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TRIANGULAR EQUILIBRIUM POINTS IN GENERALISED PHOTOGRAVITATIONAL RESTRICTED THREE BODY PROBLEM

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Abstract

We have found the locations of triangular equilibrium points in generalised photogravitational restricted three body problems. We suppose that both primaries are radiating and the bigger primary is an oblate spheroid. We obtain the locations of triangular points. These are affected by radiation and oblateness of the primaries. We find that the points L_4, L_5 form triangles with the primaries.

Keywords: Triangular points; Generalised; Photogravitational; RTBP

1. INTRODUCTION

In this paper, we propose to study location of triangular points in the generalised photogravitational restricted three body problem.

Radzievskii (1950) formulated the photogravitational restricted three body problem. This arises from the classical problem when one of the interacting masses is an intense emitter of radiation. He discussed it for three specific bodies: the sun, a planet and a dust particle. Chernikov (1970) extended his work by including aberrational deceleration (the Poynting-Robertson effect). He found that despite the absence of a Jacobi integral, the equations of motion admit of particular solutions corresponding to six libration points. He demonstrated the instability of the solutions by Lyapunov's first method.

Schueman (1980) generalized the restricted three body problem by including the force of radiation pressure and the Poynting-Robertson effect. The Poynting-Robertson effect renders the L_4 and L_5 points unstable on a time scale (T) long compared to the period of rotation of the two massive bodies. For the solar system, T is given by $T = [(1-\beta)^{2/3} / \beta] \times 544\ a^2$ year, where β is the ratio of radiation to gravitational forces, and a is the separation between the Sun and the planet in AU. He also discussed implications for space colonization and a mechanism for producing azimuthal asymmetries in the interplanetary dust complex.

Sharma (1982) studied the linear stability of triangular liberation point of the restricted three body problem when the more massive primary is a source of radiation and an oblate spheroid as well. He found that the eccentricity of the conditional retrograde elliptic periodic orbits around the triangular points at the critical mass μ_c increases with the increase in the oblateness coefficient and the radiation force and becomes unity when μ_c is zero.

Simmons et al. (1985) gave a complete solution of the restricted three-body problem. They discussed the existence and linear stabililty of the equilibrium points for all values of radiation pressures of both luminous bodies and all values of mass ratios.

Ragos and Zagouras (1988) found two families periodic solutions about the ‘out of plane’ equilibrium points in the photogravitational restricted three-body problem.

Shaboury (1990) gave a possibility of nine liberation points for small values of oblateness in the photogravitational restricted three-body problem when the infinitesimal mass is of an axisymmetric body and one of the finite masses be a spherical luminous body while the other be an axisymmetric non-luminous body.

Todoran (1993) clamied that the “out of plane” equilibrium points (out of the orbital plane of the primaries) in the restricted three-body problem as concerned radiation pressure, do not actually exist. This question was answered by Ragos and Zagouras (1993). Liou and Zook (1995) investigated asteroidal dust ring of micron-sized particles trapped in the 1: 1 mean motion resonance with Jupiter. They with Jackson (1995) examined the effects of radiation pressure, Poynting-Robertson (PR) drag, and solar wind drag on dust grains trapped in mean motion resonances with the Sun and Jupiter in the restricted three body problem. Khasan (1996) studied the existence of libration points and their stability in the photogravitational elliptic restricted three body problem.

The classical problem of three bodies was generalized by considering the various aspects such as the shape of the bodies, influence of the perturbing forces other than the forces of mutual gravitation etc., to make the problem more realistic. In the solar system, some of the planets, like Satum and Jupiter are sufficiently oblate. It has been seen that oblateness of the body plays an important role in the restricted three body problem.

Hence, the idea of the radiation pressure forces together with oblateness of the body raises a curiosity in our mind to study the location of triangular points in the generalised photogravitational restricted three-body problem. The problem is photogravitational in the sense that both the primaries are sources of radiation. The problem is generalised in the sense that one of the primaries is taken as oblate spheroid.

In section 2, we have established the equations of motion of the problem. We find that the mean motion of the primaries is affected by their oblateness. The potential and the kinetic energy for the third body are determined. By applying Lagrange’s equations of motion, we have deduced the equations of motion. These equations of motion are influenced by radiation and oblateness of the primaries.

In section 3, the locations of the triangular equilibrium points are obtained. It is proved that the locations of the triangular points are affected by radiation and oblateness of the primaries. Further we find that the points L_4, L_5 form triangles with the primaries.

2. EQUATIONS OF MOTION

Let m_1 and m_2 be masses of the bigger and smaller primaries and m are the mass of third infinitesimal body. We assume that both primaries are radiating and bigger primary as an oblate spheroid. R be the distance between the primaries.

We use A_1 for oblateness coefficients of the bigger primary.

$0 < A_1 << 1$ (Mc Cuskey, 1963) and

$$A_1 = \frac{AE_1^2 - AP_1^2}{5R^2}$$

where AE_1 is the equatorial radius and AP_1 being the radius of bigger primary.

We further denote the radiation factor by q_i (i = 1, 2), which are given by the equation

$$F_{p_i} = F_g(1-q_i)$$

F_g being the gravitational attraction force, $q_i \approx 1$ and so
$0 < 1 - q_i << 1$

We have ignored the Poynting-Robertson drag effect. We have also neglected the perturbations in the potential between m_1 and m_2 due to the radiation pressure, because m_1 is supposed to be sufficiently large.
The potential V between m_1 and m_2 is given as

$$V = -G\,m_1 m_2\left(\frac{1}{R} + \frac{A_1}{2R^3}\right) \tag{1}$$

where G is the gravitational constant.

Let (X, Y) be the coordinates of m_2 with respect to m_1. Its equations of motion are

$$\ddot{X} = -\frac{m_1+m_2}{m_1 m_2}\frac{\partial V}{\partial X},$$
$$\ddot{Y} = -\frac{m_1+m_2}{m_1 m_2}\frac{\partial V}{\partial Y}, \tag{2}$$

where $R = \sqrt{X^2+Y^2}$

Now, $\frac{\partial V}{\partial X} = -Gm_1m_2\left[-\frac{1}{R^2} - \frac{3}{2}\frac{A_1}{R^4}\right]\frac{X}{R}$

and $\frac{\partial V}{\partial Y} = -Gm_1m_2\left[-\frac{1}{R^2} - \frac{3}{2}\frac{A_1}{R^4}\right]\frac{Y}{R}$

The above equations of motion become

$$\ddot{X} = -G(m_1+m_2)\left[\frac{1}{R^2} + \frac{3}{2}\frac{A_1}{R^4}\right]\frac{X}{R}$$
$$\ddot{Y} = -G(m_1+m_2)\left[\frac{1}{R^2} + \frac{3}{2}\frac{A_1}{R^4}\right]\frac{Y}{R} \tag{3}$$

These equations of motion have the particular solution
R = constant, X = R cos θ, Y = R sin θ, θ = nt, where n is the mean motion of the primaries.

$$\dot{X} = -nR\sin\theta, \ddot{X} = -Rn^2\cos\theta$$

Putting these values in the first equations of (3), we have

$$-Rn^2\cos\theta = -G(m_1+m_2)\left[\frac{1}{R^2} + \frac{3}{2}\frac{A_1}{R^4}\right]\frac{R\cos\theta}{R}$$
$$\Rightarrow n^2 = G\left(\frac{m_1+m_2}{R}\right)\left[\frac{1}{R^2} + \frac{3}{2}\frac{A_1}{R^4}\right] \tag{4}$$

Let (x, y) be the coordinates of the third body in a rotating coordinate system with the origin at 0 and the line joining the primaries being the x-axis and the line perpendicular to it being the y-axis.

The kinetic energy T of the third body is given as

$$T = \frac{1}{2}m\left[(\dot{x} - ny)^2 + (\dot{y} + nx)^2\right]$$

$$= \frac{1}{2}m\left[\dot{x}^2 - 2n\dot{x}y + n^2y^2 + \dot{y}^2 + 2nx\dot{y} + n^2x^2\right]$$

$$= \frac{1}{2}mn^2(x^2 + y^2) + mn(x\dot{y} - \dot{x}y) + \frac{1}{2}m(\dot{x}^2 + \dot{y}^2)$$

$= T_0 + T_1 + T_2$,
with

$$T_0 = \frac{1}{2}mn^2(x^2 + y^2),$$

$$T_1 = mn(x\dot{y} - \dot{x}y),$$

$$T_2 = \frac{1}{2}m(\dot{x}^2 - \dot{y}^2) \tag{5}$$

The potential $\bar{V}$ between m and m_1 and m and m_2 is given as

$$\bar{V} = -Gm\left[m_1\left(\frac{q_1}{r_1} + \frac{A_1q_1}{2r_1^3}\right) + m_2\left(\frac{q_2}{r^2}\right)\right] \tag{6}$$

where

$$r_1^2 = (x - x_1)^2 + y^2,$$

$$r_2^2 = (x - x_2)^2 + y^2 \tag{7}$$

Here $(x_1, 0)$ and $(x_2, 0)$ are coordinates of m_1 and m_2 respectively. Let the modified potential energy be

$$\bar{U} = \bar{V} - T_0$$

$$= -Gm\left[m_1\left(\frac{q_1}{r_1} + \frac{A_1q_1}{2r_1^3}\right) + m_2\left(\frac{q_2}{r_2}\right)\right] - \frac{1}{2}mn^2(x^2 + y^2) \tag{8}$$

The Lagrangian can be put in the form

$$L = T_2 + T_1 - \bar{U}$$

$$= \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) + mn(x\dot{y} - \dot{x}y) - \bar{U}$$

From this, we have

$$\frac{\partial L}{\partial \dot{x}} = m(\dot{x} - ny),$$

$$\frac{\partial L}{\partial \dot{y}} = m(\dot{y} + nx),$$

$$\frac{\partial L}{\partial x} = mn\dot{y} - \frac{\partial \bar{U}}{\partial x},$$

$$\frac{\partial L}{\partial y} = -mn\dot{x} - \frac{\partial \bar{U}}{\partial y}$$

Hence, the equations of motion of the third body are

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0,$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) - \frac{\partial L}{\partial y} = 0$$

i.e.,

$$m(\ddot{x} - n\dot{y}) - mn\dot{y} + \frac{\partial \bar{U}}{\partial x} = 0$$

$$m(\ddot{y} - n\dot{x}) + mn\dot{x} + \frac{\partial \bar{U}}{\partial y} = 0$$

i.e.,

$$\ddot{x} - 2n\dot{y} = -\frac{1}{m}\frac{\partial \bar{U}}{\partial x},$$

$$\ddot{y} + 2n\dot{x} = -\frac{1}{m}\frac{\partial \bar{U}}{\partial y}, \qquad (9)$$

But from (8), we have

$$\frac{\partial \bar{U}}{\partial x} = -Gm\left[m_1\left(\frac{q_1}{r_1^2} - \frac{3}{2}\frac{A_1 q_1}{r_1^4}\right)\left(\frac{x - x_1}{r_1}\right) + m_2\left(-\frac{q_2}{r_2^2}\right)\left(\frac{x - x_2}{r_2}\right)\right] - mn^2 x$$

and $$\frac{\partial \bar{U}}{\partial y} = -Gm\left[m_1\left(-\frac{q_1}{r_2^2} - \frac{3}{2}\frac{A_1 q_1}{r_1^4}\right)\frac{y}{r_1} + m_2\left(-\frac{q_2}{r_2^2}\right)\frac{y}{r_2}\right] - mn^2 y$$

using these in (9) we have

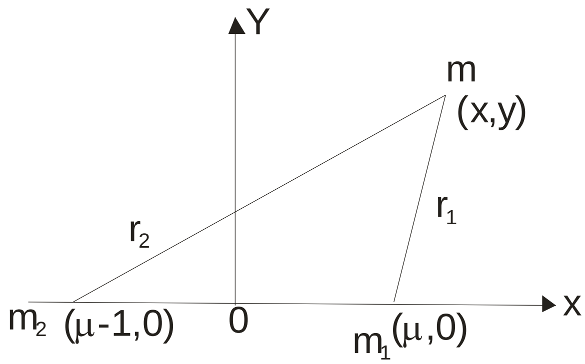
$$\ddot{x} - 2n\dot{y} = G\left[m_1\left(-\frac{q_1}{r_1^2} - \frac{3}{2}\frac{A_1 q_1}{r_1^4}\right)\left(\frac{x - x_1}{r_1}\right) + m_2\left(\frac{-q_2}{r_2^2}\right)\left(\frac{x - x_2}{r_2}\right) + n^2 x\right]$$

$$\ddot{y} + 2n\dot{x} = G\left[m_1\left(-\frac{q_1}{r_1^2} - \frac{3}{2}\frac{A_1 q_1}{r_1^4}\right)\frac{y}{r_1} + m_2\left(\frac{-q_2}{r_2^2}\right)\frac{y}{r_2}\right] + n^2 y \qquad (10)$$

Now, we choose the unit of mass equal to the sum of the primary masses. For this we taken $m_1 = 1 - \mu$ and $m_2 = \mu$, where μ is the ratio of the mass of the smaller primary to the total mass of the primaries and $0 \le \mu \le 1/2$. The unit of length is taken as equal to the distance between the primaries and the unit of time is so chosen that the gravitational constant G is unity.

Let the origin be the bary-centre of mass m_1 at $(x_1, 0)$ and m_2 at $(x_2, 0)$. Then we have $m_1x_1 + m_2 x_2 = 0$

This gives $x_1 = \mu$ and $x_2 = -(1 - \mu)$. So, the coordinates of the masses $1-\mu$ and μ are $(\mu, 0)$ and $(-(1-\mu), 0)$ respectively.



Synodic Co-ordinate System

Hence, in the dimensionless variables, the equations (10) of the third body become.

$$\ddot{x}-2n\dot{y}=(1-\mu)\left(\frac{q_1}{r_1^2}-\frac{3}{2}\frac{A_1q_1}{r_1^4}\right)\left(\frac{x-\mu}{r_1}\right)+\mu\left(-\frac{q_2}{r_2^2}\right)\left(\frac{x+1-\mu}{r^2}\right)+n^2x,$$

$$\ddot{y}+2n\dot{x}=(1-\mu)\left(-\frac{q_1}{r_1^2}-\frac{3}{2}\frac{A_1q_1}{r_1^4}\right)\frac{y}{r_1}+\mu\left(-\frac{q_2}{r_2^2}\right)\frac{y}{r_2}+n^2y,$$

i.e.,
$$\ddot{x}-2n\dot{y}=\frac{\partial U}{\partial x},$$
$$\ddot{y}+2n\dot{x}=\frac{\partial U}{\partial y} \qquad (11)$$

with

$$U=\frac{1}{2}n^2(x^2+y^2)+\frac{(1-\mu)q_1}{r_1}+\frac{\mu q_2}{r_2}+\frac{A_1q_1(1-\mu)}{2r_1^3},$$
$$r_1^2=(x-\mu)^2+y^2$$
$$r_2^2=(x+1-\mu)^2+y^2 \qquad (12)$$
$$n^2=1+\frac{3}{2}A_1 \qquad (13)$$

Thus, we find that the equations of motion are different from the classical case due to radiation and oblateness of the primaries.

Multiplying the first equation by $2\dot{x}$ and second equation by $2\dot{y}$ of (11) and then adding them, we have

$$2\dot{x}\,\ddot{x}+2\dot{y}\,\ddot{y}=2\left(\frac{\partial U}{\partial x}\dot{x}+\frac{\partial U}{\partial y}\dot{y}\right)$$
$$\frac{d}{dt}(\dot{x}^2+\dot{y}^2)=2\frac{\partial U}{dt}$$

Its integration gives

$$\dot{x}^2+\dot{y}^2=2U-C \qquad (14)$$

Here C is the Jacobian constant.

3. LOCATION OF TRIANGULAR POINTS

The locations of triangular points are the solutions of

$$\frac{\partial U}{\partial x}=0, \frac{\partial U}{\partial y}=0,\ y\neq 0$$

i.e., $$n^2x-\frac{(1-\mu)(x-\mu)q_1}{r_1^3}-\frac{\mu(x-\mu+1)q_2}{r_2^3}-\frac{3}{2}\frac{A_1(1-\mu)(x-\mu)q_1}{r_1^5}$$

and $$n^2y-\frac{(1-\mu)q_1y}{r_1^3}-\frac{\mu q_2 y}{r_2^3}-\frac{3}{2}\frac{A_1q_1(1-\mu)y}{r_1^5}$$

i.e., $$x\left[n^2-\frac{(1-\mu)q_1}{r_1^3}-\frac{\mu q_2}{r_2^3}-\frac{3}{2}\frac{A_1q_1(1-\mu)}{r_1^5}\right]+\frac{(1-\mu)\mu q_1}{r_1^3}-\frac{\mu(1-\mu)q_2}{r_2^3}+\frac{3}{2}A_1q_1\frac{\mu(1-\mu)}{r_1^5}$$

and $$y\left[n^2-\frac{(1-\mu)q_1}{r_1^3}-\frac{\mu q_2}{r_2^3}-\frac{3}{2}\frac{A_1q_1(1-\mu)}{r_1^3}\right] \tag{15}$$

The second equation of (15) gives either

$$n^2-\frac{(1-\mu)q_1}{r_1^3}-\frac{\mu q_2}{r_2^3}-\frac{3}{2}\frac{A_1q_1(1-\mu)}{r_1^5}$$

or $$y = 0 \tag{16}$$

Triangular points are the solutions of both the first equation of (15) & (16)

Making use of the first equation of (16), the first equation of (15) gives

$$\frac{(1-\mu)\mu\ q_1}{r_1^3}-\frac{\mu(1-\mu)q_2}{r_2^3}+\frac{3}{2}A_1q_1\frac{\mu(1-\mu)}{r_1^5}=0$$

which gives

$$\frac{q_1}{r_1^3}-\frac{q_2}{r_2^3}+\frac{3}{2}\frac{A_1q_1}{r_1^5}=0 \tag{17}$$

Re-writing the first equation of (16), we have

$$n^2-\frac{q_1}{r_1^3}-\frac{3}{2}\frac{A_1q_1}{r_1^5}+\mu\left[\frac{q_1}{r_1^3}-\frac{q_2}{r_2^3}+\frac{3}{2}\frac{A_1q_1}{r_1^5}\right]=0$$

Making use of (17), we have

$$n^2-\frac{q_1}{r_1^3}-\frac{3}{2}\frac{A_1q_1}{r_1^5}=0 \tag{18}$$

Combining (17) & (18), we have

$$n^2-\frac{q_2}{r_2^3}=0 \tag{19}$$

r_1 and r_2 are given by the equations (18) & (19).

Knowing r_1 and r_2, the co-ordinates of the triangular points are found by solving the equations of (12) for x and y.

Subtracting the two equations of (12), we have

$$r_2^2-r_1^2=(x+1-\mu)^2-(x-\mu)^2$$
$$=(x+1-\mu+x-\mu)(x+1-\mu-x+\mu)$$
$$=2x-2\mu+1$$
$$\Rightarrow\ 2x=2\mu-1+r_2^2-r_1^2$$

or, $$x=\mu-\frac{1}{2}+\frac{r_2^2-r_1^2}{2} \tag{20}$$

Putting the value of x in (12), i.e,

$$y^2=r_1^2-(x-\mu)^2,$$

we have

$$y^2 = r_1^2 - \left(\frac{1}{2} - \frac{r_2^2 - r_1^2}{2}\right)^2$$

or, $$y = \pm\left[\frac{r_1^2 + r_2^2}{2} - \frac{1}{4} - \left(\frac{r_2^2 - r_1^2}{2}\right)^2\right]^{1/2} \tag{21}$$

The exact coordinates of the triangular points corresponding to L_4 and L_5 are given by (20) & (21). That is

$$x = \mu - \frac{1}{2} + \frac{r_2^2 - r_1^2}{2}$$

$$y = \pm\left[\frac{r_1^2 + r_2^2}{2} - \frac{1}{4} - \left(\frac{r_2^2 - r_1^2}{2}\right)^2\right]^{1/2}$$

When the primaries are neither radiating nor oblate spheroids i.e., $A_i = 0$, $q_i = 1$ $(i = 1, 2)$, the solutions of the equations (18) & (19) are $r_i = 1$

Therefore, we can assume that the solutions of (18) & (19) are

$$r_i = 1 + \epsilon_i \tag{22}$$

Where ϵ_i are very small

$$\frac{1}{r_i^3} = \frac{1}{(1+\epsilon_i)^3} = (1+\epsilon_i)^{-3} = 1 - 3\epsilon_i + ...$$

$$\frac{1}{r_i^5} = (1+\epsilon_i)^{-5} = 1 - 5\epsilon_i + ...$$

Putting these values in (18) and (19) and putting the value of n^2 from (13) and neglecting the second and higher order terms in ϵ_i, A_i, we have

$$1 + \frac{3}{2}A_1 - q_1(1 - 3\epsilon_1) - \frac{3}{2}A_1 q_1 (1 - 5\epsilon_1) = 0$$

i.e., $$1 + \frac{3}{2}A_1 - q_1 + 3\epsilon_1 - \frac{3}{2}A_1 q_1 = 0$$

or, $$3\epsilon_1 q_1 = -1 - \frac{3}{2}A_1 + q_1\left(1 + \frac{3}{2}A_1\right)$$

or, $$3\epsilon_1 = -\frac{1 + \frac{3}{2}A_1}{q_1} + \left(1 + \frac{3}{2}A_1\right)$$

or, $$\epsilon_1 = \frac{1}{3}\left[1 + \frac{3}{2}A_1 - \frac{1 + \frac{3}{2}A_1}{q_1}\right]$$

$$= \frac{1}{3}\left[1 + \frac{3}{2}A_1 - \left\{1 + \frac{3}{2}A_1\right\}(1 - \upsilon)^{-1}\right]$$

where $q_1 = 1 - \frac{F_{p_1}}{F_g} = 1 - \upsilon$ so that $\upsilon = 1 - q_1 << 1$.

$$\therefore \epsilon_1 = \frac{1}{3}\left[1 + \frac{3}{2}A_1 - \left\{1 + \frac{3}{2}A_1\right\}(1 + \upsilon)\right]$$

$$=\frac{1}{3}\left[1+\frac{3}{2}A_1-\left\{1+\frac{3}{2}A_1+\upsilon+\frac{3}{2}\upsilon A_1\right\}\right]$$

On putting $\upsilon=1-q_1$,

$$\in_1=\frac{1}{3}\left[1+\frac{3}{2}A_1-\left\{1+\frac{3}{2}A_1+1-q_1+\frac{3}{2}(1-q_1)A_1\right\}\right]$$

$$\in_1=\frac{1}{3}\left[-\frac{3}{2}A_1-1+q_1+\frac{3}{2}A_1q_1\right], \text{ neglecting } A_2\,(1-q_1) \qquad (23)$$

Similarly, we have

$$\in_2=\frac{1}{3}\left[-\frac{1}{2}A_1-1+q_2\right]$$

Putting the value of $\in_i$ found by (23) in (22), we have

$$r_1=1-\frac{A_1}{2}-\frac{1-q_1}{3}+\frac{A_1q_1}{2}$$

$$r_2=1-\frac{A_1}{2}-\frac{1-q_2}{3} \qquad (24)$$

Putting the value of r_1 & r_2 from (24) in (20) & (21), we have

$$x=\mu-\frac{1}{2}+\in_2-\in_1$$

$$=\mu-\frac{1}{2}+\frac{1}{3}(q_2-q_1)-\frac{1}{2}A_1q_1$$

and

$$y=\pm\left[\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{3}}(\in_1+\in_2)\right]$$

$$=\pm\left[\frac{\sqrt{3}}{2}-\frac{A_1}{\sqrt{3}}+\frac{q_1+q_2}{3\sqrt{3}}+\frac{A_1q_1}{2\sqrt{3}}-\frac{2}{3\sqrt{3}}\right]$$

$$=\pm\sqrt{3}\left[\frac{1}{2}-\frac{A_1}{3}+\frac{q_1+q_2}{9}+\frac{1}{6}A_1q_1-\frac{2}{9}\right]$$

$$=\pm\sqrt{3}\left[\frac{1}{2}-\frac{1}{3}A_1+\frac{1}{6}A_1q_1-\frac{(1-q_1)}{9}-\frac{1}{9}(1-q_2)\right]$$

$$\therefore y^2=\frac{3}{4}\left[1-\frac{4}{3}A_1+\frac{2}{9}A_1q_1-\frac{4}{9}(1-q_1)-\frac{4}{9}(1-q_2)\right] \qquad (25)$$

Here, we observe that the locations of triangular equilibrium points are affected by radiation pressure forces and oblateness of the primaries.

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