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TYPES OF DOMINATION IN INTUITIONISTIC FUZZY GRAPH BY STRONG ARC AND EFFECTIVE ARC

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Abstract

In this paper, we introduce the definitions of strong arc, non- strong arc, semi μ- strong arc, semi ρ- strong arc, effective arc, semi μ-effective arc and semi ρ-effective arc of intuitionistic fuzzy graph. Further, the types of domination on intuitionistic fuzzy graph by using strong arc and effective arc are defined. Also the non-strong arc in intuitionistic fuzzy (IF) Path and IF Cycle are respectively compared with Fuzzy Path and Fuzzy cycle. Finally the relation between the effective arc domination and strong arc domination of IF graph with a suitable illustration is explained.

Keywords: Fuzzy graph, Intuitionistic fuzzy graph, Strong arc, Non strong arc, μ- strong arc , semi ρ-strong arc , Strong arc domination, Effective arc domination.

1 INTRODUCTION

In 1736, Euler introduced the concept of graph theory for the first time. In the history of mathematics, the solution given by Euler of the well known Konigsberg Bridge problem is considered to be the first theorem of graph theory. This subject is now generally regarded as a branch of combinatorics. The theory of graph is an extremely useful tool for solving combinatorial problems in different areas such as geometry, algebra, number theory, topology, operations research, optimization and computer science.

In 1999, Atanassov [1] introduced the concept of intuitionistic fuzzy set - a generalization of fuzzy sets and fuzzy relation in intuitionistic graph. Atanassov [2] added a new component (which determines the degree of non membership) in the definition of fuzzy set. The fuzzy sets give the degree of membership of an element in a given set (and the non membership degree equals one minus the degree of membership). While intuitionistic fuzzy set give both a degree of membership and a degree of non membership which more –or –less are independent of each other, the only requirement is that the sum of these two degrees is not greater than 1. Intuitionistic fuzzy sets have been applied in a wide variety of fields including computer science, engineering, mathematics, medicine, chemistry and economics.

In 1975, Rosenfeld [10] introduced the concept of fuzzy graphs. The fuzzy relations between fuzzy sets were also considered by Rosenfeld and he developed the structure of fuzzy graphs, obtaining analogs of several graph theoretical concepts. Later on, some operations on fuzzy graphs were introduced by Mondeson and Nair [4]. Mondeson and Nair [4] introduced the notion of fuzzy line graph. Bhutani and Rosenfeld [3] introduced the concept of M-strong fuzzy graphs and studied some of their properties. Nagoorgani and Chandrasekaran [6] discussed the domination in fuzzy graphs. Akram and Bijan Davvaz [5] dicussed strong intuitionistic fuzzy graphs. Atanassov [1] introduced the concept of intuitionistic fuzzy relations and intuitionistic fuzzy graphs in various areas, and further [2] studied interval-valued fuzzy graphs and intuitionistic fuzzy graphs are two different models that extend theory of fuzzy graph. Parvathi, Karunambagai and Atanassov [8] also are discussed the domination in intuitionistic fuzzy graph.

2. PRELIMINARIES

In this section, we review some definitions that are necessary for our main work and which can be found in the works [6,7,8,9,11] and [13].

Definition 2.1
A fuzzy subset μ on a set X is a map $\mu : X \to [0,1]$. A map ρ :X x X →[0,1] is called a fuzzy relation on X if $\rho(x,y) \leq \min(\mu(x), \mu(y))$ for all x,y in X. A fuzzy relation ρ is symmetric if $\rho(x,y) = \rho(y,x)$.

Definition 2.2
A mapping A = $(\mu_A, \rho_A) : X \to [0,1]\, X\, [0,1]$ is called an intuitionistic fuzzy set in X if $\mu_A(x) + \rho_A(x) \leq 1$ for all x$\in X$, where the mappings $\mu_A : X \to [0,1]$ and $\rho_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and degree of non-membership (namely $\rho_A(x)$) of each element x$\in X$ to A respectively.

Definition 2.3
Let X be a non empty set. Then we call a mapping A= (μ_A, ρ_A): X$\times X \to [0,1] \times [0,1]$ an intuitionistic fuzzy relation on X if $\mu_A(x,y) + \rho_A(x,y) \leq 1$ for all (x, y) $\in X \times X$.

Definition 2.4
Let A = (μ_A, ρ_A) and B = (μ_B, ρ_B) be intuitionistic fuzzy sets on a set X.
If A= (μ_A, ρ_A) is an intuitionistic fuzzy relation on a set X, then A= (μ_A, ρ_A) is called an intuitionistic fuzzy relation on B = (μ_B, ρ_B) if $\mu_A(x,y) \leq min\{\mu_B(x), \mu_B(y)\}$ and
$\rho_A(x,y) \leq \max\{\rho_B(x), \rho_B(y)\}$ for all x, y $\in X$.
An intuitionistic fuzzy relation A on X is called symmetric if $\mu_A(x,y) = \mu_A(y,x)$
and $\rho_A(x,y) = \rho_A(y,x)$ for all x, y $\in X$.

Definition 2.5
An intuitionistic fuzzy graph(IFG) with underlying set V is defined to be a pair
G (A,B), where (i) the functions $\mu_A : V \to [0,1]$ and $\rho_A : V \to [0,1]$ denote the the degree of membership and degree of non-membership of each element x$\in V$, respectively such that $\mu_A(x) + \rho_A(x) \leq 1$ for all x$\in V$.
(ii) the functions $\mu_B : E \subseteq$ V xV $\to [0,1]$ and $\rho_B : E \subseteq$ V xV $\to [0,1]$ are defined by
$\mu_B(x,y) \leq min\{\mu_A(x), \mu_A(y)\}$ and $\rho_B(x,y) \leq \max\{\rho_A(x), \rho_A(y)\}$ such that $0 \leq \mu_B(x,y) + \rho_B(x,y) \leq 1$ for all (x, y) $\in E$.

We call that A is the intuitionistic fuzzy vertex set and B is the intuitionistic fuzzy arc set of G.

Note : B is a symmetric intuitionistic fuzzy relation on A.

Definition 2.6
If $u, v \in V \subseteq$ G , then μ_B – strength of connectedness between u and v is ${\mu_B}^{\infty}(u,v)$.
where , ${\mu_B}^{\infty}(u,v) = \sup\{{\mu_B}^k(u,v) : k = 0,1,2,3 \ldots n\}$ and
ρ_B – strength of connectedness between u and v is ${\rho_B}^{\infty}(u,v)$ = inf { ${\rho_B}^k(u,v)$: k = 0,1,2,…n}.
If u and v are connected by means of paths of length k then
${\mu_B}^k(u,v) =$ sup{ $\mu_B(u,v_1) \wedge \mu_B(v_1,v_2) \wedge \ldots . \mu_B(v_{k-1},v)$ / $u, v_1, v_2, \ldots v_{k-1}, v \in V\}$ and ${\rho_B}^k(u,v) =$ inf { $\rho_B(u,v_1) \vee \rho_B(v_1,v_2) \vee, \ldots \rho_B(v_{k-1},v)$ / $u, v_1, v_2, \ldots v_{k-1}, v \in V$ }

Definition 2.7
An arc (u,v) is said to be a bridge if ${\mu_B}^{\infty}(u,v) < {\mu_B}'^{\infty}(u,v)$ and ${\rho_B}^{\infty}(u,v) \geq {\rho_B}'^{\infty}(u,v)$ (or)${\mu_B}^{\infty}(u,v) \leq {\mu_B}'^{\infty}(u,v)$ and ${\rho_B}^{\infty}(u,v) > {\rho_B}'^{\infty}(u,v)$ for some (u,v) $\in$ E.

In other words , deleting an arc (u,v) reduces the strength of connectedness between u and v.
(or) arc(u,v) is a bridge if there exist vertices v_i, v_j such that (u,v) is an arc of every strongest path from $v_i \ to \ v_j$.

Definition 2.8
A vertex v is said to be a cut vertex of connected intuitionistic fuzzy graph G if deleting that vertex reduces the strength of connectedness between some pairs of vertices.(or) A vertex v is a cut vertex iff there exist vertices v_i, v_j such that v is a vertex of every strongest path from $v_i \ to \ v_j$.

Definition 2.9
Let G(V,E) be an IF graph , then the vertex cardinality of V is defined by $|V| = \sum_{v \in V} \frac{1+\mu_A(v)-\rho_A(v)}{2}$ and edge cardinality of E is defined by $= \sum_{(u,v) \in E} \frac{1+\mu_B(u,v)-\rho_B(u,v)}{2}$.
The cardinality of IFG is defined by $|G| = \left| |V| + |E| \right|$.

3. DOMINATION IN INTUITIONISTIC FUZZY GRAPH BY EFFECTIVE ARC

Here, the definitions of strong arc (non-strong arc) , semi μ- strong arc , semi ρ- strong arc , effective arc , semi μ-effective arc and semi ρ-effective arc of intuitionistic fuzzy graph are introduced. Further, the types of domination on intuitionistic fuzzy graph by using strong arc and effective arc are also defined

Definition 3.1
An arc (u,v) of the intuitionstic fuzzy graph G is called a effective arc if
$\mu_B(x,y) = min\{ \mu_A(x), \mu_A(y)\}$ and $\rho_B(x,y) = \max \{ \rho_A(x), \rho_A(y)\}$ such that
$0 \leq \mu_B(x,y) + \rho_B(x,y) \leq 1$ for all (x,y) $\in E$.

Definition 3.2
An arc (u,v) of the intuitionstic fuzzy graph G is called a semi μ -effective arc if
$\mu_B(x,y) = min\{ \mu_A(xs), \mu_A(y)\}$ and $\rho_B(x,y) \neq \max \{ \rho_A(x), \rho_A(y)\}$ such that
$0 \leq \mu_B(x,y) + \rho_B(x,y) \leq 1$ for all (x,y) $\in E$.

Definition 3.3
An arc (u,v) of the intuitionstic fuzzy graph G is called a semi ρ -effective arc if
$\mu_B(x,y) \neq min\{ \mu_A(x), \mu_A(y)\}$ and $\rho_B(x,y) = \max \{ \rho_A(x), \rho_A(y)\}$ such that
$0 \leq \mu_B(x,y) + \rho_B(x,y) \leq 1$ for all (x,y) $\in E$.

Note:
(i) Effective neighborhood of u $\in V$ is $N_e(u) = \{v \in V: arc(u,v)$ is effective arc$\}$.
$N_e[u] = N_e(u) \cup \{u\}$ is the closed neighborhood of u.
(ii) Semi μ- effective neighborhood of u $\in V$ is

$$N_{\mu e}(u) = \{v \in V: arc(u,v) \text{ is semi } \mu\text{- effective arc}\} .$$

and $N_{\mu e}[u] = N_{\mu e}(u) \cup \{u\}$ is the closed neighborhood of u.
(iii) Semi ρ- effective neighborhood of $u \in V$ is

$$N_{\rho e}(u) = \{v \in V: arc(u,v) \text{ is semi } \rho\text{- effective arc}\} .$$

and $N_{\rho e}[u] = N_{\rho e}(u) \cup \{u\}$ is the closed neighborhood of u.
(iv) The minimum cardinality of effective neighborhood $\delta_e(G) = \min\{|N_e(u)|: u \in V(G)\}$.
(iv) Maximum cardinality of effective neighborhood $\Delta_e(G) = \max \{|N_e(u)|: u \in V(G)\}$.

Example 1:

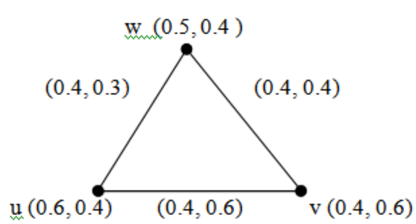


Fig. (i)

From the fig. (i)

Case (i) Consider the arc (u,v):
here $\mu_B(u,v) = min\{\mu_A(u), \mu_A(v)\}$ = min{ 0.6, 0.4} = 0.4 and
$\rho_B(u,v) =$ max { $\rho_A(u), \rho_A(v)$} = max{ 0.4, 0.6 } = 0.6.
Therefore arc (u,v) is an effective arc.

Case (ii)
Consider the arc (u,w):
here $\mu_B(u,w)$ = 0.4 $\neq min\{\mu_A(u)\ \mu_A(w)\}$ and
$\rho_B(u,w)$ = 0.3$\neq$ max { $\rho_A(u), \rho_A(w)$}, Hence arc (u,w) is not effective arc.

Case (iii) Consider the arc (v,w):
here $\mu_B(v,w)$ = 0.4 = $min\{\mu_A(v),\ \mu_A(w)\}$ and $\rho_B(v,w)$ = 0.4$\neq$ max { $\rho_A(v), \rho_A(w)$}.
Hence arc (v,w) is not an effective arc, but we can say it a semi μ- effective arc .

Definition 3.4
An arc (u,v) of the intuitionstic fuzzy graph G is called a strong arc if $\mu_B(u,v) = {\mu_B}^{\infty}(u,v)$ and $\rho_B(u,v) = {\rho_B}^{\infty}(u,v)$, else arc(u,v) is called non- strong.

Definition 3.5
An arc (u,v) of the intuitionstic fuzzy graph G is called a semi μ- strong arc if
$\mu_B(u,v) = {\mu_B}^{\infty}(u,v)$ and $\rho_B(u,v) \neq {\rho_B}^{\infty}(u,v)$.

Definition 3.6
An arc (u,v) of the intuitionstic fuzzy graph G is called a semi ρ - strong arc if
$\mu_B(u,v) \neq {\mu_B}^{\infty}(u,v)$ and $\rho_B(u,v) = {\rho_B}^{\infty}(u,v)$.

Note:
(i) Strong neighborhood of $u \in V$ is $N_s(u) = \{v \in V: arc(u,v)$ is strong} .
$N_s[u] = N_s(u) \cup \{u\}$ is the closed neighborhood of uu.
(ii) Semi μ- Strong neighborhood of $u \in V$ is $N_{\mu s}(u) = \{v \in V: arc(u,v)$ is μ- strong arc} .
$N_{\mu s}[u] = N_{\mu s}(u) \cup \{u\}$ is the closed neighborhood of u.
(iii) Semi ρ- Strong neighborhood of $u \in V$ is $N_{\rho s}(u) = \{v \in V: arc(u,v)$ is ρ- strong arc} .
$N_{\rho s}[u] = N_{\rho s}(u) \cup \{u\}$ is the closed neighborhood of u.
(iv) The minimum cardinality of strong neighborhood $\delta_s(G) = \min\{|N_s(u)|: u \in V(G)\}$.
(iv) Maximum cardinality of strong neighborhood $\Delta_s(G) = \max\{|N_s(u)|: u \in V(G)\}$.

Example 2:

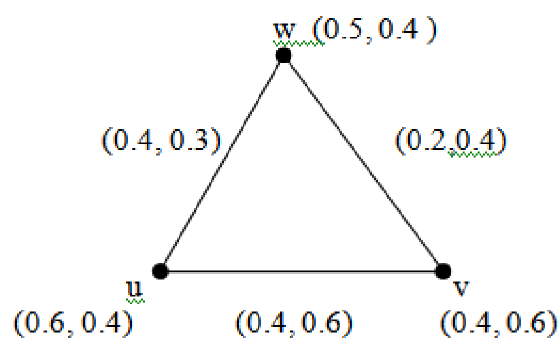


Fig. (ii)

From the fig. (ii)

Case (i) Consider the arc (u,v):
here $\mu_B(u,v)$ = 0.4 and $\rho_B(u,v)$ = 0.6.

Now, $\mu_B{}^{\infty}(u,v)$ = sup { 0.4, 0.3 } = 0.4 and $\rho_B{}^{\infty}(u,v)$ = inf{0.6, 0.4} = 0.4;
therefore, $\mu_B(u,v) = \mu_B{}^{\infty}(u,v)$ = 0.4 but $\rho_B(u,v) \neq \rho_B{}^{\infty}(u,v)$.
Hence arc (u,v) is non strong . And we can say that arc (u,v) semi μ - strong.

Case (ii)
Consider the arc (u,w):
here $\mu_B(u,w)$ = 0.4 and $\rho_B(u,w)$ = 0.3.
Now, $\mu_B{}^{\infty}(u,w)$ = sup { 0.4, 0.3 } = 0.4 and $\rho_B{}^{\infty}(u,w)$ = inf{0.3, 0.6} = 0.3;
therefore, $\mu_B(u,w) = \mu_B{}^{\infty}(u,v)$ = 0.4 and $\rho_B(u,w) = \rho_B{}^{\infty}(u,w) = 0.3$.
Hence arc (u,w) is strong arc .

Case (iii) Consider the arc (v,w):
here $\mu_B(v,w)$ = 0.2 and $\rho_B(v,w)$ = 0.4.
Now, $\mu_B{}^{\infty}(v,w)$ = sup { 0.2, 0.4 } = 0.4 and $\rho_B{}^{\infty}(v,w)$ = inf{0.4, 0.6} = 0.4;
therefore, $\mu_B(v,w) \neq \mu_B{}^{\infty}(v,w)$ but $\rho_B(v,w) = \rho_B{}^{\infty}(v,w)$.
Hence arc (v,w) is non strong arc and we can say that arc (v,w) semi ρ - strong.

From the **Case (i)**, we have
$\mu_B(u,v) = min\{ \mu_A(u), \mu_A(v)\}$ = min{ 0.6, 0.4} = 0.4 and
$\rho_B(u,v) =$ max { $\rho_A(u), \rho_A(v)$} = max{ 0.4, 0.6 } = 0.6
In this case, arc (u,v) is satisfies the effective arc condition.

Definition 3.7
Let G be a intuitionstic fuzzy graph. Let u, v be two nodes of G. We say that u dominates v if arc (u,v) is an effective arc.

A subset D of V is called a dominating set of IFG if for every $v \in V - D$, there exists $u \in D$ such that u dominates v. A dominating set D is called a minimal dominating set if no proper subset of D is a dominating set. The minimum fuzzy cardinality taken over all dominating sets of an IFG is called the effective arc domination number and is denoted by $\gamma_E(G)$ and the corresponding dominating set is called the minimum effective arc dominating set. The number of elements in the minimum effective arc dominating set is denoted by n[$\gamma_E(G)$].

Definition 3.8
(i) Let G be an intuitionstic fuzzy graph. Let u, v be two nodes of G.
We say that u (semi μ- effective) dominates v if arc (u,v) is a semi μ- effective arc .
(ii) Let G be an intuitionstic fuzzy graph. Let u, v be two nodes of G.
We say that u (semi ρ- effective) dominates v if arc (u,v) is a semi ρ- effective arc.
Note:
(i) Semi μ- effective arc domination number is denoted by $\gamma_{\mu E}(G)$ and the number of elements in the minimum (semi μ-effective arc) dominating set is denoted by n[$\gamma_{\mu E}(G)$].

(ii) Semi ρ- effective arc domination number is denoted by $\gamma_{\rho E}(G)$ and the number of elements in the minimum (semi ρ-effective arc) dominating set is denoted by n[$\gamma_{\rho E}(G)$].

Definitions 3.9
Let G be an intuitionstic fuzzy graph. Let u, v be two nodes of G. We say that u dominates v if arc (u,v) is a strong arc.

A subset D of V is called a dominating set of IFG if for every $v \in V - D$, there exists $u \in D$ such that u dominates v. A dominating set D is called a minimal dominating set if no proper subset of D is a dominating set. The minimum fuzzy cardinality taken over all dominating sets of an IFG is called the strong arc domination number and is denoted by $\gamma_s(G)$ and the corresponding dominating set is called the minimum strong arc dominating set. The number of elements in the minimum strong arc dominating set is denoted by n[$\gamma_s(G)$].

Definition 3.10
(i) Let G be an intuitionstic fuzzy graph. Let u, v be two nodes of G.
We say that u semi μ- dominates v if arc (u,v) is a semi μ- strong arc .
(ii) Let G be an intuitionstic fuzzy graph. Let u, v be two nodes of G.
We say that u semi ρ- dominates v if arc (u,v) is a semi ρ- strong arc.

Note:
(i) Semi μ- strong arc domination number is denoted by $\gamma_{\mu s}(G)$ and the number of elements in the minimum semi μ-strong arc dominating set is denoted by n[$\gamma_{\mu s}(G)$].
(ii) Semi ρ- strong arc domination number is denoted by $\gamma_{\rho s}(G)$ and the number of elements in the minimum semi ρ-strong arc dominating set is denoted by n[$\gamma_{\rho s}(G)$].

Theorem 3.1
If any two vertices of an intuitionistic fuzzy graph G are connected by exactly one path then every arc of G is a strong arc.

Proof:
Let G be a connected Intuitionstic fuzzy graph and let n be the number vertices of G.
Take n = 2 , there must be u and v adjoined by one arc (since G is a connected intuitionstic fuzzy graph).
Clearly, $\mu_B(u,v) = \mu_B{}^{\infty}(u,v)$ and $\rho_B(u,v) = \rho_B{}^{\infty}(u,v)$. Therefore, arc (u, v) is strong.
Assume that n > 2. In an intuitionstic fuzzy path, the $\mu_B{}^{\infty}(u,v)$ and $\rho_B{}^{\infty}(u,v)$ of any arc in the path will have the same fuzzy value $\mu_B(u,v)$ and $\rho_B(u,v)$ of the arc (u,v), since they are connected by one path. By the above argument, evidently it is proved that $\mu_B(u,v) = \mu_B{}^{\infty}(u,v)$ and $\rho_B(u,v) = \rho_B{}^{\infty}(u,v)$ for any number of arcs in a given path . Hence all the arcs are strong.

Corollary: 3.2
In intuitionstic fuzzy Path (IF Path) all arcs are strong.

Theorem. 3.3
Every non trivial connected Intuitionistic fuzzy graph G when $n \geq 2$, has atleast one strong arc.
Proof
Let IF graph G be a connected IF graph ,with $n \geq 2$ and let u,v be two vertices of G
Case i : When n =2:
Since G is a connected fuzzy graph, there are two vertices u and v such that (u,v) is an arc.
By above theorem, there is only one strongest path between u and v such that
$\mu_B(u,v) = \mu_B{}^{\infty}(u,v)$ and $\rho_B(u,v) = \rho_B{}^{\infty}(u,v)$.Therefore, arc (u,v) in strong.

Case ii : When n > 2:
We Claim that G has at least one strong arc.
Since G is connected with n >2 , there may be more than one path between u and v, in which at least one path is strong such that $\mu_B(u,v) = \mu_B{}^{\infty}(u,v)$ and $\rho_B(u,v) = \rho_B{}^{\infty}(u,v)$ (by the above theorem) . If not, then there is no path between u and v. Therefore G is a disconnected graph which is contradicts our initial hypothesis that G is connected. Hence every non trivial connected fuzzy graph G when ≥ 2 , has atleast one strong arc.

Corollary 3.4
Every non trivial connected IF graph for which $n \geq 2$ has at least two vertices which are not cut vertices.

Theorem 3.5
In intuitionstic fuzzy cycle, at most two non strong arcs can exist.

Proof: Let us take the minimum consideration of intuitionstic fuzzy cycle as in given Fig. (iii)

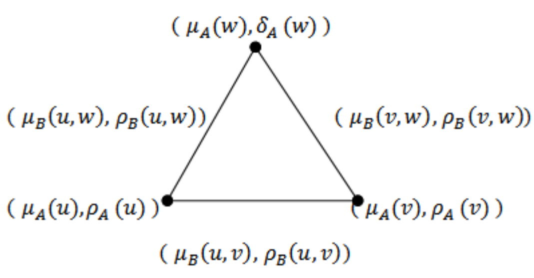


Fig. (iii)

Assume that $\mu_B(u,v)$ is the least membership fuzzy value of the arc (u, v) compared with other arcs and $\rho_B(u,w)$ is the highest non membership fuzzy value of the arc (u,w) compared with other arcs for the above Fig(iii).

Therefore, we can have $\mu_B(u,v) < \mu_B(u,w) \leq \mu_B(v,w)$ and

$$\rho_B(u,w) > \rho_B(u,v) \geq \rho_B(v,w) \qquad \text{(i)}$$

We have to prove that these two arcs (u ,v) and (u ,w) must be non strong arc.

Now, $\mu_B{}^{\infty}(u,v) = \sup\{\mu_B(u,v), \mu_B(u,w), \mu_B(v,w)\} = \mu_B(u,w)$ or $\mu_B(v,w)$ and $\rho_B{}^{\infty}(u,v)$ =Inf{ $\rho_B(u,w), \rho_B(u,v)$,$\rho_B(v,w)\} = \rho_B(u,v)$ or $\rho_B(v,w)$.

Therefore, $\mu_B(u,v) \neq \mu_B{}^{\infty}(u,v)$ and $\rho_B(u,w) \neq \rho_B{}^{\infty}(u,w)$. (ii)

Hence, arc(u,v) and arc(u,w) do not satisfy the strong arc condition.

Therefore, the two arcs of this intuitionstic fuzzy cycle are non strong. Evidently we can prove that for any intuitionstic fuzzy cycle can have at most two non strong arcs.

Example 3:
From the fig. (i) we see that the arc (u,v) and arc (v,w) are two non –strong arcs.
Note: In any Fuzzy cycle at most one non- strong can exist[11].

Theorem 3.6
Let G be an intuitionstic fuzzy graph and let (u,v) be one of the arcs of G then the following are equivalent to each other.
(i) Arc (u,v) is a strong arc in G.
(ii) Arc (u,v) must be semi μ – strong and semi ρ – strong.
(iii) Membership and non membership of (u,v) arc must lie in the closed interval [μ_{SB}, ρ_{LB}],

where μ_{SB} is the smallest value of membership and ρ_{LB} is the largest value of the non- membership of the intuitionistic fuzzy graph.

Theorem 3.7
Let G be an intuitionstic fuzzy graph. If arc (u,v) is an effective arc in G then it need not be a strong arc in G (it must be semi μ – strong).

Proof: Let (u,v) be an effective arc .
Then $\mu_B(u,v) = \min\{\mu_A(u), \mu_A(v)\}$ and $\rho_B(u,v) = \max\{\rho_A(u), \rho_A(v)\}$.
Assume that $\mu_A(u) > \mu_B(v)$ and $\rho_A(u) < \rho_A(v)$.
Then $\mu_B(u,v) = \mu_A(u)$ and $\rho_B(u,v) = \rho_A(v)$.
By the definition of an intuitionstic fuzzy graph, μ_B – strength of connectedness between u and v is $\mu_B{}^{\infty}(u,v)$, where, $\mu_B{}^{\infty}(u,v) = \sup\{\mu_B{}^k(u,v) : k = 0,1,2,3 \ldots n\}$ and
ρ_B – strength of connectedness between u and v is $\rho_B{}^{\infty}(u,v)$ = inf { $\rho_B{}^k(u,v)$: k = 0,1,2,…n}.
From the example (i), we have
$\mu_B(u,v) = \mu_B{}^{\infty}(u,v)$ and $\rho_B(u,v) \neq \rho_B{}^{\infty}(u,v)$.

Hence, it does not satisfy the strong arc condition though it is an effective arc. Therefore, an effective arc needs not be a strong arc in IF graph.

Note
(i) The effective arc must be semi μ – strong (it cannot be semi ρ – strong) in an IF graph.
(ii) In a fuzzy graph, every effective arc is a strong arc [11].
(iii) The converse of the above need not be true (i.e., a strong arc need not be an effective arc).

Example 4:

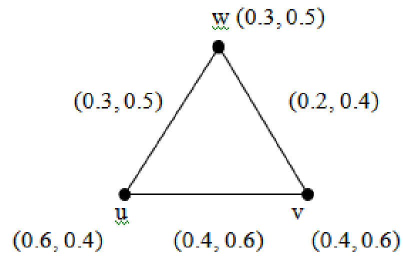


Fig. (iv)

In fig (iv), arc (u,v) and arc (u,w) are effective arcs .
For,

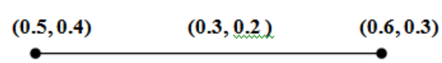
Case(i)
$\mu_B(u,v) = \min\{\mu_A(u), \mu_A(v)\} = 0.4$ and $\rho_B(u,v) = \max\{\rho_A(u), \rho_A(v)\} = 0.6$
By our definition of a strong arc,
$\mu_B(u,v) = {\mu_B}^{\infty}(u,v)$ and $\rho_B(u,v) \neq {\rho_B}^{\infty}(u,v)$.
This means that arc (u,v) is a non- strong arc (but it is called a semi μ - strong arc).
In this case arc(u,v) is an effective arc but not a strong arc.

Case (ii)
$\mu_B(u,w) = \min\{\mu_A(u), \mu_A(w)\} = 0.2$ and $\rho_B(u,v) = \max\{\rho_A(u), \rho_A(v)\} = 0.5$
But arc (u,w) satisfies the strong arc condition $\mu_B(u,v) = {\mu_B}^{\infty}(u,v)$ and $\rho_B(u,v) = {\rho_B}^{\infty}(u,v)$.
In this case, arc(u,w) is an effective arc as well as a strong arc.
From the above two cases, we conclude that an effective arc need not be a strong arc in IF graph, but in fuzzy graph all the effective arcs must be strong arcs [11].

Corollary 3.8
Let G be a fuzzy graph. Then $\gamma_S(G) \leq \gamma_E(G)$.

Proof:
Let D = $\{x_1, x_2, \ldots, x_m\}$ be a minimum effective dominating set. Let $u \epsilon V - D$ then there exist x_i in D such that the arc (x_i, u) is an effective edge. By above theorem, the arc (x_i, u) is a strong arc. Therefore D = $\{x_1, x_2, \ldots, x_m\}$ is a strong arc dominating set. Hence $\gamma_S(G) \leq \gamma_E(G)$.
We can illustrate this by the following example.

Consider the following intuitionistic fuzzy Path (IFP_2)



Clearly $\mu_B(u,v) = {\mu_B}^{\infty}(u,v) = 0.3$ and $\rho_B(u,v) = {\rho_B}^{\infty}(u,v) = 0.2$.

Therefore, arc (u,v) is a strong arc , But it does not satisfy the effective arc condition.
Therefore, we have D = { u} or D ={ v}(by strong arc dominating set).
Hence, n[$\gamma_S(G)$] = $|D| = 1$ (iii)
But we must have D = { u ,v} (by effective arc dominating set).
Therefore, n[$\gamma_E(G)$] = $|D| = 2$ (iv)
From (iii) and (iv) follows that $\gamma_S(G) \leq \gamma_E(G)$ (by definition 2.8).

4. CONCLUSION

An intuitionistic fuzzy graph is a generalization of the notion of a fuzzy graph. In this paper, the concept of strong arc and non-strong arc in intuitionistic fuzzy graph like IF path and IF cycle are discussed. Further, the definition of μ – strong, semi ρ – strong and effective arc, semi μ-effective arc and semi ρ-effective arc of intuitionistic fuzzy graph are introduced. Further, the types of domination on intuitionistic fuzzy graph by using strong arc and effective arc are defined. Finally the strong arc domination and effective arc domination are discussed and it is concluded that the strong arc domination is always less than or equal to the effective arc domination. Next the study focuses in finding the domination number of standard intuitionistic fuzzy graph by using the concepts of strong arc and effective arc.

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