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EQUATIONS OF MOTION IN OUT OF PLANE PHOTOGRAVITATIONAL ELLIPTIC RESTRICTED THREE BODY PROBLEMS WITH SMALLER PRIMARY OBLATE

Sunil Kumar Pandit[1,*], Ashok Kumar[2]

Author Affiliation:
[1]Research scholar, Department of Mathematics, Jai Prakash University, Sadhapur, Chapra (Saran), Bihar 841301, India.
[2]Department of Mathematics, D.A.V. College (Jai Prakash University), Shanti Nagar, Siwan, Bihar 841226, India.

***Corresponding Author:**
Sunil Kumar Pandit, Research scholar, Department of Mathematics, Jai Prakash University, Sadhapur, Chapra (Saran), Bihar 841301

E-mail: panditsk1991@gmail.com

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Abstract

We consider out of plane photogravitational elliptic restricted three body problem. We suppose that the bigger primary is radiating and the smaller one is an oblate spheroid. We find the forces acting due to radiation and oblateness of the primaries. We also calculate the mean motion. Finally, we derive the equations of motion for our problem. These are different from the classical elliptic restricted three body problem (ERTBP).

Keywords: equations of motion, photogravitational, ERTBP, oblate primary.

1. INTRODUCTION

In the out of plane restricted three body problem, the third body which has an infinitesimal mass moves in the gravitational field of the primaries which revolve in circle around their common centre of mass. Five libration points lie in xy-plane, of which three collinear points $L_{1,2,3}$ lies on the x-axis, while the other two are called the triangular points $L_{4,5}$. Sharma and Subba Rao [12] studied the restricted three body problem when the primaries are oblate spheroids. Sharma [11], Arrendondo et al. [2], Singh and Umar [13, 14], Safiya Beevi and Sharma [9], Abouelmed and El-Shaboury [1] studied problems involving cases amongst which the three bodies had oblateness of the primaries and with or without radiation in the circular case.

We introduce here the elliptical motion of the primaries, we get significant effect, because the orbits of most planets and stars are elliptic rather than circular, this problem is termed as elliptic ERTBP. Kumar and Ishwar [5] established the equations of motion in generalized photogravitational elliptic restricted three body problem. Also some researchers like Szebehely [15], Sahoo and Ishwar [10], Szenkovits and Mako [16], Narayan and Ramesh [7,8] have considered the ERTBP in various aspects. Douskos and Markellos [4] analyze the three dimensional problem of the out of plane equilibrium point with oblateness, they determine numerically their positions and stability, together with the effects of their existence on their topology of zero velocity curve. The out of plane equilibrium points of a passive micron size particle and their stability in the field of radiating binary systems in the photogravitational circular restricted three body problem (CRTBP) was examined by Das et al. [3].

2. EQUATIONS OF MOTION

The equations of motion of the elliptic restricted three body problem are given below following Sahoo and Ishwar [10] as

$$\xi'' - 2\eta' = \frac{\partial\bar{\Omega}}{\partial\xi}$$

$$\eta'' + 2\xi' = \frac{\partial\bar{\Omega}}{\partial\eta} \quad (1)$$

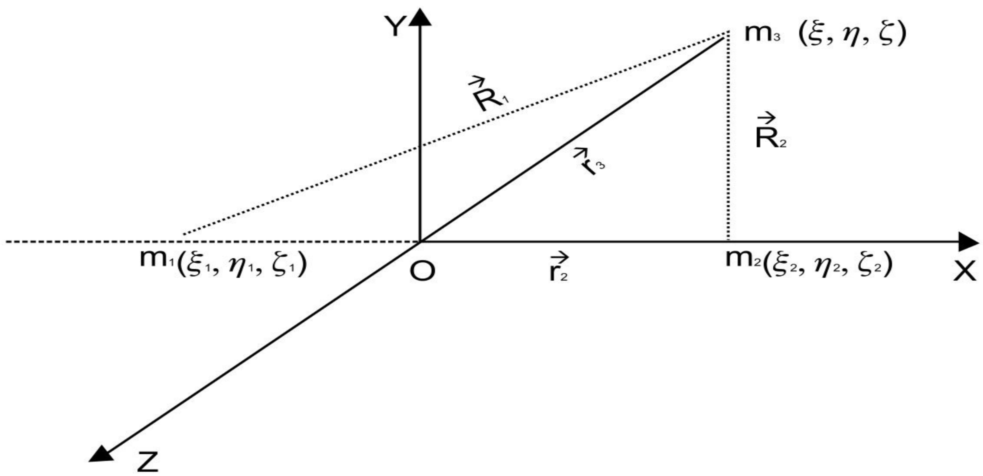
$$\zeta'' = \frac{\partial\bar{\Omega}}{\partial\zeta}$$

where the force function $\bar{\Omega} = (1-e^2)^{\frac{-1}{2}}\left\{\frac{\xi^2+n^2}{2} + \frac{\varpi}{n^2}\right\}$ (2)

where $\varpi = k^2\left(\frac{m_1}{r_1} + \frac{m_2}{r_2}\right)$

And $n_i^2 = (\xi - \xi_i)^2 + \eta^2 + \zeta^2$

here the dash(‘) denotes differentiation with respect to the eccentric anomaly.



$fig.(i)$

In the out of plane problem depicted in figure (i) above we consider the bigger primary as radiating and the smaller primary as an oblate spheroid. Now we discuss the force due to solar radiation pressure and the oblateness of the primaries.

3. FORCE DUE TO SOLAR RADIATION PRESSURE

Let f_g be the gravitational attraction force and f_p be the solar radiation pressure which acts opposite tof_g. Also f_p changes with distance by the same law as the gravitational attraction forcef_g.
So, the force of radiation is given by

$$F = f_g - f_p = f_g\left(1 - \frac{f_p}{f_g}\right) = q_1 . f_g$$

where $q_1 = 1 - \frac{f_p}{f_g}$, is the mass reduction factor constant for a given particle.

This mass reduction factor constant q_1 can be written as $q_1 = \frac{1-5.6 \text{ x } 10^{-5}}{r\rho}K$, where r is the radius of the radiating particle of density ρ, K is in e.g.s. We see that due to radiation pressure, the force function becomes lesser than the classical one. Hence instead of mass m_1, $m_1 q_1$ will be appearing in the force function.

4. FORCE DUE TO OBLATENESS WHEN ONE PRIMARY IS RADIATING AND OTHER IS OBLATE

According to McCuskey [6], the force due to oblateness of the smaller primary is

$$F = \frac{K^2 m_2}{{r_2}^2} + \frac{3K^2}{2{r_2}^4}\left(\frac{AE^2 - AP^2}{5R^2}\right) m_2$$

where K^2 is the gravitational constant and AE, AP are the dimensional equatorial and polar radii of the smaller primary.

Therefore , $$\mathrm{F} = -\frac{\partial \Omega^*}{\partial r_2}$$

where Ω^* is the potential due to oblateness.

Integrating with respect to r_2, we get

$$-\frac{K^2 m_2}{r_2} - \frac{K^2 m_2}{2{r_2}^3}\left(\frac{AE^2 - AP^2}{5R^2}\right) + c = -\Omega^*$$

Or $$-K^2\left\{\frac{m_2}{r_2} + \frac{m_2 A_2}{2{r_2}^3}\right\} = -\Omega^*$$

where, $\Omega^* = \left\{\frac{m_2}{r_2} + \frac{m_2 A_2}{2{r_2}^3}\right\}$, $A_2 = \left(\frac{AE^2 - AP^2}{5R^2}\right)$ and $K = 1$

Therefore, the force due to radiation and oblateness of m_1 and m_2 is given by

$$\Omega = (1 - \mathrm{e}^2)^{\frac{-1}{2}}\left\{\frac{\xi^2 + \mathrm{n}^2}{2} + \frac{1}{\mathrm{n}^2}\left(\frac{m_1 q_1}{r_1} + \frac{m_2}{r_2} + \frac{m_2 A_2}{2{r_2}^3}\right)\right\} \tag{3}$$

Now, We consider $m_1 = 1 - \mu$, $m_2 = \mu$ such that $\mu \ll \frac{1}{2}$

Therefore,

$$\Omega = (1 - \mathrm{e}^2)^{\frac{-1}{2}}\left\{\frac{\xi^2 + \mathrm{n}^2}{2} + \frac{1}{\mathrm{n}^2}\left(\frac{(1-\mu) q_1}{r_1} + \frac{\mu}{r_2} + \frac{\mu A_2}{2{r_2}^3}\right)\right\} \tag{4}$$

5. MEAN ANGULAR VELOCITY

The orbit of mass m_1 with respect to the centre of mass has semi major axis $a_1 = m_1 a$ and similarly, the orbit of mass m_2 with respect to the centre of mass has semi major axis $a_2 = m_2 a$ (see Szebehely [15]).
Then,

$$\frac{\mathrm{n}^2 a m_1 (1 - \mathrm{e}^2)}{(1 + \mathrm{e}^2)^{1/2}} = K^2 m_2\left\{1 + \frac{3A_2}{2}\right\}$$

and

$$\frac{\mathrm{n}^2 a m_2 (1 - \mathrm{e}^2)}{(1 + \mathrm{e}^2)^{1/2}} = K^2 m_1\left\{1 + \frac{3A_2}{2}\right\}.$$

Considering the distance between the two primaries as unity and adding the above equations we get

$$\frac{n^2 a(m_1+m_2)(1-e^2)}{(1+e^2)^{1/2}} = K^2(m_1+m_2)\left\{1+\frac{3A_2}{2}\right\}$$

or, $$\frac{n^2 a(1-e^2)}{(1+e^2)^{1/2}} = K^2\left\{1+\frac{3A_2}{2}\right\}$$

or, $$\frac{n^2 a(1-e^2)}{(1+e^2)^{1/2}} = \left\{1+\frac{3A_2}{2}\right\} \qquad [since\ K^2 = 1]$$

or, $$n^2 a(1-e^2) = (1+e^2)^{1/2}\left\{1+\frac{3A_2}{2}\right\}$$

or, $$n^2 = \frac{(1+e^2)^{1/2}}{a(1-e^2)}\left\{1+\frac{3A_2}{2}\right\} \tag{5}$$

where, a is the semi – major axis and e is the eccentricity of the ellipse, while A_2 denotes the coefficient of the oblanteness of the smaller primary.

Therefore, the equation (1) , Now takes the following form.

$$\xi'' - 2\eta' = \frac{\partial\Omega}{\partial\xi}$$

$$\eta'' + 2\xi' = \frac{\partial\Omega}{\partial\eta} \tag{6}$$

$$\zeta'' = \frac{\partial\Omega}{\partial\zeta}$$

where $$\Omega = (1-e^2)^{\frac{-1}{2}}\left[\frac{\xi^2+n^2}{2} + \frac{1}{n^2}\left\{\frac{(1-\mu)q_1}{r_1} + \frac{\mu}{r_2} + \frac{\mu A_2}{2{r_2}^3}\right\}\right] \tag{7}$$

The expression of Ω contains eccentricity, radiation coefficient and theo blateness coefficient .

6. CONCLUSION

We conclude that force function (Ω) is different from classical case in the out of plane photogravitational ERTBP with oblate primary. Mean motion is also affected by the semi-major axis and the eccentricity of ellipse. This model may be applied to find the dynamic behavior of small objects such as cosmic dust, grains etc.

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8. REFERENCES

[1] Abouelmagd, E.I., El-Shaboury, S.M. (2012). *Astrophys. Space. Sci.* 341, 331.
[2] Arrendondo, J.A., Guo, J., Jianguag, S.C., Tamayo, C. (2012). *Astrophys. Space Sci.* 341, 315.
[3] Das, M.K., Narang, P., Mahajan, S., Yuasa, M. (2009). *J. Astrophys. Astron.* 30, 177.
[4] Douskos, C.N., Markellos, V.V. (2006). *Astron. Astrophys.* 446, 357.
[5] Kumar, S., Ishwar, B.(2009). *American Inst. of Physics*, 978-0-7354-0683-4/09.

[6] McCuskey, S.W. (1963): Introduction to Celestial Mechanics, Addison-Wesley Publishing Company, Inc., New York.
[7] Narayan, A., Ramesh, C. (2011). *Int. J. Pure Appl. Math.* 68, 201.
[8] Narayan, A., Ramesh, C. (2011). *Int. J. Pure Appl. Math.* 70, 735.
[9] Safiya Beevi, A., Sharma, R.K. (2012). *Astrophys. Space Sci.* 340, 245S.
[10] Sahoo, S.K., Ishwar, B. (2000). *Bull. Astron. Soc. India* 28, 579.
[11] Sharma, R.K. (1987). *Astrophys. Space Sci.* 135, 271.
[12] Sharma, R.K., Subba Rao, P.V. (1976). *Celest. Mech.* 13, 137.
[13] Singh, J., Umar, A. (2012). *Astron. J.,* 143, 109.
[14] Singh, J., Umar, A. (2012). *Astrophys. Space Sci.* 341, 349.
[15] Szebehely, V.G. (1967). Theory of Orbits. Academic Press, New York.
[16] Szenkovits, F., Mako, Z. (2005). *PADEU* 15, 221.