Original Article

Bulletin of Pure and Applied Sciences.
Vol. 37E (Math & Stat.), No.2, 2018.P.459-470
Print version ISSN 0970 6577
Online version ISSN 2320 3226
DOI: 10.5958/2320-3226.2018.00049.8

A BATCH ARRIVAL BULK SERVICE QUEUE WITH MULTIPLE VACATIONS, OPTIONAL RE-SERVICE, CLOSEDOWN TIME AND SETUP TIME UNDER RESTRICTED ADMISSIBILITY OF ARRIVING BATCHES

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Received on 04.04.2018, Accepted on 26.09.2018

Abstract

We consider an M^X/G(a,b)/1 with multiple vacations, optional re-service, closedown time, setup time under a restricted admissibility policy of arriving batches is considered. A batch of customers arrive according to Poisson with rate λ, whereas the general bulk service is rendered by a single server with a minimum batch size of 'a' and maximum of 'b'. The service times follow a general distribution. At the completion of an essential service, the leaving batch of customers may request for a re-service with probability π. After completing re-service or if no request for re-service with probability $(1-\pi)$ is made the number of customers in the queue is less than 'a' then the server performs closedown work at its closedown time C. After that, the server leaves for multiple vacation of a random length irrespective of queue length. When the server returns from vacation and if the queue length is still less than 'a' he avails another vacation so on until the server finds at least 'a' customers in the queue. At a vacation completion epoch, if the server finds at least 'a' customers waiting for service, he requires a setup time to start the service. On service completion or on re-service completion, if the server finds at least 'a' customers waiting for service say ξ (ξ ≥ a), he serves a batch of min (ξ, b)customers, where b ≥ a. In addition we assume restricted admissibility of arriving batches in which not all batches are allowed to join the system at all time. The probability generating function of queue size at a random epoch is obtained. Some important performance measures such as expected queue size, expected busy period and idle period are derived. Along with the cost model, particular cases are discussed. Numerical illustration for particular values of parameter is presented.

Keywords: General Bulk service, multiple vacations, optional re-services, setup time, closedown time restricted admissibility policy.

1. INTRODUCTION

The service rule is assumed to operate as follows: the server starts service only when a minimum of 'a' customers is in the buffer, and the maximum capacity is 'b' customers. Such a rule for bulk service, first introduced by Neuts, may be called general bulk service rule. (Neuts (1967) introduced a general class of bulk queues and studied the queue length and busy periods)

Madan and Baklizi (2002) considered an M/G/1 queueing model, in which the server performs first essential service to all arriving customers. As soon as the first service is over, they may leave the system with the probability (1- π) and second optional service is provided with probability π. Madan et al (2004) analyzed a single server bulk arrival queue, in which the leaving batch of customers might opt for re-service. They obtained various performance measures too. Hur et al (2005) studied single server bulk arrival queueing system with vacations and server setup. Arumuganathan and Judeth Malliga (2006) analyzed a bulk queue with repair of service station and set up time. Al-khedhairi and Lotfi Tadj (2007) discussed a bulk service queue with a choice of service and re-service under Bernoulli schedule. Lotfi Tadj and Ke (2008) studied a hysteretic bulk quorum queue with a choice of service and optional re-service. Steady state analysis of a bulk queue with multiple vacations, setup times with N-policy and closedown times is analyzed by Arumuganathan, and Jeyakumar (2005). They derived the expected number of customers in the queue at an arbitrary time epoch.

In some queueing systems with batch arrival there is a restriction such that not all batches are allowed to join the system at all time. Choudhuruy and Madan proposed an M^X/G/1 queueing system with restricted admissibility of arriving batches and Bernoulli schedule server vacation.

2. MATHEMATICAL MODEL

- Let X be the group size random variable of the arrival, λ be the Poisson arrival rate when the server is busy, g_k be the probability that k customers arrive in a batch and X(z) be its PGF.
- The service follows a general (arbitrary) distribution with cumulative distribution function S(.) and density function s(x). Let $\tilde{S}(\theta)$ denote the Laplace -Stieltjes transform of S and $S^0(t)$ denote the remaining service time of a batch at an arbitrary time t.
- The server's vacation time follows a general (arbitrary) distribution with cumulative distribution function V(.) and density function v(x). Let $\tilde{V}(\theta)$ denote the Laplace -Stieltjes transform of V and $V^0(t)$ denote the remaining service time of a batch at an arbitrary time t.
- The re-service follows a general (arbitrary) distribution with cumulative distribution function R(.) and density function r(x). Let $\tilde{R}(\theta)$ denote the Laplace -Stieltjes transform of R and $R^0(t)$ denote the remaining re- service time of a batch at an arbitrary time t.
- The closedown time follows a general (arbitrary) distribution with cumulative distribution function C(.) and density function c(x). Let $\tilde{C}(\theta)$ denote the Laplace -Stieltjes transform of C and $C^0(t)$ denote the remaining closedown time of a batch at an arbitrary time t.
- The setup time follows a general (arbitrary) distribution with cumulative distribution function U(.) and density function u(x). Let $\tilde{U}(\theta)$ denote the Laplace -Stieltjes transform of U and $U^0(t)$ denote the remaining setup time of a batch at an arbitrary time t.
- There is a policy restricted admissibility of batches in which not all batches are allowed to join the system at all times. During the essential service, re-service, setup period, the arrivals are admitted with probability 'α' whereas; with probability 'β' they are admitted when the server is in vacation or in closedown period.
- N_s(t) and N_q(t) are the number of customers in the service and number of customers in the queue respectively.

The different states of the server at time t are defined as follows

$$Y(t) = \begin{cases} 0, \text{if the server is on essential servic} \\ 1, \text{if the server is on vacation service} \\ 2, \text{if the server is on re} - \text{service} \\ 3, \text{if the server is on setup work} \\ 4, \text{if the server is on closedown work} \end{cases}$$

To obtain the system of Equations, the following state probabilities are defined:

$$P_{i,n}(x,t)dt = P\{N_s(\mathrm{t}) = i, N_q(\mathrm{t}) = n, x \le S^0(t) \le x + dt, Y(t) = 0\}, a \le i \le b, n \ge 0,$$

$$Q_{i,n}(x,t)dt = P\{N_q(\mathrm{t}) = n, x \le V^0(t) \le x + dt, Y(t) = 1, Z(t) = j\}, j \ge 1, n \ge 0,$$

$R_n(x,t)dt = P\{ N_q(\mathrm{t}) = n, x \le R^0(t) \le x + dt, Y(t) = 2\}, n \ge 0$
$U_n(x,t)dt = P\{ N_q(\mathrm{t}) = n, x \le U^0(t) \le x + dt, Y(t) = 3\}, n \ge 0$
$C_n(x,t)dt = P\{ N_q(\mathrm{t}) = n, x \le U^0(t) \le x + dt, Y(t) = 4\}, n \ge 0$
In steady state, let us define for $x > 0, P_{i,j}(x) = \lim_{t\to\infty} P_{i,j}(x,t)$ for $a \le i \le b$ and $j \ge 0$

3. STEADY STATE SYSTEM EQUATIONS

The model is then, governed by the following set of differential- difference equations

$$-P_{i,0}^{'}(x) = -\lambda P_{i,0}(x) + \lambda(1-\alpha)P_{i,0}(x) + (1-\pi)\sum_{m=a}^{b} P_{m,i}(0)s(x) + R_i(0)s(x) + U_i(0)s(x);\ a \le i \le b \quad (1)$$

$$-P_{i,j}^{'}(x) = -\lambda P_{i,j}(x) + \lambda(1-\alpha)P_{i,j}(x) + \lambda\alpha \sum_{k=1}^{j} P_{i,j-k}(x)\, g_k;\ a \le i \le b-1\ \&\ j \ge 1, \quad (2)$$

$$-P_{b,j}^{'}(x) = -\lambda P_{b,j}(x) + \lambda(1-\alpha)P_{b,j}(x) + \lambda\alpha \sum_{k=1}^{j} P_{b,j-k}(x)\, g_k + R_{b+j}(0)s(x) + (1-\pi)\sum_{m=a}^{b} P_{m,b+j}(0)s(x) + U_{b+j}(0)s(x), j \ge 1 \quad (3)$$

$$-C_n^{'}(x) = -\lambda C_n(x) + \lambda(1-\beta)C_n(x) + (1-\pi)\sum_{m=a}^{b} P_{m,n}(0)c(x) + \lambda\sum_{k=1}^{n} C_{n-k}(x)\, g_k + R_n(x)c(x);\quad 1 \le n \le a-1, \quad (4)$$

$$-C_n^{'}(x) = -\lambda C_n(x) + \lambda(1-\beta)C_n(x) + \lambda\sum_{k=1}^{n-a} C_{n-k}(x)\, g_k; n \ge a \quad (5)$$

$$-Q_{1,0}^{'}(x) = -\lambda Q_{1,0}(x) + \lambda(1-\beta)Q_{1,0}(x) + C_0(0)v(x); \quad (6)$$
$$-Q_{1,n}^{'}(x) = -\lambda Q_{1,n}(x) + \lambda(1-\beta)Q_{1,n}(x) + C_n(0)v(x) + \lambda\beta\sum_{k=1}^{\infty} Q_{1,n-k}(x)g_k;\quad n \ge 1, \quad (7)$$
$$-Q_{j,0}^{'}(x) = -\lambda Q_{j,0}(x) + \lambda(1-\beta)Q_{j,0}(x) + Q_{j-1,0}(0)v(x);\ j \ge 2, \quad (8)$$

$$-Q_{j,n}^{'}(x) = -\lambda Q_{j,n}(x) + \lambda(1-\beta)Q_{j,0}(x) + Q_{j-1,n}(0)v(x) + \lambda\beta\sum_{k=1}^{\infty} Q_{j,n-k}(x)g_k;\quad j \ge 2,\ 1 \le n \le a-1, \quad (9)$$

$$-Q_{j,n}^{'}(x) = -\lambda Q_{j,n}(x) + \lambda(1-\beta)Q_{j,0}(x) + \lambda\beta\sum_{k=1}^{\infty} Q_{j,n-k}(x)g_k;\ j \ge 2, n \ge a, \quad (10)$$

$$-R_0^{'}(x) = -\lambda R_0(x) + \lambda(1-\alpha)R_0(x) + \pi\sum_{m=a}^{b} P_{m,0}(0)r(x) \quad (11)$$

$$-R_n^{'}(x) = -\lambda R_n(x) + \lambda(1-\alpha)R_n(x) + \pi\sum_{m=a}^{b} P_{m,n}(0)r(x) + \lambda\alpha\sum_{k=1}^{n} R_{n-k}(x)\, g_k;\ 1 \le n \le a-1, \quad (12)$$

$$-U_n^{'}(x) = -\lambda U_n(x) + \lambda(1-\alpha)U_n(x) + \lambda\alpha\sum_{k=1}^{n} U_{n-k}(x)\, g_k + \sum_{l=1}^{\infty} Q_{l,n}(0)u(x);\ n \ge a. \quad (13)$$

The Laplace-Stieltjes transforms of $P_{i,n}(x), Q_{j,n}(x)$ and $R_n(x)$ are defined as:

$$\tilde{P}_{i,n}(\theta) = \int_0^{\infty} e^{-\theta x}P_{i,n}(x)dx,\ \tilde{Q}_{j,n}(\theta) = \int_0^{\infty} e^{-\theta x}Q_{j,n}(x)dx,\ \tilde{C}_n(\theta) = \int_0^{\infty} e^{-\theta x}C_n(x)dx.$$
$$\tilde{U}_n(\theta) = \int_0^{\infty} e^{-\theta x}U_n(x)dx \text{ and } \tilde{R}_n(\theta) = \int_0^{\infty} e^{-\theta x}R_n(x)dx. \quad (14)$$

Taking Laplace-Stieltjes transform on both sides, we get

$$\theta\tilde{P}_{i,0}(\theta) - P_{i,0}(0) = \lambda\tilde{P}_{i,0}(\theta) - \lambda(1-\alpha)\tilde{P}_{i,0}(\theta) - \left[(1-\pi)\sum_{m=a}^{b} P_{m,i}(0) + R_i(0) + U_i(0)\right]\tilde{S}(\theta); a \le i \le b, \quad (15)$$

$$\theta\tilde{P}_{i,j}(\theta)-P_{i,j}(0)=\lambda\tilde{P}_{i,j}(\theta)-\lambda(1-\alpha)\tilde{P}_{i,j}(\theta)-\lambda\alpha\sum_{k=1}^{j}\tilde{P}_{i,j-k}(\theta)\,g_k,\ a\le i<b-1;\ j\ge 1 \tag{16}$$

$$\theta\tilde{P}_{b,j}(\theta)-P_{b,j}(0)=\lambda\tilde{P}_{b,j}(\theta)-\lambda(1-\alpha)\tilde{P}_{b,j}(\theta)-\left[(1-\pi)\sum_{m=a}^{b}P_{m,b+j}(0)+R_{b+j}(0)+U_{b+j}(0)\right]\tilde{S}(\theta)-\alpha\sum_{k=1}^{j}\tilde{P}_{b,j-k}(\theta)\,\lambda g_k;\quad j\ge 1 \tag{17}$$

$$\theta\tilde{C}_n(\theta)-C_n(0)=\lambda\tilde{C}_n(\theta)-\lambda(1-\beta)\tilde{C}_n(\theta)-\left[(1-\pi)\sum_{m=a}^{b}P_{m,n}(0)+R_n(0)\right]C(\theta)-\lambda\beta\sum_{k=1}^{j}\tilde{C}_{n-k}(\theta)\,g_k;\quad 1\le n\le a-1 \tag{18}$$

$$\theta\tilde{C}_n(\theta)-C_n(0)=\lambda\tilde{C}_n(\theta)-\lambda(1-\beta)\tilde{C}_n(\theta)-\lambda\beta\sum_{k=1}^{j}\tilde{C}_{n-k}(\theta)\,g_k;\quad n\ge a \tag{19}$$

$$\theta\tilde{Q}_{1,0}(\theta)-Q_{1,0}(0)=\lambda\tilde{Q}_{1,0}(\theta)-\lambda(1-\beta)\tilde{Q}_{1,0}(\theta)+C_0(0)\tilde{V}(\theta); \tag{20}$$

$$\theta\tilde{Q}_{1,n}(\theta)-Q_{1,n}(0)=\lambda\tilde{Q}_{1,n}(\theta)-\lambda(1-\beta)\tilde{Q}_{1,n}(\theta)-C_n(0)\tilde{V}(\theta)-\lambda\beta\sum_{k=1}^{j}\tilde{Q}_{1,n-k}(\theta)\,g_k;\ n\ge 1 \tag{21}$$

$$\lambda\theta\tilde{Q}_{j,0}(\theta)-Q_{j,0}(0)=\lambda\tilde{Q}_{j,0}(\theta)-\lambda(1-\beta)\tilde{Q}_{j,0}(\theta)-Q_{j,0}(0)\tilde{V}(\theta);\quad j\ge 2 \tag{22}$$

$$\theta\tilde{Q}_{j,n}(\theta)-Q_{j,n}(0)=\lambda\tilde{Q}_{j,n}(\theta)-\lambda(1-\beta)\tilde{Q}_{j,n}(\theta)-Q_{j,n}(0)\tilde{V}(\theta)-\lambda\beta\sum_{k=1}^{j}\tilde{Q}_{j,n-k}(\theta)\,g_k;\quad j\ge 2,1\le n\le a-1, \tag{23}$$

$$\theta\tilde{Q}_{j,n}(\theta)-Q_{j,n}(0)=\lambda\tilde{Q}_{j,n}(\theta)-\lambda(1-\beta)\tilde{Q}_{j,n}(\theta)-\lambda\beta\sum_{k=1}^{j}\tilde{Q}_{j,n-k}(\theta)\,g_k;\quad j\ge 2,n\ge a \tag{24}$$

$$\theta\tilde{R}_0(\theta)-R_0(0)=\lambda\tilde{R}_0(\theta)-\lambda(1-\alpha)\tilde{R}_0(\theta)-\pi\sum_{m=a}^{b}P_{m,0}(0)\,\tilde{R}(\theta); \tag{25}$$

$$\theta\tilde{R}_n(\theta)-R_n(0)=\lambda\tilde{R}_n(\theta)-\lambda(1-\alpha)\tilde{R}_n(\theta)-\pi\sum_{m=a}^{b}P_{m,0}(0)\,\tilde{R}(\theta)-\lambda\sum_{k=1}^{j}\tilde{R}_{n-k}(\theta)\,g_k;\quad 1\le n\le a-1, \tag{26}$$

$$\theta\tilde{U}_n(\theta)-U_n(0)=\lambda\tilde{U}_n(\theta)-\lambda(1-\alpha)\tilde{U}_n(\theta)-\lambda\sum_{k=1}^{j}\tilde{U}_{n-k}(\theta)\,g_k+\sum_{l=1}^{\infty}Q_{l,n}(0)\tilde{U}(\theta);\ n\ge a. \tag{27}$$

4. PROBABILITY GENERATING FUNCTIONS

Lee (1991) developed a new technique to find the steady state probability generating function (PGF) of the number of customers in the queue at an arbitrary time epoch. To apply the techniques, first the following probability generating functions are defined.

$$\begin{aligned}
&\tilde{P}_i(z,\theta)=\sum_{j=0}^{\infty}\tilde{P}_{i,j}(\theta)z^j; && P_i(z,0)=\sum_{j=0}^{\infty}P_{i,j}(0)z^j;\ a\le i\le b,\\
&\tilde{Q}_i(z,\theta)=\sum_{l=0}^{\infty}\tilde{Q}_{l,j}(\theta)z^j; && Q_j(z,0)=\sum_{l=0}^{\infty}Q_{l,j}(0)z^j;\ j\ge 1,\\
&\tilde{R}(z,\theta)=\sum_{n=0}^{\infty}\tilde{R}_n(\theta)z^n; && R(z,0)=\sum_{n=0}^{\infty}R_n(0)z^n;\\
&\tilde{U}(z,\theta)=\sum_{n=a}^{\infty}\tilde{U}_n(\theta)z^n; && U(z,0)=\sum_{n=a}^{\infty}U_n(0)z^n;\\
&\tilde{C}(z,\theta)=\sum_{n=0}^{\infty}\tilde{C}_n(\theta)z^n && C(z,0)=\sum_{n=0}^{\infty}C_n(0)z^n;
\end{aligned} \tag{28}$$

Multiply (20) by z^0 and (21) by z^n $(n\ge 1)$ and summing up from $n=0\ to\ \infty$ and by using (28), we get

$$\left[\theta-\beta\left(\lambda-\lambda X(z)\right)\right]\tilde{Q}_1(z,\theta)=Q_1(z,0)-\tilde{V}(\theta)\left[(1-\pi)\sum_{m=a}^{b}P_{m,n}(0)+R_n(0)\right]z^n \tag{29}$$

Multiply (22) by z^0,(23) by z^n $(1<n<a-1)$ and (24) by z^n $(n>a)$ and summing up from $n=0\ to\ \infty$ and by using (28), we get

$$[\theta-\beta(\lambda-\lambda X(z))]\tilde{Q}_j(z,\theta)= \ Q_j(z,0)-\tilde{V}(\theta)\sum_{n=0}^{a-1}Q_{j-1,n}(0)\,z^n\,; \quad j\geq 2. \tag{30}$$

Multiply (18) by z^0 (19) by z^n $(n\geq 1)$ and summing up from $n=0\ to\ \infty$ and by using (28), we get

$$[\theta-\beta(\lambda-\lambda X(z))]\tilde{C}(z,\theta)=C(z,0)-\tilde{C}(\theta)(1-\pi)\left[\sum_{n=0}^{a-1}\sum_{m=a}^{b}P_{m,n}(0)\,z^n+\sum_{n=0}^{a-1}R_n(0)z^n\right] \tag{31}$$

Multiply (25) by z^0 (26) by z^n $(n\geq 1)$ and summing up from $n=0\ to\ \infty$ and by using (28), we get

$$[\theta-\alpha(\lambda-\lambda X(z))]\tilde{R}(z,\theta)=R(z,0)-\tilde{R}(\theta)\pi\sum_{n=0}^{a-1}\sum_{m=a}^{b}P_{m,n}(0)\,z^n\,; \tag{32}$$

Multiply (27) z^n $(n>a)$ and summing up from $n=a\ to\ \infty$ and by using (28), we get

$$[\theta-\alpha(\lambda-\lambda X(z))]\tilde{U}(z,\theta)=U(z,0)-\tilde{U}(\theta)\sum_{l=1}^{\infty}\left(\ Q_l(z,0)-\sum_{n=0}^{a-1}Q_{l,n}(0)z^n\right); \tag{33}$$

Multiply (15) by z^0 , (16) by z^n $(n>a)$ and summing up from $n=0\ to\ \infty$ and by using (28), we get

$$[\theta-\alpha(\lambda-\lambda X(z))]\tilde{P}_i(z,\theta)= \ P_i(z,0)-\tilde{S}(\theta)\left[(1-\pi)\sum_{m=a}^{b}P_{m,i}(0)+R_i(0)+U_i(0)\right]z^i\,; \quad a\leq i\leq b-1 \tag{34}$$

Multiply (15) byz^0 with i=b (17) $z^j(j>a)$ and summing up from $j=0\ to\ \infty$ and by using (28), we get

$$[\theta-\alpha(\lambda-\lambda X(z))]\tilde{P}_b(z,\theta)= \ P_b(z,0)-\frac{\tilde{S}(\theta)}{z^b}\left[(1-\pi)\sum_{m=a}^{b}\left(P_m(z,0)-\sum_{j=0}^{b-1}P_{m,j}(0)z^j\right)\right.$$

$$\left.+\left(R(z,0)-\sum_{n=0}^{b-1}R_n(0)z^n\right)+\left(U(z,0)-\sum_{n=0}^{b-1}U_n(0)\,z^n\right)\right] \tag{35}$$

By substituting $\theta=\beta(\lambda-\lambda X(z))$ in (29) - (31), we get

$$Q_1(z,0)=\tilde{V}\left(\beta(\lambda-\lambda X(z))\right)C(z,0); \tag{36}$$

$$Q_j(z,0)=\tilde{V}\left(\beta(\lambda-\lambda X(z))\right)\sum_{n=0}^{a-1}Q_{j-1,n}(0)\,z^n\,; j\geq 2, \tag{37}$$

$$C(z,0)=\tilde{C}\left(\beta(\lambda-\lambda X(z))\right)\sum_{n=0}^{a-1}\left[(1-\pi)\sum_{m=a}^{b}P_{m,n}(0)+R(0)\right]z^n \tag{38}$$

By substituting $\theta=\alpha(\lambda-\lambda X(z))$ in (32) - (35) and we get

$$R(z,0)=\tilde{R}\left(\alpha(\lambda-\lambda X(z))\right)\pi\sum_{n=0}^{a-1}\sum_{m=a}^{b}P_{m,n}(0)\,z^n\,;$$

(39)

$$U(z,0)=\tilde{U}\left(\alpha\left(\lambda-\lambda X(z)\right)\right)\sum_{l=1}^{\infty}\left(\ Q_l(z,0)-\sum_{n=0}^{a-1}Q_{l,n}(0)z^n\right),$$

(40)

$$P_i(z,0)-\tilde{S}\left(\alpha\left(\lambda-\lambda X(z)\right)\right)\left[(1-\pi)\textstyle\sum_{m=a}^{b}P_{m,i}(0)+R_i(0)+U_i(0)\right];\ \ a\le i\le b-1, \qquad (41)$$

$$z^bP_b(z,0)=\tilde{S}\left(\alpha\left(\lambda-\lambda X(z)\right)\right)\Bigg[(1-\pi)\sum_{m=a}^{b-1}\left(P_m(z,0)+P_b(z,0)-\sum_{m=a}^{b}\sum_{j=0}^{b-1}P_{m,j}(0)z^j\right)$$
$$+\left(R(z,0)-\sum_{n=0}^{b-1}R_n(0)z^n\right)+\left(U(z,0)-\sum_{n=0}^{b-1}U_n(0)z^n\right)\Bigg]$$

Let us define the following,

$$p_i=\sum_{m=a}^{b}P_{m,i}(0)\,;\ \ q_i=\sum_{l=1}^{\infty}Q_{l,i}(0)\,;\ r_i\ =R_i(0);\ \ \ u_i=U_i(0);\ \ \ c_i=(1-\pi)p_i+q_i+r_i\ ;$$

and

$$k_i=(1-\pi)p_i+q_i+u_i\ ;\ \ d_i=(1-\pi)p_i+q_i$$

$$P_b(z,0)=\frac{\tilde{S}\left(\alpha\left(\lambda-\lambda X(z)\right)\right)\,f(z)}{\left(z^b-(1-\pi)\tilde{S}\left(\alpha\left(\lambda-\lambda X(z)\right)\right)-\pi\tilde{S}\left(\alpha\left(\lambda-\lambda X(z)\right)\right)\tilde{R}\left(\alpha\left(\lambda-\lambda X(z)\right)\right)\right)},$$

(42)

where

$$f(z)=\left[(1-\pi)+\pi\tilde{R}\left(\alpha\left(\lambda-\lambda X(z)\right)\right)\right]\textstyle\sum_{m=a}^{b-1}\ \ P_m(z,0)$$
$$+\tilde{U}\left(\alpha\left(\lambda-\lambda X(z)\right)\right)\left[\textstyle\sum_{l=1}^{\infty}\ Q_l(z,0)-\sum_{m=0}^{a-1}q_m\ \ z^m\right]-\ \left[\textstyle\sum_{n=0}^{b-1}[(1-\pi)p_i+r_i]z^n+\sum_{n=a}^{b-1}u_nz^n\right]$$

(43)

From equations (29) to (35) and (36) to (43), we have

$$\tilde{Q}_1(z,\theta)=\frac{\left[\tilde{V}\left(\beta\left(\lambda-\lambda X(z)\right)\right)-\tilde{V}(\theta)\right]C(z,\theta)}{\left(\theta-\beta\left(\lambda-\lambda X(z)\right)\right)},$$

(44)

$$\tilde{Q}_j(z,\theta)=\frac{\left[\tilde{V}\left(\beta\left(\lambda-\lambda X(z)\right)\right)-\tilde{V}(\theta)\right]\sum_{n=0}^{a-1}Q_{j-1,n}(0)z^n}{\left(\theta-\beta\left(\lambda-\lambda X(z)\right)\right)},j\ge 2$$

(45)

$$\tilde{C}(z,\theta)=\frac{\left(\tilde{C}\left(\alpha\left(\lambda-\lambda X(z)\right)\right)-\tilde{C}(\theta)\right)\sum_{n=0}^{a-1}[(1-\pi)p_n+r_n]z^n}{\left(\theta-\alpha\left(\lambda-\lambda X(z)\right)\right)}$$

(46)

$$\tilde{R}(z,\theta)=\frac{\left(\tilde{R}\left(\alpha\left(\lambda-\lambda X(z)\right)\right)-\tilde{R}(\theta)\right)\sum_{n=0}^{a-1}\pi p_nz^n}{\left(\theta-\alpha\left(\lambda-\lambda X(z)\right)\right)}$$

(47)

$$\tilde{U}(z,\theta)=\frac{\left(\tilde{U}\left(\alpha\left(\lambda-\lambda X(z)\right)\right)-\tilde{U}(\theta)\right)\sum_{l=1}^{\infty}\left(\ Q_l(z,0)-\sum_{j=0}^{a-1}\ Q_{i,j}(0)z^j\ \right)}{\left(\theta-\alpha\left(\lambda-\lambda X(z)\right)\right)},$$

(48)

$$\tilde{P}_i(z,\theta)=\frac{\left(\tilde{S}\left(\alpha\left(\lambda-\lambda X(z)\right)\right)-\tilde{S}(\theta)\right)c_i}{\left(\theta-\alpha\left(\lambda-\lambda X(z)\right)\right)},a\le i\le b-1$$

(49)

$$\tilde{P}_b(z,\theta)=\frac{\left(\tilde{S}\left(\alpha(\lambda-\lambda X(z))\right)-\tilde{S}(\theta)\right)f(z)}{\left(\theta-\alpha(\lambda-\lambda X(z))\right)\left(z^b-(1-\pi)\tilde{S}\left(\alpha(\lambda-\lambda X(z))\right)-\pi\tilde{S}\left(\alpha(\lambda-\lambda X(z))\right)\tilde{R}\left(\alpha(\lambda-\lambda X(z))\right)\right)}$$

(50)

Let $P(z)$ be the PGF of the queue size at an arbitrary time epoch be

$$P(z)=\sum_{m=a}^{b-1}\tilde{P}_m(z,0)+\tilde{P}_b(z,0)+\tilde{C}(z,0)+\sum_{l=1}^{\infty}\tilde{Q}_l(z,0)+\tilde{R}(z,0)+\tilde{U}(z,0).$$

By substituting $\theta=0$ on the equations (43) to (50) then the PGF of a queue size $P(z)$ at an arbitrary time epoch as

$$P(z)=\frac{\left[\begin{array}{c}\beta\left[\begin{array}{c}\left(\tilde{S}\left(\alpha(\lambda-\lambda X(z))\right)-1\right)\\+\pi\left[\tilde{R}\left(\alpha(\lambda-\lambda X(z))\right)-1\right]\tilde{S}\left(\alpha(\lambda-\lambda X(z))\right)\end{array}\right]\sum_{n=a}^{b-1}(z^b-z^n)c_n\\+\beta\left[\begin{array}{c}1-\left(\tilde{S}\left(\alpha(\lambda-\lambda X(z))\right)\right)\\-\pi\left[\tilde{R}\left(\alpha(\lambda-\lambda X(z))\right)-1\right]\tilde{S}\left(\alpha(\lambda-\lambda X(z))\right)\end{array}\right]\sum_{n=0}^{a-1}[(1-\pi)p_n+r_n]z^n\\+\beta\tilde{U}\left(\alpha(\lambda-\lambda X(z))\right)\tilde{V}\left(\beta(\lambda-\lambda X(z))\right)\tilde{C}\left(\beta(\lambda-\lambda X(z))\right)\sum_{n=0}^{a-1}d_nz^n(z^b-1)\\+\beta\tilde{U}\left(\alpha(\lambda-\lambda X(z))\right)\left(\tilde{V}\left(\beta(\lambda-\lambda X(z))\right)-1\right)\sum_{n=0}^{a-1}q_nz^n(z^b-1)\\-\left[\begin{array}{c}z^b-\left(\tilde{S}\left(\alpha(\lambda-\lambda X(z))\right)\right)\\-\pi\left[\tilde{R}\left(\alpha(\lambda-\lambda X(z))\right)-1\right]\tilde{S}\left(\alpha(\lambda-\lambda X(z))\right)\end{array}\right]\sum_{n=0}^{a-1}(\alpha k_n-\beta q_n)z^n\\+(\alpha-\beta)\left[\begin{array}{c}z^b-\left(\tilde{S}\left(\alpha(\lambda-\lambda X(z))\right)\right)\\-\pi\left[\tilde{R}\left(\alpha(\lambda-\lambda X(z))\right)-1\right]\tilde{S}\left(\alpha(\lambda-\lambda X(z))\right)\end{array}\right]\\\left[\begin{array}{c}\tilde{V}\left(\beta(\lambda-\lambda X(z))\right)\sum_{n=0}^{a-1}q_nz^n\\+\tilde{V}\left(\beta(\lambda-\lambda X(z))\right)\tilde{C}\left(\beta(\lambda-\lambda X(z))\right)\sum_{n=0}^{a-1}d_nz^n\end{array}\right]\end{array}\right]}{\left(\alpha\beta(-\lambda+\lambda X(z))\right)\left(z^b-(1-\pi)\tilde{S}\left(\alpha(\lambda-\lambda X(z))\right)-\pi\tilde{S}\left(\alpha(\lambda-\lambda X(z))\right)\tilde{R}\left(\alpha(\lambda-\lambda X(z))\right)\right)}$$

(51)

This represents the probability generating function customers in queue at an arbitrary time epoch.
The above equation (51) has b + 2a unknowns $p_0,p_1,\dots,p_{a-1},r_0,r_1,\dots,r_{a-1},q_0,q_1,\dots,q_{a-1}$ and $c_a,c_{a+1},\dots,c_{b-1}$. We express q_i and r_i ; $i=0\ to\ a-1$ in terms of p_i ; $i=0\ to\ a-1$. Hence, the equation (51) involving only b unknowns $p_0,p_1,\dots,p_{a-1},c_a,c_{a+1},\dots,c_{b-1}$. By Rouches's theorem the expression $z^b-(1-\pi)\tilde{S}\left(\alpha\left(\lambda-\lambda X(z)\right)\right)-\pi\tilde{S}\left(\alpha\left(\lambda-\lambda X(z)\right)\right)\tilde{R}\left(\alpha\left(\lambda-\lambda X(z)\right)\right)$
Has b-1 zeros inside and on the unit circle $|z|=1$. Since P(z) is analytic with in and on the unit circle, the numerator of (51) must vanish at these point, which gives b equations and b unknowns. These equations can be solved by suitable numerical techniques.

The probability generating function $P(z)$ has to satisfy $P(1)=1$. Applying L'Hospital's rule in (51), then ρ <1 is the condition to be satisfied for the existence of steady state for the model under consideration, where

$$\rho=\frac{\alpha\lambda E(X)(E(S)+\pi E(R))}{b}.$$

5. PERFORMANCE MEASURES

In this section, some useful performance measures of the proposed model like, expected number of customers in the queue E(Q), expected length of idle period E(I), expected length of busy period E(B) are derived which are useful to find the total average cost of the system. Also, probability that the server is on setup work P(U), probability that the server is on vacation P(V) and probability that the server is busy P(B) are derived.

5.1 Expected Queue Length

The expected queue length E (Q) (i.e. mean number of customers waiting in the queue) at an arbitrary time epoch, is obtained by differentiating P(z) at z=1 and given by

$$\lim_{z\to 1} P(z) = E(Q)$$

$$\mathrm{E(Q)} = \frac{\begin{pmatrix} \sum_{i=a}^{b-1} \beta(b-i)c_i f_1(X,S,R) + \sum_{i=a}^{b-1} \beta[b(b-1)-i(i-1)]c_i f_2(X,S,R) \\ + \sum_{i=0}^{a-1} \beta d_i[f_4(X,S,R,V,C)+if_6(X,S,R,V,C)] + \sum_{i=0}^{a-1} \alpha d_i[f_3(X,S,R,V,C)+if_5(X,S,R,V,C)] \\ + \sum_{i=0}^{a-1} \beta q_i[f_8(X,S,R,V)+if_{10}(X,S,R,V)] + \sum_{i=0}^{a-1} \alpha q_i[f_7(X,S,R,V)+if_9(X,S,R,V)] \end{pmatrix}}{2\alpha\beta[\lambda E(X)(b-S1+\pi R1)]^2} \quad (52)$$

where

$T1 = \lambda E(X)(b - S1 + \pi R1); T2 = \lambda X''(1)(b - T1) + \lambda E(X)[b(b-1) - T4]$

$T3 = S1 + \pi R1;\ T4 = S2 + \pi R2 + 2\pi R1S1;$

$S1 = \lambda\alpha E(X)E(S);\quad S2 = \lambda\alpha X''(1)E(S) + \lambda^2\alpha^2E^2(X)E(S^2);$

$V1 = \lambda\beta E(X)E(V);\quad V2 = \lambda\beta X''(1)E(V_1) + \lambda^2\beta^2E^2(X)E(V^2);$

$U1 = \lambda\alpha E(X)E(U);\quad U2 = \lambda\alpha X''(1)E(U) + \lambda^2\alpha^2E^2(X)E(U^2);$

$C1 = \lambda\beta E(X)E(C);\quad C2 = \lambda\beta X''(1)E(U) + \lambda^2\beta^2E^2(X)E(U^2);$

$R1 = \lambda\alpha E(X)E(R);\quad R2 = \lambda\alpha X''(1)E(R) + \lambda^2\alpha^2E^2(X)E(R^2);$

$E1 = (b - T3)(V2 + 2V1C1 + C2) + (b(b-1) - T4)(V1 - C1);$

$E2 = bU2 + 2bU1V1 + 2bU1C1 + b(b-1)U1 + T3(V2 + 2V1C1 + C2) + T4(V1 + C1)$

$E3 = (b - T3)(V1 - C1)$; $E4 = bU1 + T3(V1 - C1)$

$E5 = (b - T3)V2 + (b(b-1) - T4)V1;\ \ E6 = bV2 + 6bU1V1 + T3V2 + T4V1$

$E7 = V1(b - T3); E8 = V1T3$

$f_1(X,S,R) = [T1T4 - T3T2];\ f_2(X,S,R) = T3T1;\ f_3(X,S,R,V,C) = T1E1 - T2E3;$

$f_4(X,S,R,V,C) = T1E2 - T2E4;\ f_5(X,S,R,V,C) = T1E3;\ f_6(X,S,R,V,C) = T1E4;$

$f_7(X,S,R,V) = T1E5 - T2E7;\ f_8(X,S,R,V) = T1E6 - T2E8;$

$f_9(X,S,R,V) = T1E7;\ f_{10}(X,S,R,V) = T1E8$

5.2 Expected waiting time

The expected waiting time is obtained by using the Little's formula as

$$E(W) = \frac{E(Q)}{\lambda E(X)} \quad (53)$$

where E(Q) is given in(46)

5.3 Expected Length of Idle Period

Let I be the idle period random variable, then the expected length of the idle period is given by

$$\mathrm{E(I)} = \mathrm{E}(I_1) + \mathrm{E(C)} + \mathrm{E(U)},$$

where $I_1$is the random variable denoting ' idle period due to multiple vacation process' E(C) is the expected closedown time and E(U) is the expected length of setup time.

To find E(I_1), another random variable U_1 is defined as,

$$U_1 = \begin{cases} 0, \text{if the server finds at least 'a' customers after the first vacation} \\ 1, \text{if the server finds less than 'a' customers after the first vacation} \end{cases}$$

Now, the expected length of idle period due to multiple vacations E(I_1), is given by

$$\mathrm{E(I_1)} = \mathrm{E}(I_1/\mathrm{U_1} = 0)\mathrm{P(U_1} = 0) + \mathrm{E(I/U_1} = 1)\mathrm{P(U_1} = 1)$$

$$= \mathrm{E(V)P(U_1} = 0) + (\mathrm{E(V)} + \mathrm{E(I_1)})\mathrm{P(U_1} = 1)$$

and since $\mathrm{P(U_1 = 0) + P(U_1 = 1) = 1}$, solving for $\mathrm{E(I_1)}$, we have

$$\mathrm{E(I_1)} = \frac{\mathrm{E(V)}}{\mathrm{P(U_1 = 0)}} \quad (54)$$

To find $\mathrm{P(U_1 = 0)}$, we consider the equation (44)

$$Q_1(z,0) = \sum_{n=0}^{\infty} Q_{1,n}(0)\,\tilde{V}\left(\beta(\lambda - \lambda X(z))\right)\sum_{n=0}^{a-1}[(1-\pi)p_n + r_n]z^n$$

$$= \sum_{n=0}^{\infty} \beta_n z^n \left(\sum_{n=0}^{a-1} [(1-\pi)p_n + r_n] z^n \right)$$

Equating the coefficients of z^n $(n = 0,1,2,\dots, a-1)$ on both sides, we get

$$Q_{1,n}(0) = \sum_{i=0}^{n} \beta_i [(1-\pi)p_{n-i} + r_{n-i}]$$

Therefore,

$$\mathrm{P}(\mathrm{U}_1 = 0) = 1 - \sum_{n=0}^{a-1} Q_{1,n}(0)$$

$$= 1 - \sum_{n=0}^{a-1} \left(\sum_{i=0}^{n} \beta_i [(1-\pi)p_{n-i} + r_{n-i}] \right) \quad (55)$$

Using (48) and (49), the expected length of idle period E(I) is obtained as

$$\mathrm{E(I)} = \frac{\mathrm{E(V)}}{1 - \sum_{n=0}^{a-1} \left(\sum_{i=0}^{n} \beta_i [(1-\pi)p_{n-i} + r_{n-i}] \right)} + \mathrm{E(U)} + \mathrm{E(C)} \quad (56)$$

5.4 Expected length of busy period;

Let B be the length of busy period random variable. Let T be the residence time that the server is rendering service or under re-service. Therefore, T=S with probability $(1-\pi)$ and T=S+R with probability π.

Another random variable J is define as,

$$\mathrm{J} = \begin{cases} 0, \text{if the server finds less than 'a' customers after a residence time} \\ 1, \text{if the server finds at least 'a' customers after a residence time} \end{cases}$$

Then expected length of busy period E(B) is given by

$$E(B) = E(B/J=0)P(J=0) + E(B/J=1)P(J=1)$$
$$= E(S)P(J=0) + (E(S) + E(B))P(J=1)$$

where E(S) is the mean service time.
Solving for E(B) we get,

$$\mathrm{E(B)} = \frac{\mathrm{E(S)}}{\mathrm{P(J=0)}} + \pi \mathrm{E(R)}$$

Thus, the expected length of busy period is obtained as

$$\mathrm{E(B)} = \frac{\mathrm{E(S)}}{\sum_{n=0}^{a-1} [(1-\pi)p_n + r_n]} + \pi \mathrm{E(R)} \quad (57)$$

where E(R) is the expected re-service time and E(S) is the expected service time.

6. PARTICULAR CASES

In this section, some of the existing models are deduced as a particular case of the proposed model.

Case (i) When the closedown time is zero (i.e. $\tilde{C}\left(\alpha\left(\lambda - \lambda X(z)\right)\right) = 1$) , if all arrivals allowed to join the system, i.e., α=1 and β=1, then the equation (51) becomes

$$P(z)=\frac{\begin{bmatrix}\left(\tilde{S}\left(\lambda-\lambda X(z)\right)-1\right)\\+\pi\left[\tilde{R}\left(\lambda-\lambda X(z)\right)-1\right]\tilde{S}\left(\lambda-\lambda X(z)\right)\end{bmatrix}\sum_{n=a}^{b-1}(z^b-z^n)c_n+\begin{bmatrix}1-\tilde{S}\left(\lambda-\lambda X(z)\right)\\-\pi\left[\tilde{R}\left(\lambda-\lambda X(z)\right)-1\right]\tilde{S}\left(\lambda-\lambda X(z)\right)\end{bmatrix}\sum_{n=0}^{a-1}[(1-\pi)p_n+r_n]z^n+\sum_{n=0}^{a-1}\tilde{U}\left(\lambda-\lambda X(z)\right)\left(\tilde{V}\left(\lambda-\lambda X(z)\right)k_n-q_n\right)z^n(z^b-1)-\left[z^b-(1-\pi)\tilde{S}\left(\lambda-\lambda X(z)\right)-\pi\tilde{S}\left(\lambda-\lambda X(z)\right)\tilde{R}\left(\lambda-\lambda X(z)\right)\right]\sum_{n=0}^{a-1}(k_n-q_n)z^n}{\left(-\lambda+\lambda X(z)\right)\left(z^b-(1-\pi)\tilde{S}\left(\lambda-\lambda X(z)\right)-\pi\tilde{S}\left(\lambda-\lambda X(z)\right)\tilde{R}\left(\lambda-\lambda X(z)\right)\right)}$$

which coincides with the result A batch service queuing with multiple vacations , setup time and no server's choice of admitting re-service Haridass.M , Armuganathan.R (2012)

Case (ii): If no request for re-service (i.e. $\pi = 0$) , if all arrivals allowed to join the system, i.e., $\alpha = 1$ and $\beta=1$ and if there is no setup time (i.e., $\tilde{U}\left(\alpha\left(\lambda-\lambda X(z)\right)\right)=1$),then the equation (51) becomes

$$P(z)=\frac{1}{\left(-\lambda+\lambda X(z)\right)\left(z^b-\tilde{S}\left(\lambda-\lambda X(z)\right)\right)}\begin{bmatrix}\left[\tilde{S}\left(\lambda-\lambda X(z)\right)-1\right]\sum_{m=a}^{b}(z^b-z^m)(p_m+q_m)\\ \left[\tilde{V}\left(\lambda-\lambda X(z)\right)-1\right]\sum_{m=a}^{b}(z^b-1)(p_m+q_m)z^m\end{bmatrix}$$

which exactly coincides with the result M^X /G(a,b) /1 and multiple vacations without setup time and N-policy of Krishna Reddy et al (1998).

Case (iii): Considering single service (i.e. a = b = 1), if no request for re-service (i.e. $\pi=0$) , if all arrivals allowed to join the system, i.e., $\alpha=1$ and $\beta=1$ and if there is no setup time i.e., $\tilde{U}\left(\alpha\left(\lambda-\lambda X(z)\right)\right)=1$, then the equation (51) becomes

$$P(z)=\frac{\left[\tilde{V}\left(\lambda-\lambda X(z)\right)-1\right](Z-1)k_0}{\left(-\lambda+\lambda X(z)\right)\left(Z-\tilde{S}\left(\lambda-\lambda X(z)\right)\right)}$$

where $k_0=p_0+q_0$

which coincides with the result M^X /G /1 queueing system and multiple vacations without N-Policy of Lee et al (1994).

7. COST MODEL

Cost analysis is the most important phenomenon in any practical situation at every stage. Cost involves startup cost, operating cost, holding cost, setup cost, re-service cost and reward cost. It is quite natural that the management of the system desires to minimize the total average cost and to optimize the cost. Addressing this, in this section, the cost model for the proposed queueing system is developed and the total average cost is obtained with the following assumptions:

C_s : Startup cost per cycle
C_h : Holding cost per customer per unit time
C_o : Operating cost per unit time
C_r : Reward cost per cycle due to vacation
C_{rs} : Re-service cost per unit time
C_u : Closedown cost per unit time
C_g : Setup cost per unit time

since the length of the cycle is the sum of the idle period and busy period, expected length of the cycle $E(T_c)$ is given by

$$E(T_c)=E(I)+E(B)$$

$$E(T_c)=\frac{E(V)}{1-\sum_{n=0}^{a-1}\left(\sum_{i=0}^{n}\beta_i[(1-\pi)p_{n-i}+r_{n-i}]\right)}+E(U)+E(C)+\frac{E(S)}{\sum_{i=0}^{a-1}[(1-\pi)p_n+r_n]}+\pi E(R)$$

Now, the total average cost per unit time is obtained as

Total average cost = Start-up cost per cycle + holding cost of number of customers in the queue per unit time + Operating cost per unit time * ρ+ re-service cost per unit time + closedown time cost + Setup cost per cycle– reward due to vacation per cycle

$$TAC = \left(C_s - C_r\left(\frac{E(V)}{P(U=0)}\right) + C_u E(C) + C_g E(U) + C_{rs} E(R)\pi\right)\frac{1}{E(T_c)} + C_h E(Q) + C_0 \rho \quad (58)$$

where $\rho = \frac{\lambda\alpha E(X)[E(S)+\pi E(R)]}{b}$.

It is difficult to have a direct analytical result for the optimal value a* (minimum batch size in M^X/G(a,b)/1 queueing system) to minimize the total average cost. The simple direct search method to find optimal policy for a threshold value a* to minimize the total average cost, is defined.

Step 1: Fix the value of maximum batch size 'b'

Step 2: Select the value of 'a' which will satisfy the following relation

$$TAC(a*) \le TAC(a), \;\; 1 \le a \le b$$

Step 3: The value a* is optimum, since it gives minimum total average cost.

Using the above procedure, the optimal value of 'a' can be obtained, which minimizes the total average cost function. Some numerical example to illustrate the above procedure is presented in the next section.

8. NUMERICAL EXAMPLE

The above queueing model is analysed numerically with the following assumptions:

(i) Service time distribution is Erlang-k with k = 2

(ii) Batch size distribution of the arrival is geometric with mean 2

(iii) Vacation, re-service time, closedown time and setup time are exponential with parameters υ = 10, ω = 6, γ = 7 and η = 8

(iv) α = 0.5 and β = 0.6 respectively

(v) Service capacity with minimum a = 2 and maximum b = 4

The zero of the function $z^b - (1-\pi)\tilde{S}(\lambda - \lambda X(z)) - \pi\tilde{S}(\lambda - \lambda X(z))\tilde{R}(\lambda - \lambda X(z))$are got using MATLAB software professional and the simultaneous equations are solved. Results are presented for the service rate μ = 5 and the arrival rate ranging from 1.0 to 3.5 in the following data.

μ= 5

Table 1.1: Computed values of various queue characteristics

λ	ρ	E(Q)	E(I)	E(B)	E(W)
1.0	0.1714	0.0932	0.3962	3.8054	0.0466
1.5	0.2572	0.4374	0.3960	3.8637	0.1458
2.0	0.3429	1.1972	0.3957	4.0008	0.2982
2.5	0.4287	2.6072	0.3956	4.0246	0.5214
3.0	0.5143	4.8856	0.3955	4.1047	0.8143
3.5	0.6000	8.0872	0.3950	4.1132	1.1553

μ = 6

Table 1.2: Computed values of various queue characteristics

λ	ρ	E(Q)	E(I)	E(B)	E(W)
1.0	0.1448	0.0976	0.3963	3.0808	0.0488
1.5	0.2171	0.4634	0.3962	3.0808	0.1545
2.0	0.2895	1.3862	0.3960	3.1994	0.3464
2.5	0.3619	2.8235	0.3956	3.2427	0.5647
3.0	0.4343	5.4817	0.3955	3.2995	0.9136
3.5	0.5067	9.5805	0.3950	3.3634	1.3686

The Table 1.1 and 1.2 clearly shows as long as increasing the arrival rate λ, the expected length of idle period decreases while the utilization factor, the expected queue length, expected waiting time, expected length of busy period of our queueing model are all increases.

9. CONCLUSION

An M^X/G(a,b)/1 queue with multiple vacations, optional re-service closedown time and setup time under restricted admissibility of arriving batches has been studied. The PGF of queue size at arbitrary time epoch is obtained. Some performance measures are also derived. Table1.1 and 1.2 shows that the effect of arrival rate λ over E(Q). From the table 1.1 and 1.2, it is clear that the average number of customer in the system increase as arrival rate λ increases.

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