0.997	**0**	0.745	0.012	0.857	0.003
West Bengal	0.971	**0.001**	0.581	0.046	1	**0**	0.71	0.017	0.828	0.004

Table 5 shows that in all the models like the quadratic model,the cubic model, the power model and the S model all the states are significant . Only in the compound model the states like Andhra Pradesh, Delhi, Haryana,HimachalPradesh,Karnataka,MadhyaPradesh,Maharashtra, Punjab,and Tamil Nadu are not significant.

Table 6: R^2 and P value for the fitted models:(BASFR)

State name	Quadratic model		Compound model		Cubic model		Power model		"S" model	
Basfr(2011)	R	Sig	R	Sig	R	Sig	R	Sig	R	Sig
India	0.958	0.002	0.945	0	0.98	0.005	0.86	0.003	0.746	0.012
Andhra Pradesh	0.952	0.002	0.942	0	0.959	0.014	0.857	0.003	0.742	0.013
Assam	0.97	0.001	0.917	0.001	0.989	0.002	0.825	0.005	0.707	0.018
Bihar	0.962	0.001	0.918	0.001	0.987	0.002	0.823	0.005	0.703	0.018
Chhattisgarh	0.959	0.002	0.913	0.001	0.977	0.006	0.817	0.005	0.696	0.02
Delhi	0.945	0.003	0.892	0.001	0.979	0.005	0.79	0.007	0.666	0.025
Gujarat	0.951	0.002	0.953	0	0.978	0.006	0.875	0.002	0.763	0.01
Haryana	0.943	0.003	0.868	0.002	0.962	0.012	0.767	0.01	0.646	0.029
Himachal Pradesh	0.942	0.003	0.953	0	0.967	0.01	0.878	0.002	0.769	0.01
Jammu & Kashmir	0.951	0.002	0.903	0.001	0.996	0	0.802	0.006	0.677	0.023
Jharkhand	0.963	0.001	0.941	0	0.988	0.002	0.854	0.003	0.738	0.013
Karnataka	0.953	0.002	0.953	0	0.965	0.011	0.876	0.002	0.765	0.01
Kerala	0.951	0.002	0.906	0.001	0.979	0.005	0.806	0.006	0.682	0.022
Madhya Pradesh	0.953	0.002	0.958	0	0.973	0.007	0.882	0.002	0.772	0.009
Maharashtra	0.952	0.002	0.95	0	0.966	0.011	0.868	0.002	0.755	0.011
Odisha	0.96	0.002	0.941	0	0.984	0.003	0.854	0.003	0.738	0.013
Punjab	0.941	0.003	0.937	0	0.972	0.008	0.849	0.003	0.731	0.014
Rajasthan	0.957	0.002	0.948	0	0.98	0.005	0.865	0.002	0.751	0.012
Tamil Nadu	0.947	0.003	0.935	0	0.971	0.008	0.848	0.003	0.731	0.014
Uttar Pradesh	0.958	0.002	0.928	0	0.988	0.002	0.836	0.004	0.716	0.016
West Bengal	0.974	0.001	0.968	0	0.979	0.005	0.898	0.001	0.793	0.007

Table 6 shows that for all the five models all the states are significant.

The estimated cross validity prediction power and shrinkage corresponding to models fitted in Table 4,5 and 6 are presented in Tables 7, 8 and 9.

Table 7: ρ^2v and Shrinkage – ASFR

State name	Quadratic model		Compound model		Cubic model		Power model		"S" model	
Asfr 2011	**P**	**shr**	**P**	**shr**	**ρ**	**Shr**	**P**	**Shr**	**ρ**	**shr**
India	0.141	0.358	0.292	0.295	0.82	0.075	0.052	0.395	-0.19	0.496
Andhra Pradesh	0.013	0.411	0.662	0.141	0.5681	0.1799	0.443	0.232	0.1978	0.334
Assam	0.328	0.28	0.356	0.268	0.9057	0.0393	0.116	0.368	-0.126	0.469
Bihar	0.26	0.308	-0.03	0.429	0.892	0.045	-0.26	0.524	-0.459	0.608
Chhattisgarh	0.133	0.361	0.41	0.246	0.7789	0.0921	0.169	0.346	-0.078	0.449
Delhi	0.131	0.362	0.086	0.381	0.8869	0.0471	-0.15	0.481	-0.373	0.572
Gujarat	0.104	0.373	0.225	0.323	0.8423	0.0657	-0.01	0.419	-0.241	0.517
Haryana	-0.039	0.433	0.261	0.308	0.6658	0.1392	0.023	0.407	-0.208	0.503
Himachal Pradesh	-1E-03	0.417	0.239	0.317	0.76	0.1	0.016	0.41	-0.217	0.507
Jammu & Kashmir	0.496	0.21	-0.6	0.667	0.952	0.02	-0.69	0.704	-0.712	0.713
Jharkhand	0.59	0.171	0.164	0.348	0.9246	0.0314	-0.07	0.448	-0.304	0.543
Karnataka	0.04	0.4	0.609	0.163	0.6538	0.1442	0.398	0.251	0.1533	0.353
Kerala	0.148	0.355	0.378	0.259	0.88	0.05	0.133	0.361	-0.114	0.464
MadhyaPradesh	0.044	0.398	0.256	0.31	0.7343	0.1107	0.025	0.406	-0.212	0.505
Maharashtra	0.028	0.405	0.553	0.186	0.6726	0.1364	0.328	0.28	0.0779	0.384
Odisha	0.206	0.331	0.325	0.281	0.8834	0.0486	0.083	0.382	-0.16	0.483

Punjab	0.045	0.398	0.182	0.341	0.8235	0.0735	-0.06	0.441	-0.289	0.537
Rajasthan	0.131	0.362	0.2	0.333	0.8269	0.0721	-0.04	0.432	-0.27	0.529
Tamil Nadu	0.052	0.395	0.448	0.23	0.7892	0.0878	0.215	0.327	-0.034	0.431
Uttar Pradesh	0.268	0.305	-0.12	0.467	0.9314	0.0286	-0.34	0.557	-0.522	0.634
West Bengal	0.27	0.304	0.727	0.114	0.724	0.115	0.536	0.193	0.299	0.292

ρ= $\rho^2 v$ **shr=Shrinkage**

Table 8: $\rho^2 v$ and Shrinkage – FASFR

State name	Quadratic model		Compound model		Cubic model		Power model		"S" model	
fasfr 2011	**P**	**Shr**	**P**	**shr**	**ρ**	**Shr**	**P**	**Shr**	**P**	**Shr**
India	0.9811	0.008	0.29	0.296	0.998	0.0007	0.512	0.203	0.712	0.12
Andhra Pradesh	0.9074	0.039	0.165	0.348	0.997	0.0014	0.388	0.255	0.602	0.1656
Assam	0.9914	0.004	0.402	0.249	0.997	0.0014	0.616	0.16	0.799	0.0835
Bihar	0.9949	0.002	0.347	0.272	0.997	0.0014	0.563	0.182	0.755	0.1021
Chhattisgarh	0.9743	0.011	0.273	0.303	0.998	0.0007	0.494	0.211	0.698	0.1257
Delhi	0.9829	0.007	0.236	0.318	0.993	0.0029	0.458	0.226	0.666	0.1392
Gujarat	0.9777	0.009	0.261	0.308	0.995	0.0021	0.484	0.215	0.69	0.1292
Haryana	0.9503	0.021	0.17	0.346	0.998	0.0007	0.392	0.253	0.606	0.1642
Himachal Pradesh	0.9606	0.016	0.186	0.339	0.997	0.0014	0.409	0.246	0.621	0.1578
Jammu & Kashmir	0.9811	0.008	0.392	0.253	0.986	0.0057	0.607	0.164	0.793	0.0864
Jharkhand	0.9914	0.004	0.376	0.26	0.995	0.0021	0.592	0.17	0.781	0.0914
Karnataka	0.9332	0.028	0.194	0.336	0.998	0.0007	0.417	0.243	0.63	0.1542
Kerala	0.9777	0.009	0.272	0.303	0.993	0.0029	0.496	0.21	0.7	0.125
Madhya Pradesh	0.9691	0.013	0.237	0.318	0.998	0.0007	0.457	0.226	0.664	0.1399
Maharashtra	0.9434	0.024	0.2	0.333	0.998	0.0007	0.422	0.241	0.633	0.1528
Odisha	0.9846	0.006	0.32	0.283	0.997	0.0014	0.539	0.192	0.738	0.1092
Punjab	0.9691	0.013	0.206	0.331	0.993	0.0029	0.429	0.238	0.64	0.1499
Rajasthan	0.9829	0.007	0.287	0.297	0.998	0.0007	0.506	0.206	0.709	0.1214
Tamil Nadu	0.9606	0.016	0.222	0.324	0.995	0.0021	0.446	0.231	0.655	0.1435
Uttar Pradesh	0.9931	0.003	0.345	0.273	0.995	0.0021	0.563	0.182	0.755	0.1021
West Bengal	0.9503	0.021	0.282	0.299	1	0	0.503	0.207	0.705	0.1228

Table 9: $\rho^2 v$ and Shrinkage - BASFR

State name	Quadratic model		Compound model		Cubic model		Power model		"S" model	
basfr 2011	ρ	**Shr**	ρ	**Shr**	ρ	**Shr**	ρ	**Shr**	ρ	**Shr**
India	0.928	0.03	0.906	0.0393	**0.9657**	0.014	0.8356	0.024	0.565	0.181
Andhra Pradesh	0.9177	0.0343	0.901	0.0414	**0.9297**	0.029	0.8321	0.025	0.558	0.184
Assam	0.9486	0.0214	0.858	0.0593	**0.9811**	0.008	0.7946	0.03	0.498	0.209
Bihar	0.9349	0.0271	0.859	0.0585	**0.9777**	0.009	0.7922	0.031	0.491	0.212
Chhattisgarh	0.9297	0.0293	0.851	0.0621	**0.9606**	0.016	0.7852	0.032	0.479	0.217
Delhi	0.9057	0.0393	0.815	0.0771	**0.964**	0.015	0.7535	0.037	0.428	0.238
Gujarat	0.916	0.035	0.919	0.0336	**0.9623**	0.016	0.8533	0.022	0.594	0.169
Haryana	0.9023	0.0407	0.774	0.0942	**0.9349**	0.027	0.7265	0.041	0.393	0.253
HimachalPradesh	0.9006	0.0414	0.919	0.0336	**0.9434**	0.024	0.8568	0.021	0.604	0.165
Jammu&Kashmir	0.916	0.035	0.834	0.0693	**0.9931**	0.003	0.7675	0.034	0.446	0.231
Jharkhand	0.9366	0.0264	0.899	0.0421	**0.9794**	0.009	0.8286	0.025	0.551	0.187

Karnataka	0.9194	0.0336	0.919	0.0336	**0.94**	0.025	0.8544	0.022	0.597	0.168
Kerala	0.916	0.035	0.839	0.0671	**0.964**	0.015	0.7722	0.034	0.455	0.227
Madhya Pradesh	0.9194	0.0336	0.928	0.03	**0.9537**	0.019	0.8615	0.021	0.609	0.163
Maharashtra	0.9177	0.0343	0.914	0.0357	**0.9417**	0.024	0.845	0.023	0.58	0.175
Odisha	0.9314	0.0286	0.899	0.0421	**0.9726**	0.011	0.8286	0.025	0.551	0.187
Punjab	0.8989	0.0421	0.892	0.045	**0.952**	0.02	0.8227	0.026	0.539	0.192
Rajasthan	0.9263	0.0307	0.911	0.0371	**0.9657**	0.014	0.8415	0.023	0.573	0.178
Tamil Nadu	0.9092	0.0378	0.889	0.0464	**0.9503**	0.021	0.8216	0.026	0.539	0.192
U.P	0.928	0.03	0.877	0.0514	**0.9794**	0.009	0.8075	0.029	0.513	0.203
West Bengal	0.9554	0.0186	0.945	0.0228	**0.964**	0.015	0.8803	0.018	0.645	0.148

A summary of the significant states identified by fitting models to ASFR/BASFR/FASFR is given in the Tables 10, 11 and 12.

Table 10: Significant States in all the models for Age specific fertility rate in India:

ASFR 2011	N	P	significant state	R	$\rho^2 v$	Shrinkage
Quadratic model	7	1	Nil	Nil	Nil	Nil
Cubic model	7	1	Assam	0.945	0.906	0.039
			Gujarat	0.908	0.842	0.066
			Jammu &Kashmir	0.972	0.952	0.02
			Jharkhand	0.956	0.9246	0.0314
			Kerala	0.93	0.88	0.05
			Odisha	0.932	0.883	0.049
			Punjab	0.897	0.824	0.074
			Rajasthan	0.899	0.827	0.072
			Uttar Pradesh	0.96	0.931	0.029
Compound model	7	1	India	0.587	0.292	0.294
			Andhra Pradesh	0.803	0.662	0.14
			Assam	0.624	0.355	0.268
			Chhattisgarh	0.656	0.41	0.308
			Haryana	0.569	0.261	0.317
			Himachal Pradesh	0.556	0.239	0.246
			Karnataka	0.772	0.609	0.163
			Kerala	0.637	0.378	0.259
			Madhya Pradesh	0.566	0.256	0.31
			Maharashtra	0.739	0.553	0.186
			Odisha	0.606	0.325	0.281
			Tamil Nadu	0.678	0.448	0.23
			West Bengal	0.841	0.727	0.114
Power model	7	1	Andhra Pradesh	0.675	0.443	0.232
			Karnataka	0.649	0.398	0.251
			Maharashtra	0.608	0.328	0.28
			West Bengal	0.729	0.536	0.193
“s” model	7	1	West Bengal	0.591	0.299	0.292

Table 11: Significant States in all the models for Backward Age specific fertility rate in India:

Basfr(2011)	n	P	significant state	R value	$\rho^2 v$	Shrinkage
Quadratic model	7	1	India	0.958	0.928012	0.029988
			Andhra Pradesh	0.952	0.917728	0.034272
			Assam	0.97	0.94858	0.02142
			Bihar	0.962	0.934868	0.027132
			Chhattisgarh	0.959	0.929726	0.029274
			Delhi	0.945	0.90573	0.03927
			Gujarat	0.951	0.916014	0.034986
			Haryana	0.943	0.902302	0.040698
			Himachal Pradesh	0.942	0.900588	0.041412
			Jammu & Kashmir	0.951	0.916014	0.034986
			Jharkhand	0.963	0.936582	0.026418
			Karnataka	0.953	0.919442	0.033558
			Kerala	0.951	0.916014	0.034986
			Madhya Pradesh	0.953	0.919442	0.033558
			Maharashtra	0.952	0.917728	0.034272
			Odisha	0.96	0.93144	0.02856
			Punjab	0.941	0.898874	0.042126
			Rajasthan	0.957	0.926298	0.030702
			Tamil Nadu	0.947	0.909158	0.037842
			Uttar Pradesh	0.958	0.928012	0.029988
			West Bengal	0.974	0.955436	0.018564
Cubic model	7	1	India	0.98	0.96572	0.01428
			Andhra Pradesh	0.959	0.929726	0.029274
			Assam	0.989	0.981146	0.007854
			Bihar	0.987	0.977718	0.009282
			Chhattisgarh	0.977	0.960578	0.016422
			Delhi	0.979	0.964006	0.014994
			Gujarat	0.978	0.962292	0.015708
			Haryana	0.962	0.934868	0.027132
			Himachal Pradesh	0.967	0.943438	0.023562
			Jammu & Kashmir	0.996	0.993144	0.002856
			Jharkhand	0.988	0.979432	0.008568
			Karnataka	0.965	0.94001	0.02499
			Kerala	0.979	0.964006	0.014994
			Madhya Pradesh	0.973	0.953722	0.019278
			Maharashtra	0.966	0.941724	0.024276
			Odisha	0.984	0.972576	0.011424
			Punjab	0.972	0.952008	0.019992
			Rajasthan	0.98	0.96572	0.01428
			Tamil Nadu	0.971	0.950294	0.020706
			Uttar Pradesh	0.988	0.979432	0.008568
			West Bengal	0.979	0.964006	0.014994
Compound model	7	1	India	0.945	0.90573	0.03927
			Andhra Pradesh	0.942	0.900588	0.041412
			Assam	0.917	0.857738	0.059262
			Bihar	0.918	0.859452	0.058548
			Chhattisgarh	0.913	0.850882	0.062118
			Delhi	0.892	0.814888	0.077112
			Gujarat	0.953	0.919442	0.033558
			Haryana	0.868	0.773752	0.094248
			Himachal Pradesh	0.953	0.919442	0.033558
			Jammu & Kashmir	0.903	0.833742	0.069258

			Jharkhand	0.941	0.898874	0.042126
			Karnataka	0.953	0.919442	0.033558
			Kerala	0.906	0.838884	0.067116
			Madhya Pradesh	0.958	0.928012	0.029988
			Maharashtra	0.95	0.9143	0.0357
			Odisha	0.941	0.898874	0.042126
			Punjab	0.937	0.892018	0.044982
			Rajasthan	0.948	0.910872	0.037128
			Tamil Nadu	0.935	0.88859	0.04641
			Uttar Pradesh	0.928	0.876592	0.051408
			West Bengal	0.968	0.945152	0.022848
Power model	7	1	India	0.86	0.83564	0.02436
			Andhra Pradesh	0.857	0.832118	0.024882
			Assam	0.825	0.79455	0.03045
			Bihar	0.823	0.792202	0.030798
			Chhattisgarh	0.817	0.785158	0.031842
			Delhi	0.79	0.75346	0.03654
			Gujarat	0.875	0.85325	0.02175
			Haryana	0.767	0.726458	0.040542
			Himachal Pradesh	0.878	0.856772	0.021228
			Jammu & Kashmir	0.802	0.767548	0.034452
			Jharkhand	0.854	0.828596	0.025404
			Karnataka	0.876	0.854424	0.021576
			Kerala	0.806	0.772244	0.033756
			Madhya Pradesh	0.882	0.861468	0.020532
			Maharashtra	0.868	0.845032	0.022968
			Odisha	0.854	0.828596	0.025404
			Punjab	0.849	0.822726	0.026274
			Rajasthan	0.865	0.84151	0.02349
			Tamil Nadu	0.848	0.821552	0.026448
			Uttar Pradesh	0.836	0.807464	0.028536
			West Bengal	0.898	0.880252	0.017748
" S " Model	7	1	India	0.746	0.564644	0.181356
			Andhra Pradesh	0.742	0.557788	0.184212
			Assam	0.707	0.497798	0.209202
			Bihar	0.703	0.490942	0.212058
			Chhattisgarh	0.696	0.478944	0.217056
			Delhi	0.666	0.427524	0.238476
			Gujarat	0.763	0.593782	0.169218
			Haryana	0.646	0.393244	0.252756
			Himachal Pradesh	0.769	0.604066	0.164934
			Jammu & Kashmir	0.677	0.446378	0.230622
			Jharkhand	0.738	0.550932	0.187068
			Karnataka	0.765	0.59721	0.16779
			Kerala	0.682	0.454948	0.227052
			Madhya Pradesh	0.772	0.609208	0.162792
			Maharashtra	0.755	0.58007	0.17493
			Odisha	0.738	0.550932	0.187068
			Punjab	0.731	0.538934	0.192066
			Rajasthan	0.751	0.573214	0.177786
			Tamil Nadu	0.731	0.538934	0.192066
			Uttar Pradesh	0.716	0.513224	0.202776
			West Bengal	0.793	0.645202	0.147798

Table 12: Significant States in all the models for Forward Age specific fertility rate in India

Fasfr(2011)	n	P	Significant state	R value	ρ^2v	Shrinkage
Quadratic model	7	1	India	0.989	0.981	0.007
			Andhra Pradesh	0.946	0.907	0.038
			Assam	0.995	0.991	0.004
			Bihar	0.997	0.994	0.002
			Chhattisgarh	0.985	0.974	0.010
			Delhi	0.99	0.982	0.007
			Gujarat	0.987	0.977	0.009
			Haryana	0.971	0.950	0.021
			Himachal Pradesh	0.977	0.960	0.016
			Jammu & Kashmir	0.989	0.981	0.007
			Jharkhand	0.995	0.991	0.003
			Karnataka	0.961	0.933	0.027
			Kerala	0.987	0.977	0.009
			Madhya Pradesh	0.982	0.969	0.012
			Maharashtra	0.967	0.943	0.023
			Odisha	0.991	0.984	0.006
			Punjab	0.982	0.969	0.012
			Rajasthan	0.99	0.982	0.007
			Tamil Nadu	0.977	0.960	0.016
			Uttar Pradesh	0.996	0.993	0.002
			West Bengal	0.971	0.950	0.020
Cubic model	7	1	India	0.999	0.998	0.001
			Andhra Pradesh	0.998	0.996	0.001
			Assam	0.998	0.996	0.001
			Bihar	0.998	0.996	0.001
			Chhattisgarh	0.999	0.998	0.001
			Delhi	0.996	0.993	0.0028
			Gujarat	0.997	0.994	0.0021
			Haryana	0.999	0.998	0.0007
			Himachal Pradesh	0.998	0.996	0.0014
			Jammu & Kashmir	0.992	0.986	0.0057
			Jharkhand	0.997	0.99485	0.0021
			Karnataka	0.999	0.998	0.0007
			Kerala	0.996	0.993	0.0028
			Madhya Pradesh	0.999	0.998	0.0007
			Maharashtra	0.999	0.998	0.0007
			Odisha	0.998	0.996	0.0014
			Punjab	0.996	0.993	0.0028
			Rajasthan	0.999	0.998	0.0007
			Tamil Nadu	0.997	0.994	0.0021
			Uttar Pradesh	0.997	0.994	0.0021
			West Bengal	1	1	0
Compound model	7	1	India	0.586	0.2904	0.2955
			Assam	0.651	0.4018	0.2491
			Bihar	0.619	0.3469	0.2720
			Chhattisgarh	0.576	0.2732	0.3027
			Gujarat	0.569	0.2612	0.3077
			Jammu & Kashmir	0.645	0.3915	0.2534
			Jharkhand	0.636	0.3761	0.2598
			Kerala	0.575	0.2715	0.3034
			Odisha	0.603	0.3195	0.2834

			Rajasthan	0.584	0.2869	0.2970
			Uttar Pradesh	0.618	0.3452	0.2727
			West Bengal	0.581	0.2818	0.2991
Power model	7	1	India	0.715	0.5115	0.2034
			Andhra Pradesh	0.643	0.3881	0.2548
			Assam	0.776	0.6160	0.1599
			Bihar	0.745	0.5629	0.1820
			Chhattisgarh	0.705	0.4943	0.2106
			Delhi	0.684	0.4583	0.2256
			Gujarat	0.699	0.4840	0.2149
			Haryana	0.645	0.3915	0.2534
			Himachal Pradesh	0.655	0.4086	0.2463
			Jammu & Kashmir	0.771	0.6074	0.1635
			Jharkhand	0.762	0.5920	0.1699
			Karnataka	0.66	0.4172	0.2427
			Kerala	0.706	0.4960	0.2099
			Madhya Pradesh	0.683	0.4566	0.2263
			Maharashtra	0.663	0.4223	0.2406
			Odisha	0.731	0.5389	0.1920
			Punjab	0.667	0.4292	0.2377
			Rajasthan	0.712	0.5063	0.2056
			Tamil Nadu	0.677	0.4463	0.2306
			Uttar Pradesh	0.745	0.5629	0.1820
			West Bengal	0.71	0.5029	0.2070
" S " Model	7	1	India	0.832	0.7120	0.1199
			Andhra Pradesh	0.768	0.6023	0.1656
			Assam	0.883	0.7994	0.0835
			Bihar	0.857	0.7548	0.1021
			Chhattisgarh	0.824	0.6983	0.1256
			Delhi	0.805	0.6657	0.1392
			Gujarat	0.819	0.6897	0.1292
			Haryana	0.77	0.6057	0.1642
			Himachal Pradesh	0.779	0.6212	0.1577
			Jammu & Kashmir	0.879	0.7926	0.0863
			Jharkhand	0.872	0.7806	0.0913
			Karnataka	0.784	0.6297	0.1542
			Kerala	0.825	0.7000	0.1249
			Madhya Pradesh	0.804	0.6640	0.1399
			Maharashtra	0.786	0.6332	0.1527
			Odisha	0.847	0.7377	0.1092
			Punjab	0.79	0.6400	0.1499
			Rajasthan	0.83	0.7086	0.1213
			Tamil Nadu	0.799	0.6554	0.1435
			Uttar Pradesh	0.857	0.7548	0.1021
			West Bengal	0.828	0.7051	0.1228

4. RESULTS AND FINDINGS

To check the suitability of models on population data obtained from different SRS reports of India and its major states, similar analysis was done for the years 2012 to 2015 for ASFR, FASFR, BASFR.From the analysis for the years 2012 and 2013 all the states are significant in all the models. Other thanthecompound model in 2014 and 2015 for all the states the models are significant in forward age specific fertility rate. Jharkhand and Jammu & Kashmir are the significant states and incase of power modelAndhra Pradesh, Karnataka, Maharashtra and West Bengal are the significant states in the year 2012.

Andhra Pradesh, Assam, Chhattisgarh, Gujarat, Himachal Pradesh, Kerala, Madhya Pradesh, Maharashtra, Rajasthan, TamilNadu and West Bengal are the significant states in the compound model. By using power model Andhra Pradesh, Karnataka, Maharashtra and West Bengal are the states which are significant. West

Bengal alone is the significant state in the model 'S' for age specific fertility rate in the year 2012. All the states are not significant by using the linear model, logarithmic model, inverse model and quardratic model in the year 2014.

In thecubic model other than the states like Andhra Pradesh, Chhattisgarh, Karnataka, Madhya Pradesh, Maharashtra and West Bengal all other states are significant. In the compound model Andhra Pradesh, Assam, Chhattisgarh, Gujarat, Himachal Pradesh, Kerala, Madhya Pradesh, Maharashtra, Rajasthan, Tamil Nadu and West Bengal are the significant states. Assam, TamilNadu and West Bengal are the three states which are significant in the power model in age specific fertility rate in the year 2014.

In all the four models for the age specific fertility rate in the year 2015 all the states are not significant. In the year 2015 using power model and S model all the states are not significant. In the compound model West Bengal alone is the significant state but in the cubic model India, Assam, Bihar, Delhi, Gujarat, Himachal Pradesh, Jammu & Kashmir, Jharkhand, Odisha, Punjab, Rajasthan, Tamil Nadu and Uttar Pradesh are the significant states.

5.CONCLUSIONS

The age specific fertility rate, forward age specific fertility rate and backward age specific fertility rate for the year 2011 to 2015 are fitted for the five models. In the year 2012 forward age specific fertility rate and backward age specific fertility rate for all the models as well as all the states are siginificant. In age specific fertility rate the states like Assam,Himachal Pradesh, Jharkhand, Kerala and Odisha are significant in the cubic model and compound model.In the year 2013 forward age specific fertility rate and backward age specific fertility rate for all the models as well as all the states are siginificant. In age specific fertility rate the states like Assam, Jharkhand, Kerala and Odisha are significant in the cubic model and compound model.

In the year 2014forward age specific fertility rate and backward age specific fertility rate for all the models as well as all the states are siginificant but in age specific fertility rate the states like Assam, Gujarat, Himachal Pradesh , Kerala, Rajasthan and Tamil Nadu are significant in the cubic model and compound model.

In the year 2015 forward age specific fertility rate and backward age specific fertility rate for all the modelsas well as all the states are siginificant but in the age specific fertility rate other than the states like Andhra Pradesh, Chhattisgarh, Haryana, Karnataka, Madhya Pradesh, West Bengal are significant states in the cubic model.

Compared to the age specific fertility rate fitted model in all the states and also in all the years, the forward age fertility rate and the backward age specific fertility rate gives the best fitted model. Based on the cross validity prediction it is observed that a cubic trend in ASFR/FASFR and BASFR exhibited by states in India is the best predictive model.

6. REFERENCES

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