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BOUNDS ON PARITY CHECK OF LINEAR CODES DETECTING REPEATED SOLID BURSTS

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Abstract

The paper presents lower and upper bounds for linear codes detecting two types of repeated solid burst error. Simultaneous detection and correction of such errors are also studied. An example of such a code is provided. Further, we also obtain some improvement/correction of previous results dealing with repeated solid burst error.

Keyword: Linear codes, parity check digits, syndromes, solid bursts, cosets.

AMS Subject Classification (2010): 94B05, 94B65.

1. INTRODUCTION

Investigations in coding theory are done in various directions for optimum efficiency in communications. For the improvement of efficiency, one direction is to establish the nature of the communication channel. Once it is established the nature of the communication channel i.e., once the type of error occurred during communication is known, remedy can be done accordingly instead of dealing with error that does not occur and thus saving the capacity of the system.

Solid burst errors are found to be in many memory channels (viz. semiconductor memory data [6], supercomputer storage system [1]). A good amount of work on codes dealing with solid burst can be found in [11, 12, 13 etc].

Definition 1: *A solid burst of length b is a vector whose all the b-consecutive components are non zero and rest are zero.*

In busy communication channels, it is found by Dass, Verma and Berardi [4] that errors repeat themselves. They have initiated the idea of repeated errors and introduced 2-repeated burst. In very busy communication channels, errors repeat themselves more frequently. In view of this, Dass and Verma [5] defined m-repeated burst and obtained results regarding the number of parity-check digits required for codes detecting such errors.

Extending this idea, '2-repeated solid burst of length b' and 'm-repeated solid burst of length b' has been studied by Das [2, 3]. Further, Rohtagi and Sharma [9] studied the codes that are capable of correcting '2-repeated solid burst' and 'm-repeated solid burst'. Cyclic codes for the detection of '2-repeated solid burst' and 'm-repeated solid burst' are also studied.

However, there is a difference between the definition of repeated solid burst of length b given by Das, and given by Rohtagi and Sharma. In the definition of 2-repeated (or m-repeated) solid burst of length b, given by Das, nonzero components are confined consecutively to 2 distinct (or m distinct) sets of b consecutive components. Here the nonzero components are in *any* consecutive positions within each set of b consecutive components. But in the definition given by Rohtagi and Sharma, the nonzero components are in all b positions in each set of b consecutive components.

To distinguish the two definitions, we call the definition given by Das as *repeated solid burst of length b (Type I)* and given by Rohtagi and Sharma as *repeated solid burst of length b* and redefined as follows.

Definition 2: *A 2-repeated solid burst of length b is a vector whose only nonzero components are confined consecutively to two distinct sets of b consecutive components, all the components of each set being nonzero.*

Example: (0011000110) is a 2-repeated solid burst of length 2 over GF(2) .

Definition 3: *An m-repeated solid burst of length b is a vector of length n whose only nonzero components are confined consecutively to m distinct sets of b consecutive components, all the components of each set being nonzero.*

Example: (01101100110) is a 3-repeated solid burst of length 2 over GF (2).

Definition 4: *A 2-repeated solid burst of length b (Type I) is a vector of length n whose only nonzero components are confined to two distinct sets of b consecutive components, nonzero components in each set are consecutive.*

For example, (00112002100000000) is a 2-repeated solid burst of length 3 (*Type I*) over GF (3).

In 2*-repeated solid burst of length b (Type I)*, it may be noted that, in the last $2b$-1 components only a single solid burst of length b or less can exist. Further, it is allowed to have one set not containing any nonzero component.

Definition 5: *An m-repeated solid burst of length b (Type I) is a vector of length n whose only nonzero components are confined to m distinct sets of b consecutive components, nonzero components in each set are consecutive.*

For example, (00112002410030003100) is a 4-repeated solid burst of length 3 (*Type I*) over GF (5).

In *m-repeated solid burst of length b (Type I),* the number of starting positions of this error is among the first n-mb+1 components. It may be noted also that, in the last mb-1 components, (m-1)-repeated solid burst of length b (Type I) can exist. Further, it is also allowed to have some sets not containing nonzero components i.e., consecutive errors are in m or less sets of b consecutive components. The following situation motivates the author to consider *m-repeated solid burst of length b (Type I)* and to deal with them.

Consider a long scratch in a compact disc. The scratch is continuous upto a certain length, then no scratch, then again a continuous scratch and so on. Such type of error falls in this category of *repeated solid burst errors of length b (Type I).*

Note: *repeated solid burst of length b (Type I) is a generalisation of repeated solid burst of length b.*

The paper is organised as follows. The introduction of the paper is given in Section 1. In Section 2, lower bounds on the number of parity check digits for linear codes detecting both types of repeated solid bursts are presented. In Section 3, upper bound on the number of parity check digits required for linear codes detecting 2-repeated solid burst of length b or less is given and this is followed by an example of such a code. In Section 4, lower bounds on parity check number of linear codes simultaneously detecting and correcting both types of errors are obtained.

In what follows, by an (*n, k*) linear code, we mean a subspace of the space of all *n*-tuples over *GF (q)* and $\lfloor y \rfloor$ means the greatest integer less than or equal to *y*.

2. LOWER BOUNDS ON CODES DETECTING REPEATED SOLID BURST

In this section, we obtain lower bounds on the number of parity check digits required for a code that detects both types of repeated solid bursts and also gives some improvement/correction of previous results.

Theorem 1: *The number of parity check digits for an (n, k) linear code over GF(q) that detects any 2-repeated solid burst of length b or less in the space of all n-tuples is bounded from below by*

$$q^{n-k} \geq \begin{cases} 2+b \quad for \quad q=2 \\ 1+(q-1)^2 \sum_{i=0}^{b-1} \left(\left\lfloor \frac{q-1}{2} \right\rfloor \right)^{2i} \quad for \quad q \neq 2. \end{cases}$$

Proof. Since the (*n*, *k*) linear code over *GF*(*q*) that detects any 2-repeated solid bursts of length *b* or less, any 2-repeated solid burst of length *b* or less can not be a codeword.

For binary case, let us consider a set *X* of all those vectors of which all the first i $(1 \leq i \leq b+1)$ positions of any two fixed distinct sets of $b+1$ consecutive components are the nonzero element of the field and zero elsewhere. Each set has the same number of consecutive nonzero components.

Then the elements of X should be in different cosets, otherwise for any x_1; x_2 in X, the syndrome of x_1- x_2 will be a zero syndrome, which is not possible as x_1- x_2 is a 2-repeated solid burst of length *b* or less. The number of such vectors over *GF* (2), excluding the zero vector is clearly $b+1$.

We know that the total number of cosets in an (*n*, *k*) linear code equals q^{n-k}. So we must have

$$q^{n-k} \geq 1+(b+1).$$

For non-binary case, let us consider *X* is a set of all those vectors of which the first position of any two fixed distinct sets of *b* consecutive components is any nonzero component and its following i $(0 \leq i \leq b-1)$ consecutive positions in each set are any nonzero element belonging to $\left\{1,\ 2,\ .....,\ \left\lfloor \frac{q-1}{2} \right\rfloor\right\}$ and zero elsewhere. Each set has the same number of consecutive nonzero components.

Here also the elements of X should be in different cosets, otherwise for any x_1; x_2 in X, x_1-x_2 is a code vector and its syndrome will be a zero syndrome. This is a contradiction as x_1- x_2 is a 2-repeated solid burst of length *b* or less.

The number of elements of X, excluding the vector of all zeros is clearly

$$(q-1)^2 \sum_{i=0}^{b-1} \left(\left\lfloor \frac{q-1}{2} \right\rfloor \right)^{2i}.$$

So we must have

$$q^{n-k} \geq 1+(q-1)^2 \sum_{i=0}^{b-1} \left(\left\lfloor \frac{q-1}{2} \right\rfloor \right)^{2i}.$$

In the similar way, we can prove the following results for *m*-repeated solid bursts of length *b* or less.

Theorem 2: *The number of parity check digits for an (n, k) linear code over GF(q) that detects any m-repeated solid burst of length b or less in the space of all n-tuples is bounded from below by*

$$q^{n-k} \geq \begin{cases} 2+b \quad for \quad q=2 \\ 1+(q-1)^m \sum_{i=0}^{b-1} \left(\left\lfloor \frac{q-1}{2} \right\rfloor \right)^{mi} \quad for \quad q \neq 2. \end{cases}$$

Now, we present here the improvement/correction of the result (Theorem 1) obtained in [2].

Theorem 3: *The number of parity check digits for an (n, k) linear code over GF(q) that detects any 2-repeated solid burst of length b (Type I) in the space of all n-tuples is bounded from below by*

$$q^{n-k} \geq \begin{cases} 1+(b+1)^2 & for \quad q=2 \\ 1+\left[(q-1)\sum_{i=0}^{b-1}\left(\left\lfloor \frac{q-1}{2} \right\rfloor\right)^i\right]^2 & for \quad q \neq 2. \end{cases}$$

Proof. **For binary case,** let X be the set X of all those vectors whose the only nonzero element of the field are confined consecutively to the first b+1 or less consecutive positions of any two fixed distinct sets of b+1 consecutive components It is not necessary to have the same number of consecutive nonzero components in both sets.

In this case also, the elements of X should be in different cosets, otherwise this lead to contradiction to the fact that syndrome of a detecting error vector is a nonzero syndrome.

The number of such vectors over GF (2), excluding the vector of all zeros is clearly $(b+1)^2$. Since the maximum number of cosets in an (n, k) linear code is q^{n-k} , so we must have

$$q^{n-k} \geq 1+(b+1)^2 .$$

For non-binary case, let X be a set of all those vectors of which the first position of any two fixed distinct sets of b consecutive components is any nonzero component and its following $b-1$ or less consecutive positions in each set are any nonzero element belonging to $\left\{1, \; 2, \; \ldots.., \left\lfloor \frac{q-1}{2} \right\rfloor\right\}$ and zero elsewhere. Here also, it is not necessary that the two sets have the same number of consecutive nonzero components.

For the same reason as above, the elements of X should be in different cosets. The number of such vectors over GF (q), excluding the vector of all zeros is clearly

$$\left[(q-1)\sum_{i=0}^{b-1}\left(\left\lfloor \frac{q-1}{2} \right\rfloor\right)^i\right]^2 .$$

Therefore, we must have

$$q^{n-k} \geq 1+\left[(q-1)\sum_{i=0}^{b-1}\left(\left\lfloor \frac{q-1}{2} \right\rfloor\right)^i\right]^2 .$$

In the similar way, the result (Theorem 3.1) of [3] can be improved/corrected as follows.

Theorem 4: *The number of parity check digits for an (n, k) linear code over GF(q) that detects any m-repeated solid burst of length b (Type I) in the space of all n -tuples is bounded from below by*

$$q^{n-k} \geq \begin{cases} 1+(b+1)^m & for \quad q=2 \\ 1+\left[(q-1)\sum_{i=0}^{b-1}\left(\left\lfloor \frac{q-1}{2} \right\rfloor\right)^i\right]^m & for \quad q \neq 2. \end{cases}$$

3. UPPER BOUNDS ON CODES DETECTING REPEATED SOLID BURST

In this section, we obtain an upper bound on parity check for existence of a linear detecting 2-repeated solid burst of length b or less and this is followed by an example.

Theorem 5: *There exists an (n, k) linear code over GF(q) that has no 2-repeated solid bursts of length b or less as a code word provided that*

$$q^{n-k} > 1 + \sum_{\ell=0}^{b-1} (q-1)^{2\ell+1}(n-1-2\ell).$$

Proof. The theorem is proved by the well known technique used in Varshomov-Gilbert Sacks bound by constructing a parity check matrix for such a code (refer Sacks [10], also theorem 4.7 Peterson and Weldon [7]). The construction of the appropriate $(n\text{-}k)\times n$ parity-check matrix H is given as follows.

Choose any nonzero $(n\text{-}k)$–tuple as the first column h_1 of H. Subsequent columns are added to H such that after selecting first $(j-1)$ columns $h_1, h_2, \ldots, h_{j-1}$, the j^{th} column h_j is added provided that h_j is not a linear sum of immediately preceding ℓ columns of H, (where $\ell \le b-1$) together with any previous $\ell+1$ consecutive columns amongst the first $(j-1-\ell)$ columns. In other words,

$$h_j \neq (u_1 h_{j-1} + u_2 h_{j-2} + \ldots\ldots + u_\ell h_{j-\ell}) + (v_i h_i + v_{i+1} h_{i+1} + \ldots\ldots + v_{i+\ell} h_{i+\ell}) \qquad (1)$$

where u_i, v_i are any nonzero coefficients and h_i's in the second bracket are any $\ell+1$ consecutive columns among the first $j\text{-}1\text{-}\ell$ columns, $\ell = 0, 1, 2, \ldots, (b\text{-}1)$.

Note that for $\ell = 0$, there is no term in the first bracket of R.H.S. of (1).

Thus coefficients u_i form a solid burst of length ℓ and the coefficients v_i form a solid burst of length $\ell+1$ in a $(j\text{-}1\text{-}\ell)$-tuple. This condition ensures that no 2-repeated solid burst of length b or less can be a code word.

The different number of choices of these coefficients u_i *and* v_i of (1) is given by

$$\sum_{\ell=0}^{b-1} (q-1)^{2\ell+1}(j-1-2\ell).$$

At worst, all these linear combinations might give distinct sums.
Therefore a column h_j can be added to H provided that

$$q^{n-k} > 1 + \sum_{\ell=0}^{b-1} (q-1)^{2\ell+1}(j-1-2\ell).$$

For a code of length n, replacing j by n gives the result.

Example 1: Consider a (9, 4) binary code with parity check matrix

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

This matrix has been constructed by the synthesis procedure, outlined in the proof of Theorem 5, by taking $q = 2$, $n = 9$ and $b = 3$. It can be seen from Error pattern-Syndrome Table 1 that the syndromes of the different 2-repeated solid bursts of length 3 or less are nonzero, showing thereby that the code that is the null space of this matrix can detect all 2-repeated solid bursts of length 3 or less.

Error pattern-Syndrome Table 1

Error pattern	Syndrome	Error pattern	Syndrome
2-repeated solid burst of length 1		000000101	10101
110000000	11000	000000011	11010
101000000	10100	2-repeated solid burst of length 2	
100100000	10010	111100000	11110
100010000	10001	110110000	11011
100001000	00100	110011000	01101
100000100	11010	110001100	00110

100000010	10101	110000110	10111
100000001	01111	110000011	00010
011000000	01100	011110000	01111
010100000	01010	011011000	11001
010010000	01001	011001100	10010
010001000	11100	011000110	00011
010000100	00010	011000011	10110
010000010	01101	001111000	10011
010000001	10111	001101100	11000
001100000	00110	001100110	01001
001010000	00101	001100011	11100
001001000	10000	000111100	11101
001000100	01110	000110110	01100
001000010	00001	000110011	11001
001000001	11011	000011110	11010
000110000	00011	000011011	01111
000101000	10110	000001111	00100
000100100	01000	2-repeated solid burst of length 3	
000100010	00111	111111000	01011
000100001	11101	111011100	00011
000011000	10101	111001110	00111
000010100	01011	111000111	01100
000010010	00100	011111100	10001
000010001	11110	011101110	10101
000001100	11110	011100111	11110
000001010	10001	001111110	11100
000001001	01011	001110111	10111
000000110	01111	000111111	00111

4. SIMULTANOUS DETECTION AND CORRECTION OF REPEATED SOLID BURST

In this section, we obtain Reiger's type of bound (refer [8]; also Theorem 4.15, Peterson and Weldon[7]) for a linear code for simultaneous detecting and correcting both types of repeated solid burst error. First, we give Reiger's type of bound for 2-repeated solid bursts of length b or less.

Theorem 6: *The number of parity-check digits for an (n, k) linear code over GF(q) that corrects all 2-repeated solid bursts of length b or less must is bounded from below by*

$$q^{n-k} \geq \begin{cases} 2+2b & \text{for} \quad q=2 \\ 1+(q-1)^2 \displaystyle\sum_{i=0}^{2b-1}\left(\left\lfloor \frac{q-1}{2} \right\rfloor\right)^{2i} & \text{for} \quad q \neq 2. \end{cases}$$

Further, if the code corrects all 2-repeated solid bursts of length b or less and simultaneously detects 2-repeated solid bursts of length d or less (d > b), then the number of parity-check digits for the code must is bounded from below by

$$q^{n-k} \geq \begin{cases} 2+(b+d) & \text{for} \quad q=2 \\ 1+(q-1)^2 \displaystyle\sum_{i=0}^{b+d-1}\left(\left\lfloor \frac{q-1}{2} \right\rfloor\right)^{2i} & \text{for} \quad q \neq 2. \end{cases}$$

Proof. To prove the first part, consider a 2-repeated solid burst of length $2b$ or less. Such a vector is expressible as a sum or difference of two vectors, each of which is a 2-repeated solid burst of length b or less. These component vectors must belong to different cosets of the standard array because both such errors are correctable errors. Accordingly, such a vector viz. 2-repeated solid burst of length $2b$ or less can not be a code vector. Applying Theorem 1, the minimum number of parity check digits required for such a code can be obtained by

$$q^{n-k} \geq \begin{cases} 2+2b \quad for \quad q=2 \\ 1+(q-1)^2 \sum_{i=0}^{2b-1} \left(\left\lfloor \frac{q-1}{2} \right\rfloor \right)^{2i} \quad for \quad q \neq 2. \end{cases}$$

Further, consider a 2-repeated solid burst of length ($b + d$) or less. Such a vector is expressible as a sum or difference of two vectors, one of which is a 2-repeated solid burst of length b or less and the other is a 2-repeated solid burst of length d or less. Both such component vectors, one being a detectable error and the other being a correctable error, can not belong to the same coset of the standard array. Therefore such a vector can not be a code vector, i.e., any 2-repeated solid burst of length $b+d$ or less can not be a code vector. Then again applying Theorem 1, the minimum number of parity check digits required for such a code is obtained by

$$q^{n-k} \geq \begin{cases} 2+(b+d) \quad for \quad q=2 \\ 1+(q-1)^2 \sum_{i=0}^{b+d-1} \left(\left\lfloor \frac{q-1}{2} \right\rfloor \right)^{2i} \quad for \quad q \neq 2. \end{cases}$$

Again applying Theorem 2, we can extend the above result for *m-repeated solid bursts of length b or less* as follows.

Theorem 7: *The number of parity-check digits for an (n, k) linear code over GF(q) that corrects all m-repeated solid bursts of length b or less must is bounded from below by*

$$q^{n-k} \geq \begin{cases} 2+2b \quad for \quad q=2 \\ 1+(q-1)^m \sum_{i=0}^{2b-1} \left(\left\lfloor \frac{q-1}{2} \right\rfloor \right)^{mi} \quad for \quad q \neq 2. \end{cases}$$

Further, if the code corrects all m-repeated solid bursts of length b or less and simultaneously detects m-repeated solid bursts of length d or less (d > b), then the number of parity-check digits for the code must is bounded from below by

$$q^{n-k} \geq \begin{cases} 2+(b+d) \quad for \quad q=2 \\ 1+(q-1)^m \sum_{i=0}^{b+d-1} \left(\left\lfloor \frac{q-1}{2} \right\rfloor \right)^{mi} \quad for \quad q \neq 2. \end{cases}$$

Based on the argument and logic in the above two theorems, we can improve/correct Theorem 3 of [2] and Theorem 4.1 of [3]. They are given below and can be proved in the similar way.

Theorem 8: *The number of parity-check digits for an (n, k) linear code over GF(q) that corrects all 2-repeated solid bursts of length b (Type I) must is bounded from below by*

$$q^{n-k} \geq \begin{cases} 1+(2b+1)^2 \quad for \quad q=2 \\ 1+\left[(q-1) \sum_{i=0}^{2b-1} \left(\left\lfloor \frac{q-1}{2} \right\rfloor \right)^{i} \right]^2 \quad for \quad q \neq 2. \end{cases}$$

Further, if the code corrects all 2-repeated solid bursts of length b (Type I) and simultaneously detects 2-repeated solid bursts of length d (Type I) (d > b), then the number of parity-check digits for the code must is bounded from below by

$$q^{n-k} \geq \begin{cases} 1+(b+d+1)^2 \quad for \quad q=2 \\ 1+\left[(q-1) \sum_{i=0}^{b+d-1} \left(\left\lfloor \frac{q-1}{2} \right\rfloor \right)^{i} \right]^2 \quad for \quad q \neq 2. \end{cases}$$

Theorem 9: *The number of parity-check digits for an (n, k) linear code over GF(q) that corrects all m-repeated solid bursts of length b (Type I) must is bounded from below by*

$$q^{n-k} \geq \begin{cases} 1+(2b+1)^m & for \quad q=2 \\ 1+\left[(q-1)\sum_{i=0}^{2b-1}\left(\left\lfloor \frac{q-1}{2} \right\rfloor\right)^i\right]^m & for \quad q \neq 2. \end{cases}$$

Further, if the code corrects all m-repeated solid bursts of length b (Type I) and simultaneously detects m-repeated solid bursts of length d (Type I) (d > b), then the number of parity-check digits for the code must is bounded from below by

$$q^{n-k} \geq \begin{cases} 1+(b+d+1)^m & for \quad q=2 \\ 1+\left[(q-1)\sum_{i=0}^{b+d-1}\left(\left\lfloor \frac{q-1}{2} \right\rfloor\right)^i\right]^m & for \quad q \neq 2. \end{cases}$$

CONCLUSION

The paper presents lower bound for linear code detecting both types of repeated solid burst error, also presents upper bound for detection of 2-repeated solid burst of length *b* or less. Existence of codes detecting/correcting both types of *m*-repeated solid burst will remain a further study which the author will take up separately.

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