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A VARIANT OF CHEBYSHEV'S METHOD

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Abstract

In this paper, we present a new two step iterative method to solve the nonlinear equation $f(x)=0$ and discuss about its convergence. Few numerical examples are considered to show the efficiency of the new method in comparison with the other methods considered in this paper.

Keywords: Chebyshev's method, Convergence, Iterative method, Newton's method nonlinear equation.

1. INTRODUCTION

Many of the complex problems in Science and Engineering contains the function of nonlinear equation of the form

$$f(x)=0 \tag{1.1}$$

Where $f: I \rightarrow R$ for an open interval I is a scalar function.

Let x_{n+1} be the root of the equation (1.1) i.e., $f(x_{n+1})=0$ while $f'(x_{n+1}) \neq 0$.

The classical quadratic convergent Newton's method [2] for finding the root of (1.1) is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad (1.2)$$

$$(n = 0, 1, 2, \ldots)$$

The third order two step Chebyshev's method [3] is

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = y_n - \frac{(y_n - x_n)^2}{2} \cdot \frac{f''(x_n)}{f'(x_n)} \quad (1.3)$$

$$(n = 0, 1, 2, \ldots)$$

In this paper, we present a two step variant of Chebyshev's method in section 2. In section 3, the convergence criterion of the new method is discussed where as in the concluding section several numerical examples are considered to exhibit the efficiency of the developed method.

2. A VARIANT OF CHEBYSHEV'S METHOD

The homotopy analysis method (HAM)[1, 4-7] is used to construct a more efficient family of iterative methods for solving nonlinear equations from

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} + \frac{h}{2}(y_n - x_n)^2 \cdot \frac{f''(x_n)}{f'(x_n)} \quad (2.1)$$

$$(n = 0, 1, 2, \ldots)$$

For $h = -1$, HAM method (2.1) gives rise to Chebyshev's method given by (1.5).

Rewriting $f''(x_n) = \frac{f'(y_n) - f'(x_n)}{y_n - x_n}$ in (2.1) gives the two step variant of Chebyshev's method as

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = y_n - \frac{(y_n - x_n)}{2} \cdot \frac{f'(y_n) - f'(x_n)}{f'(x_n)} \quad (2.2)$$

$$(n = 0, 1, 2, \ldots)$$

3. CONVERGENCE CRITERIA

Theorem 3.1. Let $\alpha \in I$ be a simple zero of a sufficiently differentiable function $f : I \to R$ for an open interval I. Then, the new method that is defined by equation (2.2) has the third order convergence and satisfies the following error equation,

$$\varepsilon_{n+1} = \left(c_2^{\,2} + c_2 + \tfrac{1}{2}c_3\right)\varepsilon_n^{\,3} + o\left(\varepsilon_n^{\,4}\right)$$

Where, $x_{n+1} = \varepsilon_{n+1} + \alpha$ and $x_n = \varepsilon_n + \alpha$

Proof: Let α be a simple zero of equation (1.1). By the Taylor's expansions

$$f(x_n) = f'(\alpha)\left[\varepsilon_n + c_2\varepsilon_n^2 + c_3\varepsilon_n^3 + o(\varepsilon_n^4)\right] \tag{3.1}$$

and $f'(x_n) = f'(\alpha)\left[1 + 2c_2\varepsilon_n + 3c_3\varepsilon_n^2 + 4c_4\varepsilon_n^3 + o(\varepsilon_n^4)\right]$ (3.2)

Dividing the equation (3.2) by (3.1), we have

$$\left[\frac{f(x_n)}{f'(x_n)}\right] = \left[\varepsilon_n - (c_2)\varepsilon_n^2 + 2(c_2^2 - c_3)\varepsilon_n^3 + 0(\varepsilon_n^4)\right] \tag{3.3}$$

Now, $y_n = x_n - \dfrac{f(x_n)}{f'(x_n)}$

$$y_n = \alpha + c_2\varepsilon_n^2 + 2(c_3 - c_2^2)\varepsilon_n^3 + o(\varepsilon_n^4) \tag{3.4}$$

Hence,

$$\frac{y_n - x_n}{2} = \frac{1}{2}\left[-\varepsilon_n + c_2\varepsilon_n^2 + 2(c_3 - c_2^2)\varepsilon_n^3 + o(\varepsilon_n^4)\right] \tag{3.5}$$

From the equation (3.4), we get

$$f'(y_n) = f'(\alpha)\left[1 + 2c_2\varepsilon_n^2 + 4c_2(c_3 - c_2^2)\varepsilon_n^3 + o(\varepsilon_n^4)\right]$$

Thus,

$$\frac{f'(x_n) - f'(y_n)}{f'(x_n)} = \left[2c_2\varepsilon_n - 2c_2\varepsilon_n^2 + 3c_3\varepsilon_n^2 - 4c_2(c_3 - c_2^2)\varepsilon_n^3 + o(\varepsilon_n^4)\right].$$
$$\left[1 - 2c_2\varepsilon_n + (-3c_3 + 4c_2)\varepsilon_n^2 + (-4c_4 + 12c_2c_3)\varepsilon_n^3 + o(\varepsilon_n^4)\right]$$

$$= 2c_2\varepsilon_n + (-4c_2^2 - 2c_2 + 3c_2)\varepsilon_n^2 + (-16c_2c_3 + 12c_2^3 + 4c_2^2 + 4c_4)\varepsilon_n^3 + 0(\varepsilon_n^4) \tag{3.6}$$

Multiplying the equation (3.5) by (3.6), we have

$$\left(\frac{y_n - x_n}{2}\right)\left(\frac{f'(x_n) - f'(y_n)}{f'(x_n)}\right) = \frac{1}{2}\left[-2c_2\varepsilon_n^2 + (6c_2^2 + 2c_2 - 3c_3)\varepsilon_n^3 + o(\varepsilon_n^4)\right] \tag{3.7}$$

Thus, $x_{n+1} = y_n - \dfrac{(y_n - x_n)}{2} \cdot \dfrac{f'(y_n) - f'(x_n)}{f'(x_n)}$ becomes

$$\varepsilon_{n+1} + \alpha = \left[\alpha + c_2\varepsilon_n^2 + 2(c_3 - c_2^2)\varepsilon_n^3 + o(\varepsilon_n^4)\right] + \frac{1}{2}\left[-2c_2\varepsilon_n^2 + (6c_2^2 + 2c_2 - 3c_3)\varepsilon_n^3 + o(\varepsilon_n^4)\right]$$

$$\varepsilon_{n+1} = \left(c_2^2 + c_2 + \frac{1}{2}c_3\right)\varepsilon_n^3 + o(\varepsilon_n^4) \tag{3.8}$$

Equation (3.8) establishes the third order convergence of the method that is defined by equation (2.2).

4. NUMERICAL EXAMPLES

We consider few numerical examples considered by [8, 9] and the method (2.2) is compared with the methods (1.2) and (1.3). The computational results are tabulated below and the results are correct up to an error less than ε as indicated for each of the problems.

Example 1: Consider the following equation $f(x) = e^{x} - 3x^{2} - = 0$.

Table 1: The results obtained by three methods for solving $f(x) = e^{x} - 3x^{2} - = 0$ with $x_0 = 1.5$ and $\varepsilon = 0.5E - 20$

Formula	Root x_n	No. of iterations (n)
Newton	0.91000757248870907904	7
Chebyshev	0.91000757248870907904	7
A Variant of Chebyshev	0.91000757248870907904	4

Example 2: Consider the following equation, $f(x) = x^{3} - e^{-x} = 0$

Table 2: The results obtained by four methods for solving $f(x) = x^{3} - e^{-x} = 0$ with $x_0 = -1$ and $\varepsilon = 0.5E - 20$

Formula	Root x_n	No. of iterations (n)
Newton	0.77288295914921017344	8
Chebyshev	0.77288295914921017344	7
A Variant of Chebyshev	0.77288295914921017344	7

Example 3: Consider the following equation, $f(x) = \sin x - 0.5x = 0$

Table 3: The results obtained by four methods for solving $f(x) = \sin x - 0.5x = 0$ with $x_0 = 3$ and $\varepsilon = 0.5E - 20$

Formula	Root x_n	No. of iterations (n)
Newton	1.89549426703398109184	6
Chebyshev	1.89549426703398109184	5
A Variant of Chebyshev	1.89549426703398109184	5

Example 4: Consider the following equation, $f(x) = x^{3} - 2x - 5 = 0$

Table 4: The results obtained by four methods for solving $f(x) = x^{3} - 2x - 5 = 0$ with $x_0 = 2.5$ and $\varepsilon = 0.5E - 20$

Formula	Root x_n	No. of iterations (n)
Newton	2.09455148154232668160	6
New Iteration	2.09455148154232668160	4
A Variant of Chebyshev	2.09455148154232668160	4

Example 5: Consider the following equation, $f(x) = \sin x = 0$

Table 5: The results obtained by four methods for solving $f(x) = \sin x = 0$ with $x_0 = 0.5$ and $\varepsilon = 0.5E-20$

Formula	Root x_n	No. of iterations (n)
Newton	0	5
Chebyshev	0	5
A Variant of Chebyshev	0	4

5. CONCLUSION

With the number of iterations tabulated for each of the methods for five non-linear equations, we conclude that the method (2.2) is efficient one compared to the methods considered in this paper.

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