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STABILITY ANALYSIS OF A NON-LINEAR HARVESTING MODEL WITH TAXATION

Aktar Saikh[1], Md. Manarul Haque[2]

Author Affiliation:
[1]Department of Mathematics and Statistics, Aliah University, IIA/27, New Town, Kolkata-700160, India
E-mail: asaikh.math@gmail.com
[2]Department of Mathematics and Statistics, Aliah University, IIA/27, New Town, Kolkata-700160, India
E-mail: mana.math90@gmail.com

***Corresponding Author:**
Md. Manarul Haque, Department of Mathematics and Statistics, Aliah University, IIA/27, New Town, Kolkata-700160, India

E-mail: mana.math90@gmail.com

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Abstract

This paper formulates and analyzes a non-linear harvesting of prey, considered in a prey-predator system. Here, the effort is taken as a dynamic variable. Mathematical analysis of the model system includes the existence, uniqueness and uniform boundedness of the solutions in the positive octant. Authoritarian agency has power over exploitation by striking a tax per unit biomass of the prey species. The conditions for the existence of the positive steady states of the model system are derived. Also the local and global stabilities of several equilibrium points are studied using the Routh Hurwitz criteria and Lyapunov function respectively. Numerical simulations are performed by making use of MATLAB to justify the analytical conclusions.

Keywords: Non-linear harvesting, Uniform boundedness, Local stability, Global stability, Numerical simulations.

1. INTRODUCTION

World population is expanding at an ever increasing rate, making more demand for products and goods. Some of these products come from various renewable resources. Fisheries are renewable resources because fish reproduce. Although these resources are renewable, the quality of the resource may of course differ and sometimes the utility of the resource is wasted. Haphazard fishing will lead to a collapse in fisheries. Therefore, the question comes off how we can productively govern as to these resources to prevent a catastrophic bust to our global economy. Thus our study is focused on to manage the renewable resources, considering the economic questions such as revenue, cost and also the impact of this demand on the resource to manage the ecosystem in a sustainable way.

The finest plausible organization of restorable resources like fishery, which has a direct alliance to sustainable development, has been deliberated by Clark [1]. In addition, financial and ecological management of restorable resources has been reviewed by Bhattacharya and Begum [2], Clark [3], Goh [4], Leung and Wang [5], Mesterton-Gibbons [6, 7] and Kitubatake [8]. The problem of joined yielding of two contending fish species is analyzed by Chaudhuri [9]. Ganguli and Chaudhuri [10] deliberated a model to reconcile the instructions of single species fishery through taxes. Rent of asset rights, excise, permit charge, regular harvesting etc. are continuously regarded as effective prevailing tools for the proper guidelines of utilization of natural resources which have presently become a problem of major concern. Harvesting problems with excise as a control key are studied by many authors like Chaudhuri and Johnson [11], Krishna et al. [12], Kar and Chaudhuri [13] and Dubey et al. [14]. Here, we consider a prey predator model with harvesting and taxation [15, 16], which seems to be more realistic. In the present paper, we consider a predator- prey model with non-linear harvesting on prey while the effort is taken as a dynamic variable. The present study concerns with the stability of a dynamic reaction model in the case of a prey-predator type fishery system, where only the prey species is subjected to non-linear harvesting.

2. THE MATHEMATICAL MODEL

Let $x(t), y(t)$ be the dynamic variables of the densities of the prey population, predator population respectively and $E(t)$ be the harvesting effort at time t. The logistic growth is taken for the prey species. The predator population consumes the prey by the rule of Holling type-II functional response and non-linear harvesting is considered for the prey species. Also we assume hyperbolic mortality for the predator population. The above postulates are governed by the system of three first order nonlinear differential equations:

$$\left.\begin{aligned}\frac{dx}{dt}&=rx\left(1-\frac{x}{k}\right)-\frac{\alpha xy}{a+x}-\frac{qEx}{lE+mx},\\ \frac{dy}{dt}&=\frac{e\alpha xy}{a+x}-\frac{\beta y^2}{a+x},\\ \frac{dE}{dt}&=\eta E\left\{\frac{q(p-T)x}{lE+mx}-c\right\}\end{aligned}\right\}\qquad (2.1)$$

with initial conditions $x(0)>0, y(0)>0, E(0)>0$; where, $r,k,\alpha,a,q,l,m,e,\beta,\eta,p,T$ and c all are positive constants having the following biological meanings: $r\equiv$ intrinsic growth rate of the prey population, $k\equiv$ environmental carrying capacity of the prey population, $\alpha\equiv$ encounter rate of predator to prey, $a\equiv$ half-saturation constant, $q\equiv$ catchability coefficient of the prey population, $e\equiv$ conversion factor $(0<e\le 1)$, $\beta\equiv$ death rate of predator (hyperbolic), $\eta\equiv$ stiffness parameter measuring the intensity of reaction between the effort and the perceived rent, $p\equiv$ the profit or fixed price per unit of the prey biomass, $T\equiv$ tax per unit of prey and $c\equiv$ the constant cost of the harvesting effort. Also from the biological definitions of a,k,p and T we must have $(p-T)>0$ and $\frac{a}{k}<1$ always.

3. UNIFORM BOUNDEDNESS

Uniform boundedness implies that the system is ecologically consistent. Biologically, an unbounded population size is an indication of population extinction. To study a biological system, it is essential to verify that the amount of all population of the system always stays within some bound. The following theorem confirms the uniform boundedness of the system (2.1).

Theorem 3.1: All the solutions of the system (2.1) which start in $\mathbb{R}^3_+$ are uniformly bounded.

Proof: Let $\left(x(t),\ y(t),E(t)\right)$ be any solutions of the system with positive initial conditions. Now we consider the following function:

$$W(t)=x(t)+y(t)+\frac{1}{\eta(p-T)}E(t)$$

$$\Rightarrow \frac{dW}{dt}=rx\left(1-\frac{x}{k}\right)+(e-1)\frac{\alpha xy}{a+x}-\frac{\beta y^2}{a+x}-\frac{cE}{p-T}.$$

Now using $0<e\le 1$ and $0<x\le k$ we get,

$$\frac{dW}{dt}\le\left(rx-\frac{rx^2}{k}\right)-\frac{\beta y^2}{a+k}-\frac{cE}{p-T}.$$

Now we choose $0<N<c\eta$. Therefore,

$$\frac{dW}{dt}+NW\le\left\{(r+N)x-\frac{rx^2}{k}\right\}+\left(Ny-\frac{\beta y^2}{a+k}\right)-\frac{(c\eta-N)E}{\eta(p-T)}$$

$$\le\frac{-r}{k}\left\{x-\frac{k(r+N)}{2r}\right\}^2-\frac{\beta}{a+k}\left\{y-\frac{N(a+k)}{2\beta}\right\}^2+M\le M.$$

Where, $M=\frac{k(r+N)^2}{4r}+\frac{N^2}{4\beta}$. Now, applying the theory of differential inequality [17], we obtain $0\le W(x,y,E)\le\frac{M}{N}+W\left(x(0),y(0),E(0)\right)e^{-Nt}$ and for $t\to\infty, 0\le W\le\frac{M}{N}$. Thus, all the solutions of the system (2.1) enter into the region $B=\left\{(x,y,E):0\le W\le\frac{M}{N}+\varepsilon,\forall\varepsilon>0\right\}$. This completes the proof.

4. EQUILIBRIA AND THEIR FEASIBILITY CONDITIONS

We find equilibrium points of the system (2.1) by solving $\dot{x}=\dot{y}=\dot{E}=0$ which are given below:

(i) The axial equilibrium point $P_1(k,0,0)$ is always feasible.

(ii) The boundary equilibrium point $P_2(x_2,y_2,0)$, where $y_2=\frac{e\alpha x_2}{\beta}$ and x_2 is the unique positive root of the quadratic equation $A_1x^2+B_1x+C_1=0$, where $A_1=-r\beta<0, B_1=r\beta(k-a)-e\alpha^2k$ and $C_1=ra\beta k>0.$ Clearly, without any parametric restriction the equilibrium point P_2 is always feasible.

(iii) The boundary equilibrium point $P_3(x_3,0,E_3)$, where $x_3=\frac{clE}{q(p-t)-cm}$ and $E_3=\frac{rmx_3(k-x_3)}{(lrx_3+kq)-krl}$ is feasible if $rl\left(1-\frac{x_3}{k}\right)<q<\frac{cm}{p-T}$.

(iv) The interior equilibrium point $P_*(x^*,y^*,E^*)$, where, $y^*=m_1x^*, E^*=m_2x^*$ and $m_1=\frac{e\alpha}{\beta}, m_2=\frac{q(p-T)-cmx^*}{cl}>0,$ provided $q(p-T)>cm.$ And x^* is a positive root of the quadratic equation $Ax^{*2}+Bx^*+C=0,$ where $A=\frac{-r}{k}<0, B=r\left(1-\frac{a}{k}\right)-\left(\alpha m_1+\frac{qm_2}{lm_2+m}\right)$ and $C=\frac{-qam_2}{lm_2+m}<0.$ Clearly, by the Descartes' rule of signs the above quadratic has either two positive roots or no positive root if $B>0$ *i.e.* if $r\left(1-\frac{a}{k}\right)>\left(\alpha m_1+\frac{qm_2}{lm_2+m}\right)$. Therefore, the coexisting equilibrium point

P^* will be feasible if the aforementioned two conditions hold simultaneously. In the numerical simulation section we show that there exist two plausible sets of parameter values for which two interior equilibrium points exist.

5. STABILITY ANALYSIS

We now study the stability behaviour of the system (2.1) around several equilibria. This section consists of the study of local and global stability behaviour around the several equilibrium points of the model system (2.1).

5.1 Local analysis

Here we discuss about the local behaviour of the model system (2.1) near the equilibrium points. For this purpose, we linearize the model equations around the equilibrium points and calculate the corresponding Jacobian matrix J. If all the eigenvalues of J about an equilibrium point P have negative real parts, then the system is locally asymptotically stable around P [18]. Now, the Jacobian matrix at P is given by

$$J(P)=\begin{pmatrix} r-\frac{2rx}{k}-\frac{a\alpha y}{(a+x)^2}-\frac{lqE^2}{(lE+mx)^2} & \frac{-\alpha x}{a+x} & \frac{-mqx^2}{(lE+mx)^2} \\ \frac{ea\alpha y+\beta y^2}{(a+x)^2} & \frac{e\alpha x-2\beta y}{a+x} & 0 \\ \frac{l\eta q(p-T)E^2}{(lE+mx)^2} & 0 & \frac{m\eta q(p-T)x^2}{(lE+mx)^2}-\eta c \end{pmatrix}.$$

The characteristic equation of $J(P)$ in λ is $\det\left(J(P)-\lambda I_3\right)=0,$ where I_3 is a third order identity matrix.

Theorem 5.1.System (2.1) is

(i) always unstable around the axial equilibrium point P_1 for all parametric values,

(ii) locally asymptotically stable around the harvesting free equilibrium point P_2 if $a_{11}<0$ and $a_{33}<0,$ where $J(P_2)=(a_{ij})_{3\times 3}$,

(iii) always unstable around the boundary equilibrium point P_3 for all parametric values.

(iv) locally asymptotically stable around the interior equilibrium point P_* if $c_{11}<0,$ where $J(P_*)=(c_{ij})_{3\times 3}$.

Proof: (i) Here $J\left(P_1\right)=\begin{pmatrix} -2r & \frac{-\alpha k}{a+k} & \frac{-q}{m} \\ 0 & \frac{e\alpha k}{a+k} & 0 \\ 0 & 0 & \frac{q\eta\left(p-T\right)}{m}-\eta c \end{pmatrix}.$

Clearly, the eigenvalues of the Jacobian matrix calculated at P_1 are $-2r, \frac{e\alpha k}{a+k}$ and $\frac{\eta q(p-T)}{m}-\eta c.$ Since, $\frac{e\alpha k}{a+k}>0,$ the result is obvious.

(ii) The characteristic equation of $J(P_2)=(a_{ij})_{3\times 3}$ is given by

$$(\lambda-a_{33})\left\{\lambda^2-(a_{11}+a_{22})\lambda+(a_{11}a_{22}-a_{12}a_{21})\right\}=0. \tag{5.1}$$

where,

$$a_{11} = r - \frac{2rx_2}{a+k} - \frac{a\alpha y_2}{(a+x_2)^2},\ a_{12} = \frac{-\alpha x_2}{a+x_2} < 0,\ a_{13} = \frac{-q}{m} < 0,$$

$$a_{21} = \frac{ea\alpha y_2 + \beta y_2^2}{(a+x_2)^2} > 0,\ a_{22} = \frac{e\alpha x_2 - 2\beta y_2}{a+x_2},\ a_{23} = 0,$$

$$a_{31} = 0,\ a_{32} = 0,\ a_{33} = \frac{\eta q(p-T)}{m} - \eta c.$$

Now, using the fact that $P_2(x_2, y_2, 0)$ is an equilibrium point of the system (2.1), one can easily verify that $a_{22} = \frac{-\beta y_2}{a+x_2} < 0.$

Clearly, the roots of the equation (5.1) will have negative real parts if $a_{33} < 0, a_{11} + a_{22} < 0$ and $a_{12}a_{21} - a_{11}a_{22} < 0$, i.e. if $a_{11} < 0, a_{33} < 0$ (using the signs of a_{ij}'s).

(iii) The characteristic equation of $J(P_3) = (b_{ij})_{3\times 3}$ is given by

$$(\lambda - b_{22})\left\{\lambda^2 - (b_{11} + b_{33})\lambda + (b_{11}b_{33} - b_{13}b_{31})\right\} = 0. \qquad (5.2)$$

where,

$$b_{11} = r - \frac{2rx_3}{k} - \frac{lqE_3^2}{(lE_3 + mx_3)^2},\ b_{12} = \frac{-\alpha x_3}{a+x_3} < 0,\ b_{13} = \frac{-mqx_3^2}{m} < 0,$$

$$b_{21} = 0,\ b_{22} = \frac{e\alpha x_3}{a+x_3} > 0,\ b_{23} = 0,$$

$$b_{31} = \frac{l\eta q(p-T)}{(lE_3 + mx_3)^2},\ b_{32} = 0,\ b_{33} = \frac{m\eta q(p-T)x_3^2}{(lE_3 + mx_3)^2} - \eta c.$$

Now, the eigenvalue b_{22} of $J(P_3)$ remains always positive. Hence, the theorem follows.

(iv) The characteristic equation of $J(P_*)$ is given by

$$\lambda^3 - \text{trace}\left(J(P_*)\right)\lambda^2 + \text{trace}\left(\text{adj}\left(J(P_*)\right)\right)x - \det\left(J(P_*)\right) = 0. \qquad (5.3)$$

where, $J(P_*) = (c_{ij})_{3\times 3}$ and $\text{adj}\left(J(P_*)\right) \equiv$ adjoint matrix of the Jacobian matrix $J(P_*)$, and

$$c_{11} = r - \frac{2rx^*}{k} - \frac{a\alpha y^*}{(a+x^*)^2} - \frac{lqE^{*2}}{(lE^* + mx^*)^2},\ c_{12} = \frac{-\alpha x^*}{a+x^*} < 0,\ c_{13} = \frac{-mqx^{*2}}{(lE^* + mx^*)^2} < 0,$$

$$c_{21} = \frac{ea\alpha y^* + \beta y^{*2}}{(a+x^*)^2} > 0,\ c_{22} = \frac{e\alpha x^* - 2\beta y^*}{a+x^*},\ c_{23} = 0,$$

$$c_{31} = \frac{l\eta q(p-T)E^{*2}}{(lE^* + mx^*)^2} > 0,\ c_{32} = 0,\ c_{33} = \frac{m\eta q(p-T)x^{*2}}{(lE^* + mx^*)^2} - \eta c.$$

Now, using the fact that $P_*(x^*, y^*, E^*)$ is an equilibrium point of the system (2.1), one can easily verify that

$$c_{22} = \frac{-\beta y^*}{a+x^*} < 0 \text{ and } c_{33} = \frac{\eta q(p-T)x^{*2}}{(lE^* + mx^*)}\left[\frac{mx^*}{(lE^* + mx^*)} - 1\right] < 0.$$

Routh-Hurwitz [18, 19] criteria says that the real parts of the roots of the equation (5.3) are negative if

$\text{trace}\left(J(P_*)\right)<0$, $\det\left(J(P_*)\right)<0$ and $\text{trace}\left(J(P_*)\right)\cdot\text{trace}\left(\text{adj}\left(J(P_*)\right)\right)-\det\left(J(P_*)\right)<0$ such that $\text{trace}\left(J(P_*)\right)=c_{11}+c_{22}+c_{33}$, $\det\left(J(P^*)\right)=c_{11}c_{22}c_{33}-c_{13}c_{31}c_{22}-c_{12}c_{21}c_{33}$ and $\text{trace}\left(J(P_*)\right)\cdot\text{trace}\left(\text{adj}\left(J(P_*)\right)\right)-\det\left(J(P_*)\right)=3c_{11}c_{22}c_{33}+c_{11}^2(c_{22}+c_{33})+c_{22}^2(c_{11}+c_{33})+c_{33}^2(c_{11}+c_{22})-c_{13}c_{31}(c_{11}+c_{22})-c_{12}c_{21}(c_{11}+c_{22})-c_{33}(c_{12}c_{21}+c_{13}c_{31})$.

Hence, just using the signs of c_{ij}'s, it can be easily shown that the above three criteria due to Routh-Hurwitz for getting negative real parts of the three roots of (5.3) hold simultaneously only if $c_{11}<0$. This completes the proof.

5.2 Global Analysis

In this section, we analyze the system (2.1) from the global perspective, only around the interior equilibrium point because of its biological importance.

Theorem 5.2.The model system (2.1) is globally asymptotically stable around the interior equilibrium point $P_*\left(x^*,y^*,E^*\right)$ if $\frac{r}{k}>\frac{\alpha y^*}{G}+\frac{qmE^*}{H}$ with $G=(a+x)(a+x^*)$ and $H=(lE+mx)(lE^*+mx^*)$.

Proof: To show the global stability of the system (2.1) around $P_*(x^*,y^*,E^*)$ we have to construct a suitable Lyapunov function. We define a Lyapunov function

$$L(x,y,E)=L_1\int_{x^*}^{x}\frac{t_1-x^*}{t_1}dt_1+L_2\int_{y^*}^{y}\frac{t_2-y^*}{t_2}dt_2+L_3\int_{E^*}^{E}\frac{t_3-E^*}{t_3}dt_3. \tag{5.4}$$

where L_1, L_2 and L_3 are positive constants to be determined in the subsequent steps. It can be easily verified that the function L is zero at the equilibrium $P_*(x^*,y^*,E^*)$ and is positive for all other positive values of x, y and E. Now, using the fact that $P_*(x^*,y^*,E^*)$ is an equilibrium point of the system (2.1), we get

$$\begin{aligned}\frac{dL}{dt}&=L_1(x-x^*)^2\left[\frac{-r}{k}+\frac{\alpha y^*}{(a+x)(a+x^*)}+\frac{qmE^*}{(lE+mx)(lE^*+mx^*)}\right]\\&+\frac{(x-x^*)(E-E^*)}{(lE+mx)(lE^*+mx^*)}\left[L_3\eta ql(p-T)E^*-L_1qmx^*\right]\\&+L_2(y-y^*)^2\left[-\frac{a\beta}{(a+x)(a+x^*)}-\frac{\beta x^*}{(a+x)(a+x^*)}\right]\\&+L_3(E-E^*)^2\left[-\frac{\eta ql(p-T)x^*}{(lE+mx)(lE^*+mx^*)}\right].\end{aligned} \tag{5.5}$$

Now, we choose $L_1=\eta l(p-T)E^*$, $L_2=\frac{L_1\alpha(a+x^*)}{ea\alpha+\beta}$ and $L_3=mx^*$. Then,

$$\frac{dL}{dt}=L_1\left[\frac{-r}{k}+\frac{\alpha y^*}{(a+x)(a+x^*)}+\frac{qmE^*}{(lE+mx)(lE^*+mx^*)}\right]\left(x-x^*\right)^2$$
$$-L_2\left[\frac{a\beta}{(a+x)(a+x^*)}+\frac{\beta x^*}{(a+x)(a+x^*)}\right]\left(y-y^*\right)^2$$
$$-L_3\left[\frac{\eta ql(p-T)x^*}{(lE+mx)(lE^*+mx^*)}\right]\left(E-E^*\right)^2 \quad (5.6)$$

$$=-X^tQX \text{ , where } X^t=\left[x-x^*,y-y^*,E-E^*\right] \text{ and}$$

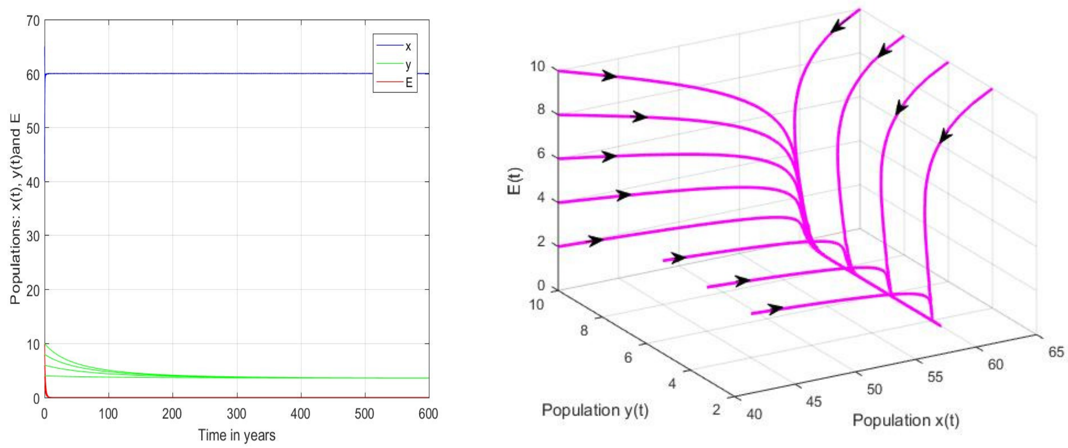
$$Q=\begin{pmatrix} L_1\left(\frac{r}{k}-\frac{\alpha y^*}{G}-\frac{qmE^*}{H}\right) & 0 & 0 \\ 0 & \frac{L_2\beta}{G}(a+\beta) & 0 \\ 0 & 0 & \frac{L_3\eta ql(p-T)x^*}{H}\end{pmatrix},$$

with $G=(a+x)(a+x^*)$ and $H=(lE+mx)(lE^*+mx^*)$.

Hence by LaSalle's theorem [20], the model system (2.1) is globally asymptotically stable around the coexisting equilibrium point $P_*(x^*,y^*,E^*)$ if $\frac{dL}{dt}<0$ i.e. if Q is positive definite. Now, Q is positive definite if $\frac{r}{k}>\frac{\alpha y^*}{G}+\frac{qmE^*}{H}$. This completes the proof.

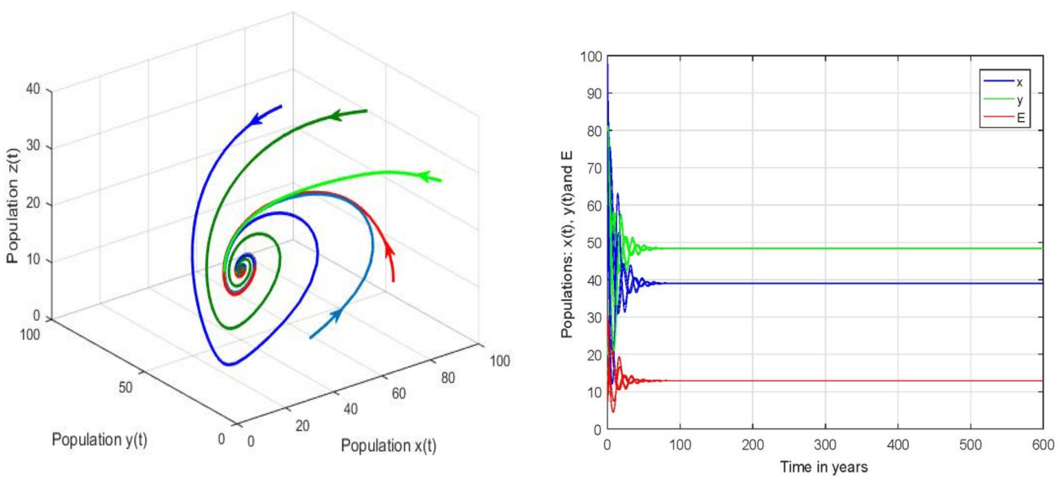
Table 1: Schematic representation of the feasibility and stability conditions of the equilibria of proposed model (2.1): LAS $\equiv$ Locally asymptotically stable; GAS $\equiv$ Globally asymptotically stable

Equilibria	**Feasibility conditions**	**Stability conditions**	**Nature**
P_1	No condition	No condition	Unstable
P_2	No condition	$a_{11}<0,\ a_{33}<0$	LAS
P_3	$rl\left(1-\frac{x_3}{k}\right)<q<\frac{cm}{p-T}$	No condition	Unstable
P_*	$r\left(1-\frac{a}{k}\right)>\alpha m_1+\frac{qm_2}{lm_2+m}$	$c_{11}<0$	LAS
P_*	....	$\frac{r}{k}>\frac{\alpha y^*}{G}+\frac{qmE^*}{H}$	GAS



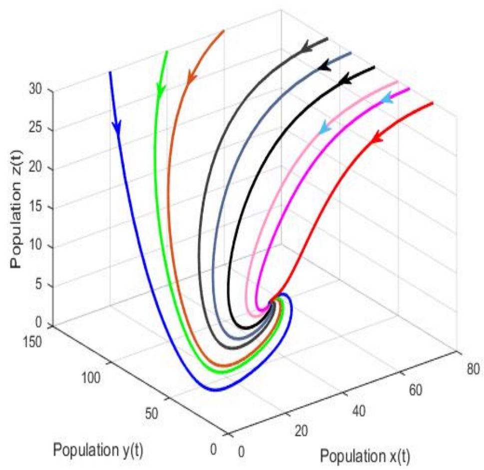
(a) (b)

Figure 1: (a)-(b) Demonstrates the local asymptotical stability (LAS) of the system (2.1) around the harvesting free equilibrium point $P_2(x_2, y_2, 0) = (59.9902, 3.5994, 0)$ corresponding to the parameter values: $r = 4.9$, $a = 30$, $l = 0.5$, $m = 0.5$, $\beta = 0.2$, $\alpha = 0.02$, $p = 4$, $q = 0.315$, $k = 60$, $e = 0.6$, $\eta = 1$, $c = 2.5$, $T = 1$.

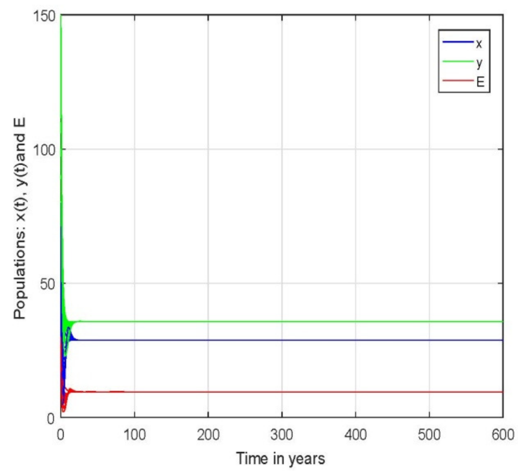


(a) (b)

Figure 2: (a)-(b) Demonstrates the local asymptotical stability (LAS) of the system (2.1) around the harvesting free equilibrium point $P_*(x^*, y^*, E^*) = (38.9926, 48.3501, 12.9041)$ corresponding to the parameter values: $r = 1.2$, $a = 2$, $l = 0.4$, $m = 0.6$, $\beta = 0.25$, $\alpha = 0.5$, $p = 7$, $q = 0.315$, $k = 100$, $e = 0.62$, $\eta = 2$, $c = 2$, $T = 2.35$.



(a)



(b)

Figure 3: (a)-(b) Solution plots showing the global asymptotical stability (GAS) of the system (2.1) around interior equilibrium point $P_*(x^*, y^*, E^*) = (28.8191, 35.7358, 9.5373)$ corresponding to the parameter values: $a = 20,\ k = 50$ and the other parameters are the same as in Figure 2.

6. NUMERICAL SIMULATIONS

By making use of MATLAB and Maple, simulation work with plausible hypothetical set of parameter values is performed to understand the analytical results which are concluded in the various sections of this paper. To justify the local asymptotical stability around the harvesting free equilibrium point $P_2(x_2, y_2, 0)$ of the system (2.1), we take a set of biologically plausible system parameter values: $r = 4.9$, $a = 30$, $l = 0.5$, $m = 0.5$, $\beta = 0.2$, $\alpha = 0.02$, $p = 4$, $q = 0.315$, $k = 60$, $e = 0.6$, $\eta = 1$, $c = 2.5$, $T = 1$. For this set of parameter values, the model system (2.1) possesses a unique harvesting free equilibrium point $P_2(59.9902, 3.5994, 0)$ (Figure 1:(a)-(b)). Again to justify the global asymptotical stability of the coexisting or interior equilibrium point $P_*(x^*, y^*, E^*)$, interestingly we see that there are two sets of hypothetical parameter values for which the system (2.1) possesses two feasible interior equilibrium points as we see in Section 4 that the system (2.1) has two feasible interior equilibrium points. For this purpose, we choose the following set of parameter values: $r = 1.2$, $a = 2$, $l = 0.4$, $m = 0.6$, $\beta = 0.25$, $\alpha = 0.5$, $p = 7$, $q = 0.315$, $k = 100$, $e = 0.62$, $\eta = 2$, $c = 2$, $T = 2.35$, which give the coexisting equilibrium point $P_*(38.9926, 48.3501, 12.9041)$ (Figure 2:(a)-(b)) and for the same set of parameter values except $a = 20$, $k = 50$ gives another interior equilibrium point $P_*(28.8191, 35.7358, 9.5373)$ (Figure 3:(a)-(b)) as expected.

7. CONCLUDING REMARKS

The present study highlights the dynamics of a biological system under consideration by means of stability. The research essentially focuses on understanding the real complexity in the natural phenomena especially in the presence of non-linear harvesting of the prey population. We state several steady states of the model system as well as find their feasibility conditions. Also we analyze the system for local and global stability around several equilibria. The ecological balance is controlled by maintaining the harvesting effort through tax. Most importantly, it is observed that, in the presence of predation and non-linear harvesting of the prey, all the system variables persist at a stable equilibrium state in the long run. The dynamical analysis on non-linear harvesting of prey has exhibited that harvesting under optimal schemes really influences the species exaggeration. All the analytical results are justified by the given numerical simulations. This being a generalized model system; any specific system can be developed and may be studied. The results that we obtain in the present study are generic type and may be fitted to any specific classes of model.

8. FUTURE SCOPE

There are so many scopes to do with this model such as (i) Hopf-bifurcation, (ii) Saddle-node bifurcation, (iii) Trans-critical bifurcation, (iv) Pitch-fork bifurcation and optimal control like (a) Determination of optimal effort, (b) Optimal taxation policy. Also, one can consider combined harvesting of prey and predator both. In our future work we want to perform these all.

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