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RELIABILITY ANALYSIS OF A SYSTEM BY USING SUGENO (TSK) FUZZY MODEL

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Abstract

In the present paper the binary assumption is replaced by a fuzzy state assumption, thereby leading to fuzzy reliability estimates. In reliability estimation failure rate estimation is an important parameter. Available methods for the estimation of such parameters do not cover up such type of uncertainty in the failure data collection involving human judgment, evaluation and decision. In this paper we introduce a new approach based on Sugeno [TSK] fuzzy model to estimate such system parameters and analyze the fuzzy reliability of a three unit system by Sugeno's fuzzy model. We also obtain the fuzzy effect on the system reliability.

Keywords: Reliability, Fuzzification, Fuzzy numbers, Defuzzification, Sugeno Fuzzy Models, Profust reliability, Probist reliability.

1. INTRODUCTION

In gracefully degradable systems [2] it is unrealistic to assume that the system possesses only two states that is, 'working' or 'failed'. Such systems may be considered working to certain degrees at different states of its degradation during its transfer from fully working state to completely failed state. The grading may be any real number in the closed interval [0 1]. Grading zero would represent the system in completely failed state while the grading one represents the fully working state of the system. The allocation of the grading may rely upon the tolerance limit of the user about the adequate performance of the system. Zadeh [6] suggested a paradigm shift from the theory of total denial and affirmation to a theory of grading, to give a new concept of sets called fuzzy sets. Fuzzy logic can express the progressing slow process of the system from operating state to non-operative state. The classical set logic only dichotomizes the system in operative state and non-operative state but fuzzy state theory can cover up all possible states between a fully working state and completely failed state. This approach to the reliability theory is known as profust reliability [8] wherein the binary state assumption [1] is replaced by Fuzzy state assumption.

In the present paper we study the reliability of a three unit system. This work also includes the fuzzy failure rate estimation. The fuzzy failure rate estimation is based on Sugeno (TSK) fuzzy models. In fact the failure rate is one of the important parameters in reliability estimation which involves uncertainties of different kinds. In some cases the relationship between the failure mechanism and the failure rate function may be used in making a choice of failure distribution. Sometimes two or more types of failures occur at once. It is the exponential distribution that has been mostly explored by the researchers for failure distribution because it has a number of desirable mathematical properties. It has a constant failure rate. To estimate the constant parameter λ of the exponential distribution, the system is to be tested for failure times under certain desired operating conditions. It may impart different failure characteristics in different trials. At this stage assignment of a crisp number to the parameter λ involves uncertainties in actual observations of failure time and also in statistical approximation in getting a single value from several close options. In this study we use fuzzy numbers to find out a crisp number from these fuzzy sets by using the Sugeno fuzzy models and precisely covering up the involved uncertainties in estimation of failure rates we obtain a crisp output from a fuzzy input.

2. MODEL DESCRIPTION

Suppose the system comprises of three independent operative units, which can perform their own tasks in parallel. Evidently we can take the system to be fully functioning when all the three processing units are failed. However, when two processing units are functioning and the other one is failed, or when one processing unit is functioning unit and the other two are failed; the system operates as degraded throughout. At this point the system is neither fully functioning nor fully failed, but is in some intermediate states.

We consider a gracefully degradable system [5] with three identical and independent modules. Each module has only two states: non-functioning or functioning. The time to break down for each module follows an exponential distribution with parameter λ. This system is activated with three active modules. When failure comes to some module, the system immediately takes reconfiguration operation, with negligible time, to remove the faulty module, whereas the other fault-free modules continue to do this work if the reconfiguration operation is performed successfully.

Let S_3 represents the system state when three active (operational) modules are available. The system may have states: S_0, S_1, S_2 and S_3. The figure 1 below depicts the Markovian transitions among the system states, where c represent the coverage factor i.e. success probability of a reconfiguration operation.

Defuzzification: In certain situations one needs a crisp output when the input number is fuzzy. Defuzzification is the tool that makes it possible. We have several methods of defuzzication [2, 9] in the literature. Here in this work we use the centroid method, which is given by the following expression.

$$x^* = \frac{\int \mu_{\tilde{A}}(x).x\ dx}{\int \mu_{\tilde{A}}(x).\ dx}$$

where $\tilde{A}$ is a fuzzy set, which is the union of two or more fuzzy sets i.e. $\tilde{A} = \tilde{A}_1 \cup \tilde{A}_2 \cup \ldots \cup \tilde{A}_n$ and $\cup$ is a standard fuzzy union.

Sugeno Fuzzy Models: The Sugeno fuzzy model was proposed by Takagi, Sugeno and Kang [10, 11]. This method is similar to the Mamdani's method but in this method the first two parts fuzzify the inputs and apply the fuzzy operators. In Sugeno's method the output is linear or constant.

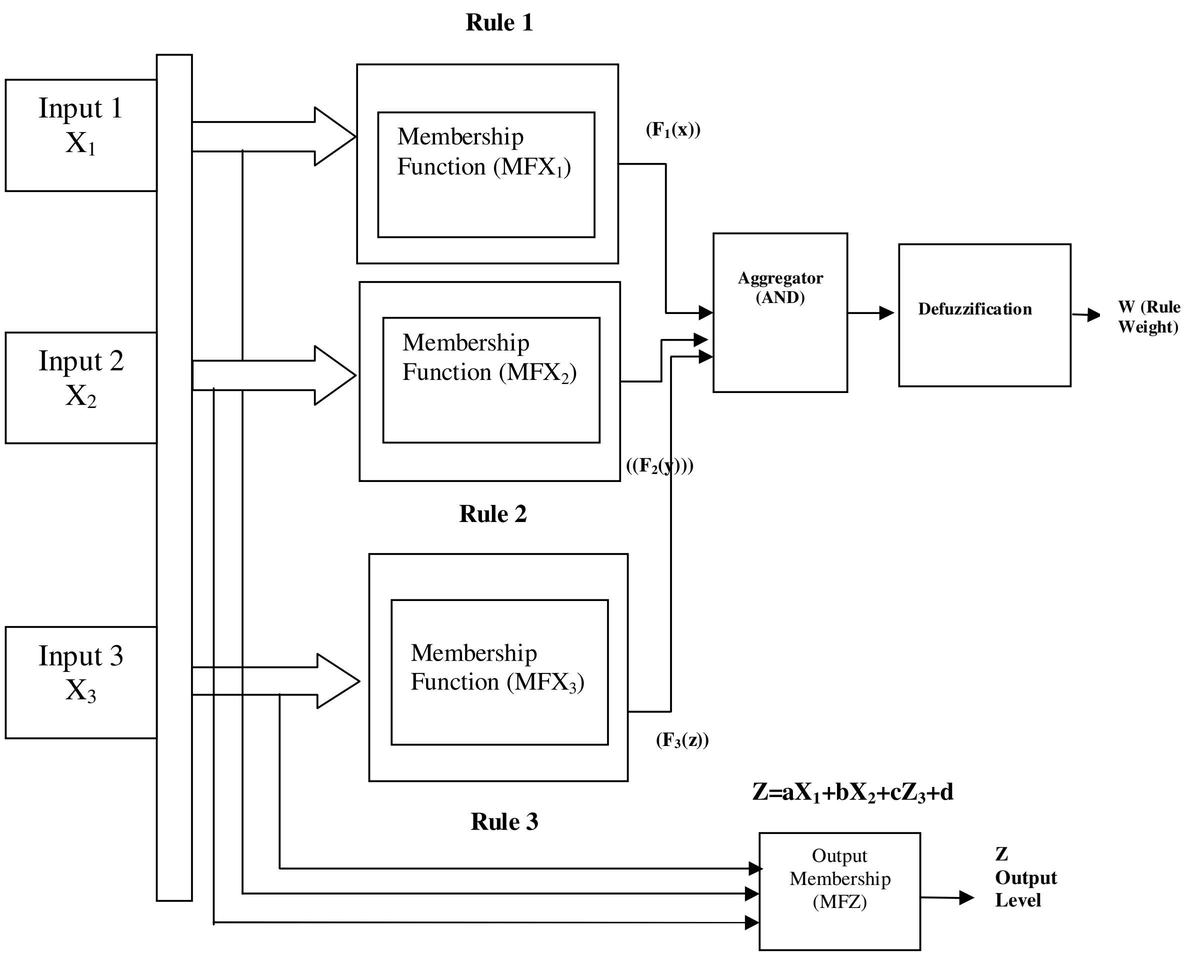


Fig 1:

Failure Rate Parameter Estimation By Sugeno's Fuzzy Model: Failure / Repair rates [7] are important parameters in the estimation of reliability characteristics of any system. A small error in failure rate may lead to over / under estimation of system reliability. For systems having very sensitive applications, this risk must be avoided to the maximum possible extent. A standard method for determining a failure rate parameter is the maximum likelihood utilizing estimation from multiple data sets. Collection of failure data may involve following uncertainties:

1. Failure exactly occurs, but the failure time is not accurately observed or might be missed.
2. Failure doesn't occur or occurs partially. So, the reported failure time is based on censored observation.
3. Multiple failure data values need to be obtained under similar operating conditions. Operating Conditions, in all cases, cannot be uniquely explained and contain hazziness concerning the description.
4. It may involve human judgment, evaluation and decision at certain stages that may be vague.

Under the above-mentioned situations, it is appropriate to deal with the failure data by fuzzy techniques. We propose a method based on Sugeno fuzzy model to estimate failure rate parameter by using the concepts of fuzzy numbers, fuzzy aggregation and defuzzification. Defuzzification is the process that creates a single assessment from the fuzzy conclusion set. The philosophy of the method is based on the two basic concepts of Sugeno's fuzzy model:-

(i) In first step we will find out the weight for the failure data set.
(ii) In second step we will find the out the output from the input given from the various data sets.

On the basis of these two steps we will get a weighted average of all the rules for output computed as:

$$Final\ output = \frac{\sum_{m=1}^{N} W_m Z_m}{\sum_{m=1}^{N} W_m}$$

On the basis of the following two steps we generate the algorithm for the evaluation of the failure rate as follows:

(a) Failure rate is first estimated according to the existing procedure. The process must be done so many times that more than one number is available for estimating the failure rate.

(b) Numbers obtained in (a) will be fuzzified by using the fuzzufication process.

(c) Now by using the fuzzy operations i.e. (OR=max) fuzzy union we will get a single fuzzy number.

(d) Now we will use the defuzzification method to get a single crisp number.

(e) On the basis of (a) we will provide the weights for the inputs and also fuzzify the weights.

(f) Numbers obtained in (e) will be supplied to the fuzzy operations (AND=min) fuzzy intersection and will get a single fuzzy number.

(g) Now we will use the defuzzufication process to get a single output.

On the basis of this algorithm we will get various weights and various outputs which will be based on the number of fuzzy rules which we have defined for our system.

For finding out the final output we will use

$$Final\ output = \frac{\sum_{m=1}^{N} W_m Z_m}{\sum_{m=1}^{N} W_m}$$

on the basis of which we will get a single (crisp) value which will be used as failure rate.

To demonstrate the process of fuzzy failure rate estimation, it would be in the fitness of the things, if we use the same crisp failure rates for fuzzy failure rate estimation and also for fuzzy reliability estimation in this paper. This will be beneficial in making a comparative study of the reliability estimates in the two cases.

Let λ^1=0.0002, λ^2=0.0008 and λ^3=0.0016 be three numbers obtained as the estimated failure rates of the components of unit, by the existing method in three repetitions of the process. Instead of taking their average λ= 0.00086 as the final value for failure rate, step (b) suggests to define three fuzzy numbers $\tilde{\lambda}_1$ $\tilde{\lambda}_2$ and $\tilde{\lambda}_3$ about 0.0002, 0.0008 and 0.0016 respectively.

$$\tilde{\lambda}' = \begin{cases} \dfrac{\lambda}{.0002} & 0 \le \lambda \le .0002 \\ 1 & .0002 \le \lambda \le .0004 \\ \dfrac{.0008-\lambda}{.0002} & .0004 \le \lambda \le .0008 \end{cases} \quad \tilde{\lambda}'' = \begin{cases} \dfrac{\lambda - .0004}{.0002} & .0004 \le \lambda \le .0006 \\ 1 & .0006 \le \lambda \le .0012 \\ \dfrac{.0014-\lambda}{.0002} & .0012 \le \lambda \le .0014 \end{cases}$$

$$\tilde{\lambda}''' = \begin{cases} \dfrac{\lambda - .0012}{.0002} & .0012 \le \lambda \le .0014 \\ 1 & .0014 \le \lambda \le .0016 \\ \dfrac{.0030-\lambda}{.0014} & .0016 \le \lambda \le .0030 \end{cases}$$

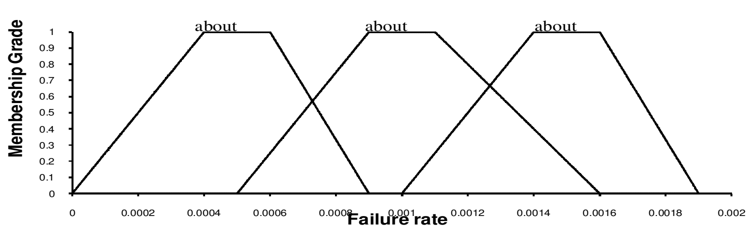


Fig 2: Failure rates

As suggested in step (c), the fuzzy union of $\tilde{\lambda}_1(x)$, $\tilde{\lambda}_2(x)$ and $\tilde{\lambda}_3(x)$ is given by $\tilde{\lambda}$ where,

$$\tilde{\lambda}(x) = \begin{cases} \dfrac{x}{.0002} & if\ 0 \le x < .0002 \\ 1 & if\ .0004 \le x < .0006 \\ \dfrac{.0006 - x}{.00024} & if\ .0006 \le x < .000847 \\ \dfrac{x - .0007}{.000371} & if\ .000729 \le x < .0009 \\ 1 & if\ .00085 \le x < .0011 \\ \dfrac{.0011 - x}{.0004} & if\ .0011 \le x < .001145 \\ \dfrac{x - .001}{.0003} & if\ .001145 \le x < .00145 \\ 1 & if\ .0014 \le x < .0016 \\ \dfrac{.0024 - x}{.0008} & if\ .0016 \le x < .00240 \end{cases}$$

Fig.3 shows the fuzzy union of $\tilde{\lambda}_1(x)$, $\tilde{\lambda}_2(x)$ and $\tilde{\lambda}_3(x)$.

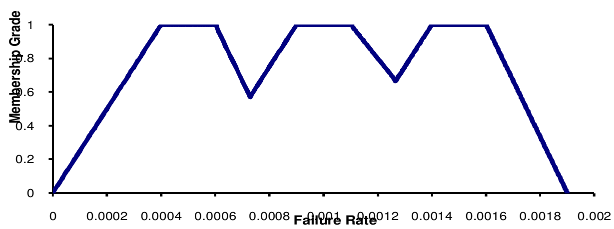


Fig 3: Failure rates

The weights for the failure rates are defined as follows:

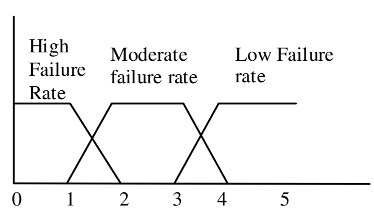
High failure rate: $0 \le x_i \le 2$;

Medium failure rate: $1 \le x_i \le 4$;

Low failure rate: $3 \le x_i \le 5$.

Fuzzy sets for high failure rate, moderate failure rate and low failure rate are also defined with the following trapezoidal membership functions:

The membership functions for high, moderate and low failure rate are as follows:



Using the centroid method for defuzzification of λ, we consider the area covered by the fuzzy number and return the center of gravity of the covered area as the required non-fuzzy number.

$$\lambda \text{ (Non-fuzzy number)} = Final\ output = \frac{\sum_{m=1}^{N} W_m Z_m}{\sum_{m=1}^{N} W_m}$$

The defuzzification of $\tilde{\lambda}(x)$ yields the crisp value 0.000521 for $\tilde{\lambda}$, from which we can get the fuzzy reliability evaluation of the system.

3. MATHEMATICAL FORMULATION AND SOLUTION OF THE MODEL

The differential equations of the system are: -

$$\frac{dp_3}{dt}+3\lambda p_3=\mu p_1, \qquad \frac{dp_2}{dt}+2\lambda p_2=\mu p_1, \qquad \frac{dp_1}{dt}+(\lambda+\mu)p_1=2\lambda c p_2,$$

$$\frac{dp_0}{dt}=\lambda(1-c)p_1+2\lambda(1-c)p_2+3\lambda(1-c)p_3, \quad p_3(0)=1$$

Solution of the Model

$$P_i(t)=c^{3-i}e^{-i\lambda t}\binom{3}{i}\left(1-e^{-\lambda t}\right)^{3-i}$$

i = 0, 1, 2, 3

$$P_0(t)=1-\sum_{i=1}^{3}P_i(t) \quad , \quad \text{where,} \quad \binom{3}{i}=\frac{3!}{i!(3-i)!}$$

Now, we define one fuzzy success state S and one fuzzy failure state F with membership function,

$$\mu_s(S_i)=\frac{i}{3}; i=0,1,2,3.$$

$$\mu_F(S_i)=\frac{3-i}{3}; i=0,1,2,3.$$

Then, we have a transition from success to failure states,

$$\mu_{T_{SF}}(m_{ij})=\begin{cases} i-\dfrac{j}{3}, & \text{if } i>j \\ 0, & \text{if } i\le j \end{cases}$$

We have $p_3(0)=1$, so the system profust reliability becomes,

$$R(t)=\sum_{j=1}^{3}\frac{j}{3}P_j(t) \qquad =\sum_{j=1}^{3}\frac{j}{3}c^{3-j}c^{-j\lambda t}\frac{3!}{j!(3-j)!}\left(1-e^{-\lambda t}\right)^{3-j}$$

If the system success / failure is defined clearly i.e.

$$\mu_S(S_i)=\begin{cases}1; i\ge k\\ 0; i\le k\end{cases}$$

$$\mu_F(S_i)=\begin{cases}0; i\ge k\\ 1; i\le k\end{cases}$$

where k is some positive integer, then

$$R(t)=\sum_{i=k}^{3}P_i(t)$$

i.e. the system profust reliability reduces to the system probist reliability.

4. EFFECT OF FUZZINESS

We have the system profust reliability

$$R(t) = \sum_{j=1}^{3}\frac{j}{3}c^{3-j}c^{-j\lambda t}\frac{3!}{j!(3-j)!}\left(1-e^{-\lambda t}\right)^{3-j}$$

$$= c^2 e^{-\lambda t}(1 - e^{-\lambda t})^2 + 2ce^{-\lambda t}(1 - e^{-\lambda t}) + e^{-3\lambda t}$$

$$= c^2\left[e^{-\lambda t} + e^{-3\lambda t} - 2e^{-2\lambda t}\right] + 2c\left[e^{-2\lambda t} - e^{-3\lambda t}\right] + e^{-3\lambda t}$$

$$R(t) = c^2 e^{-\lambda t} + (c-1)^2 e^{-3\lambda t} + 2c(1-c)e^{-2\lambda t}$$

Again for a non - fuzzy case,

$$R_k(t) = \sum_{k=1}^{3} P_k(t), \quad \text{where} \quad P_k(t) = c^{3-k} e^{-k\lambda t}\binom{3}{k}(1 - e^{-\lambda t})^{3-k}$$

Therefore $R_k(t) = \sum_{k=1}^{3} 3c^{3-k} e^{-k\lambda t}\binom{3}{k}(1 - e^{-\lambda t})^{3-k}$, k= 1, 2, 3.

Or, $R_1(t) = c^2 e^{-\lambda t}\binom{3}{1}(1-e^{-\lambda t})^2$, $R_2(t) = c\ e^{-2\lambda t}\binom{3}{2}(1-e^{-\lambda t})$, $R_3(t) = e^{-3\lambda t}\binom{3}{3}(1-e^{-\lambda t})^0$

For the effect of fuzziness attached to the system success / failure state of the system on the system reliability, we have

$\Delta R_k(t) = R(t) - R_k(t)$; k=1, 2, 3

Therefore

$$\Delta R_1(t) = -2c^2 e^{-\lambda t} + (-c + 4c^2)e^{-2\lambda t} + (c - 2c^2)e^{-3\lambda t}$$

$$\Delta R_2(t) = c^2 e^{-\lambda t} + (-c + 2c^2)e^{-2\lambda t} + (c + c^2)e^{-3\lambda t}$$

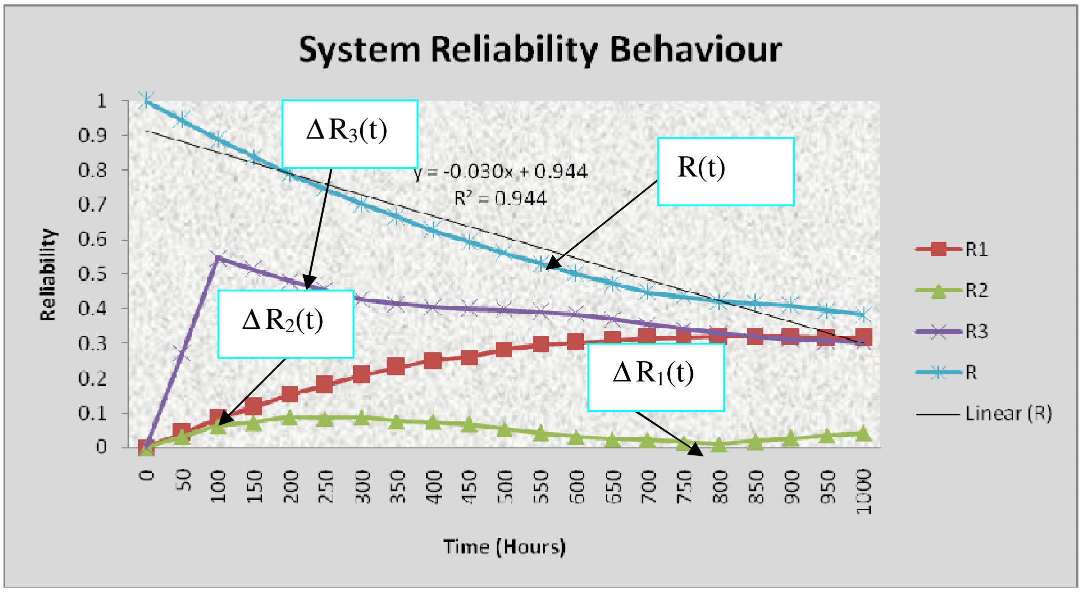
and, $\Delta R_3(t) = c^2 e^{-\lambda t} + (-2c + c^2)e^{-2\lambda t} + 2c(1 - c\ )e^{-3\lambda t}$.

Now failure rate from fuzzy failure rate estimation

$$\lambda = 0.000521 \ / hour$$

TABLE 1: Effect of fuzziness- Numerical results of profust Reliability and effect of fuzziness on the system when (λ =.000521 / hour, t=500 hours, c=0.7928)

Time X(hours)	$\Delta R_1(t)$	$\Delta R_2(t)$	$\Delta R_3(t)$	R(t)
0	0	0	0	1
50	0.0421	0.0312	0.2734	0.9438
100	0.0834	0.0635	0.5469	0.8877
150	0.1182	0.0718	0.5132	0.8385
200	0.1529	0.0876	0.4805	0.7893
250	0.1808	0.0856	0.4534	0.7461
300	0.2087	0.0876	0.4263	0.7029
350	0.2312	0.0758	0.4144	0.6649
400	0.2514	0.0742	0.4056	0.6268
450	0.2614	0.0682	0.4011	0.5934
500	0.2823	0.0543	0.3966	0.5597
550	0.2967	0.0424	0.3908	0.5301
600	0.3033	0.0318	0.3851	0.5004
650	0.3098	0.0258	0.3702	0.4742
700	0.3156	0.0219	0.3554	0.4479
750	0.3187	0.0155	0.3429	0.4344
800	0.3207	0.0102	0.3305	0.4209
850	0.3205	0.0183	0.3215	0.4154
900	0.3203	0.0277	0.3126	0.4098
950	0.3178	0.0345	0.3079	0.3964
1000	0.3152	0.0422	0.3031	0.3829



Graph 1: Fuzzy Reliability behavior for a gracefully degradable system
(λ =.000521 / hour, t=500 hours, c=0.7928)

5. CONCLUSION

The two state assumptions does not seem very effective to cover up most of the uncertainties that occurr in the data and various statistical methods used for failure rate estimation and reliability evaluation of a system. Here in this paper we introduce a new approach for failure rate estimation based on Sugeno's fuzzy model. The degraded state of the system is considered to be fuzzy in nature rather than crisp, i.e., many system states are taken between a fully working and fully failed state to evaluate the reliability of the system. The probist reliability of the system, which is a particular case of fuzzy reliability, is evaluated. This consideration also shows that the curves representing fuzzy reliabilities are very smooth, that suits the actual degradation of the system. From the numerical values of the results we see that lower the value of the coverage factor no reconfiguration of the system takes place and the fuzzy reliability becomes probist reliability.

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