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Notions of Projective Bounded Sets in Sequence Spaces

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Abstract

In this article we generalise some notions in the context of "Infinite Matrices and sequence spaces" by Dr. R.G. Cooke [3], such as projective bounded sets. Examples are included to illustrate the theoretical results .In particular our purpose is to generalise some Lemma on projective bounded sets.

Keywords: Projective bounded sets, the smallest sequence space, P-convergence, P-limit, C-convergence.

AMS classification No………[46A45]

1. INTRODUCTION

The notion of projective bounded sets is quite old and can be s found in Cooke [3].

2. DEFINITIONS AND PRELIMINARIES

If $\phi \le \beta \le \alpha^*$, and if the set of projections of sequences in a set X in α on every fixed direction in β is bounded, i.e., if $\left|\sum_{k=1}^{\infty} u_k^r x_k^r\right| \le s(u)$ for every x in X and a fixed u in β, we say that X is projective bounded (p-bd) relative to β, or $\alpha\beta$-bd. When $\beta = \alpha^*$, we say that X is p-bd in α or α-bd.

Clearly every $\alpha\beta$-cgt sequence is $\alpha\beta$-bd. The projection of *x* on $e^{(k)}$ is x_k^r, and so, if *X* is $\alpha\beta$-bd, then $|x_k|^r \le s_k$ for every *k* and for every *x* in *X*. (since $\beta \ge \phi$, it contains $e^{(k)}$.)

Definition: We denote by $\left(x^{(n)}\right)^s$ the sequence represented by $\left\{\left(x_k^{(n)}\right)^s\right\}$ where *s* > 0. Also if the sequences $x^{(1)}, x^{(2)}, \ldots$ are in α, we denote the smallest space containing the sequences

$$\left(x^{(1)}\right)^s, \left(x^{(2)}\right)^s, \ldots, \sum_{i=1}^{n} b_i \left(x^{(i)}\right)^s, \left(\sum_{i=1}^{n} b_i x^{(i)}\right)^s$$

etc. and similar combinations as α^s where *n* is a positive integer. By $\alpha^s_{1/s}$ we denote the smallest sequence space which contains sequences such as $x^{(1)}, x^{(2)}, \ldots, \left[\sum_{i=1}^{n} b_i \left(x^{(i)}\right)^s\right]^{\frac{1}{s}}, \sum_{i=1}^{n} b_i x^{(i)}$ and such other sequences which go in making a space.

It is clear that the sequence $\left[\sum_{i=1}^{n} b_i \left(x^{(i)}\right)^s\right]^{\frac{1}{s}}$ does not necessarily belong to α.

Hence $\alpha^s_{1/s} \ge \alpha$

There are some spaces where α^s coincide with $\alpha, (s > 0)$ e.g σ, ϕ, S_0, $\sigma_\infty, C, O_1, O_2, \overline{O_1}, \overline{O_2}$. It is also clear that in all such spaces except *C*, $\alpha^s_{1/5} = \alpha$.

Lemma: If $x^{(n)}$ in α is $\alpha\beta$-bd, and $\sum |b_k|^r$ converges, then *y* defined by

$$\left(y_k^{(n)}\right) = \left[\sum_{i=1}^{n} b_i^r \left(x_k^{(i)}\right)^r\right]^{\frac{1}{r}}$$ is $\alpha^r_{1/r}\beta$-cgt

where $\phi \le \beta \le \left(\alpha^r_{1/r}\right)^* \le \alpha^*$

For *u* in β, $\left|\sum_{k=1}^{\infty} u_k^r \left(x_k^{(n)}\right)^r\right| \le R(u)$ for every *n*.

Given $\in > 0$, we can choose *p* so that $\sum_{i=p+1}^{\infty} |b_i|^r \le \frac{\in}{R}$; Then if *m* > *n* > *p*, we have

$$\left|\sum_{k=1}^{\infty} u_k^r \left[\left(y_k^{(m)}\right)^r - \left(y_k^{(n)}\right)^r\right]\right| = \left|\sum_{k=1}^{\infty} u_k^r \left[\sum_{i=1}^{m} b_i^r \left(x_k^{(i)}\right)^r - \sum_{i=1}^{n} b_i^r \left(x_k^{(i)}\right)^r\right]\right|$$

$$= \left|\sum_{k=1}^{\infty} u_k^r \left[\sum_{i=n+1}^{m} b_i^r \left(x_k^{(i)}\right)^r\right]\right| = \left|\sum_{i=n+1}^{m} b_i^r \sum_{k=1}^{\infty} u_k^r \left(x_k^{(i)}\right)^r\right|$$ changing the order of double series

$$\leq \sum_{i=n+1}^{m} \left| b_i^r \left[\sum_{k=1}^{\infty} u_k^r \left(x_k^{(i)}\right)^r\right]\right| = \sum_{i=n+1}^{m} |b_i|^r \left|\sum_{k=1}^{\infty} \left(u_k x_k^{(i)}\right)^r\right|$$

$$\leq R \sum_{i=n+1}^{m} |b_i|^r \leq R.\frac{\in}{R} = \in$$

Hence as given in [5] y^n is $\alpha^r_{1/r}\beta$-cgt.

So The Lemma has been proved.

Now $\left|x_k^{(n)}\right|^r \leq s_k$ for every n;

Hence $\sum_{n=1}^{\infty} \left|b_n x_k^{(n)}\right|^r$ converges for every k.

If c-lim $y^{(n)} = y$,

Then $y_k = \left[\sum_{n=1}^{\infty} \left(b_n x_k^{(n)}\right)^r\right]$

and we then write

$$y = \left[\sum_{n=1}^{\infty} \left(b_n x^{n)}\right)^r\right]$$

For example, suppose that $x_k^{(n)} = (-1)^{k+1}$ for $1 \leq k \leq n$ and $x_k^{(n)} = 0$ for $k > n$.

Then $x^{(n)}$ is in ϕ. It is also ϕC-bd.

Take $b_k = \dfrac{1}{[k(k+1)]^{1/r}}$, clearly $\sum |b_k|^r$ is convergent.

By the lemma, $\left[\sum_{s=1}^{n} \left\{\dfrac{1}{(s(s+1))^{1/r}}.x^{(s)}\right\}^r\right]^{1/r} = y^{(n)}$

is $\phi^r_{1/r} C$-cgt i.e ϕC-cgt.

Also $y_k^{(n)} = \left[\sum_{s=1}^{n} \left\{ \frac{1}{(s(s+1))^{1/r}} . x_k^{(s)} \right\}^r \right]^{1/r}$

$$= \left[\sum_{s=1}^{k-1} \left\{ \frac{1}{(s(s+1))^{1/r}} . x_k^{(s)} \right\}^r + \sum_{s=k}^{n} \left\{ \frac{1}{(s(s+1))^{1/r}} . x_k^{(s)} \right\}^r \right]^{1/r}$$

According to the supposition,

$x_k^{(s)} = (-1)^{k+1}$ for $1 \le k \le s$ i.e $s \ge k \ge 1$ $x_k^{(s)} = 0$ for $k > s$ i.e $s < k$.

When s varies from 1 to k-1, $s < k$ and $x_k^{(s)} = 0$

When s varies from k to n, $s \ge k$ and $x_k^{(s)} = (-1)^{k+1}$.

$$\therefore\ y_k^{(n)} = (-1)^{k+1} \left[\sum_{s=k}^{n} \frac{1}{s(s+1)} \right]^{1/r}$$

$$= (-1)^{k+1} \left[\sum_{s=k}^{n} \left(\frac{1}{s} - \frac{1}{s+1} \right) \right]^{1/r}$$

$$= (-1)^{k+1} \left(\frac{1}{k} - \frac{1}{n+1} \right)^{1/r}$$

$$\therefore\ \left[y_k^{(n)} \right]^r = (-1)^{r(k+1)} \left(\frac{1}{k} - \frac{1}{n+1} \right)$$

hence $y_k = \lim_{n \to \infty} \left[y_k^{(n)} \right]^r = \frac{(-1)^{r(k+1)}}{k}$

In this case, $\sum_{k=1}^{\infty} |y_k|^r = \sum_{k=1}^{\infty} \frac{1}{k^r}$ which converges.

Hence c-limit is in $C^* = \sigma_r$, and so satisfies the condition (i) necessary for ϕC -limit, It also satisfies condition (ii) for ϕC -*Lt*, as can be easily verified.

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