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SOLUTION OF FUZZY MULTI OBJECTIVE NON-LINEAR PROGRAMMING PROBLEM (FMONLPP) USING FUZZY PROGRAMMING TECHNIQUES BASED ON EXPONENTIAL MEMBERSHIP FUNCTIONS

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Abstract

In this paper a Fuzzy Multi Objective Non Linear Programming Problem (FMONLPP) is first reduced to crisp MONLPP using ranking function. The crisp MONLPP is then solved by Zimmerman Technique using Exponential membership functions. The comparison of the results obtained using this method with those using trapezoidal membership function method is presented.

Keywords: Multi Objective Non Linear Programming Problem; Fuzzy Multi-objective Non Linear Programming Problem; Exponential Membership Function; Ranking function; Fuzzy Multi objective linear programming problem.

1. INTRODUCTION

Most of the real world problems are inherently characterized by multiple, conflicting and incommensurate aspect of evaluation. These area of evolution are generally operationalized by objective functions to be optimized in the framework of multiple objective non linear programming models. Furthermore, when addressing real world problems, frequently the parameters are imprecise numerical quantities. Fuzzy quantities are very adequate for modeling these situations. Bellmann and Zadeh [1] introduced the concept of fuzzy quantities and also the concept of fuzzy decision making. The most common approach to solve fuzzy non linear programming problem is to change them into corresponding deterministic non linear program. Some methods based on comparison of fuzzy numbers have been suggested by H.R. Maleki [10], A. Ebrahimnejad, S.H. Nasser[12]i, F. Roubens[7]. A. Munoz. Zimmermann [2,3] has introduced fuzzy programming approach to solve crisp multi objective linear programming problem. H.M. Nehi et. al [11]., used ranking function suggested by Delgodo et.al [9]., to solve fuzzy MOLPP. Leberling [4] used a special type non-linear (hyperbolic) membership function for the vector maximum linear programming problem. Dhingra and Moskowitz [6] defined other type of non-linear (exponential, quadratic and logarithmic) membership functions and applied them to an optimal design problem. Verma, Bishwal and Biswas [8] used the fuzzy programming technique with some non-linear (hyperbolic and exponential) membership function to solve a multi objective transportation problem. R.B. Dash and P.D.P Dash [13] introduced a method in which a fuzzy MONLLP is first reduced to crisp MONLLP using ranking function suggested by F. Roubens [7].Then he solved crisp MONLPP using Zimmerman technique based on trapezoidal membership function.

In this paper, following R.B. Dash [13] we reduce Fuzzy MONLPP to crisp MONLPP using Roben's Ranking function. Then we solve The crisp problem applying exponential membership functions and results obtained using this method are compared with those obtained using Trapezoidal membership function method through a numerical study.

2. MULTI OBJECTIVE NON LINEAR PROGRAMMING

The problem to optimize multiple conflicting objective functions simultaneously under given constraints is called multi objective non linear programming problem and can be formulated by following optimization problem.

$$\text{Max } f(x) = (\ f_1(x), f_2(x).......f_k(x)\ )^T$$

$$\text{s.t. } x \in X = \{x \in R^n \mid g_j(x) \leq 0\ ,\ j = 1, 2.....m\}.........(2.1)$$

where $f_1(x), f_2(x)......f_k(x)$ are k-distinct non linear objective fuctions of the decision variables and X is the feasible set of constrained decision

Definition 2.1:

x* is said to be a complete optimize solution for (2.1) if there exist x* ϵ X

s. t. $f_i(x^*) \geq f_i(x)$, i = 1, 2, 3….k. For all x ϵ X

1. Exponential membership function for fuzzy numbers :-

An exponential membership function is defined by

$$\mu^E Z_p(x) = \begin{cases} 1 & \text{if } Z_p \leq L_p \\ \dfrac{e^{-s\psi_p(x)} - e^{-s}}{1 - e^{-s}} & \text{if } L_p \leq Z_p \leq U_p \\ 0 & \text{if } Z_p \leq U_p \end{cases} \qquad (3.1)$$

$$\text{Where } \psi_p(x) = \frac{Z_p(x) - L_p}{U_p - L_p}$$

P = 1, 2, 3…..p and s is a non-zero parameter prescribed by the decision maker

If the membership function $\mu^H Z_p(\mathrm{x})$ is piecewise linear, then it is referred as trapezoidal fuzzy number and is usually denoted by $A = (\mathrm{a}^1, \mathrm{a}^2, \mathrm{a}^3, \mathrm{a}^4)$. If $a^2 = a^3$ then trapezoidal fuzzy number is turned into a triangular fuzzy number $A = (\mathrm{a}^1, \mathrm{a}^3, \mathrm{a}^4)$

A fuzzy number A = (a, b, c) is said to be a triangular fuzzy number if its membership function is given by

$$\mu_A^E(\mathrm{x}) = \begin{cases} \dfrac{x-a}{b-a} & a \le x \le b \\ 1 & x = b \\ \dfrac{x-b}{b-c} & b \le x \le c \\ 0 & \text{Otherwise} \end{cases}$$

Assume that R: F → R. R is linear ordered function that maps each fuzzy number into the real number, in which F denotes the whole fuzzy numbers. Accordingly for any two fuzzy numbers $\tilde{a}$ and $\tilde{b}$ we have.

$$\tilde{a} \underset{R}{\ge} \tilde{b} \quad iff \;\; R(\tilde{a}) \ge R(\tilde{b})$$

$$\tilde{a} \underset{R}{>} \tilde{b} \quad iff \;\; R(\tilde{a}) > R(\tilde{b})$$

$$\tilde{a} \underset{R}{=} \tilde{b} \quad iff \;\; R(\tilde{a}) = R(\tilde{b})$$

We restrict our attention to linear ranking function, that is a ranking function R such that $R(\mathrm{k}\tilde{\mathrm{a}} + \tilde{\mathrm{b}}) = \mathrm{k}\, R(\tilde{\mathrm{a}}) + R(\tilde{\mathrm{b}})$ For any $\tilde{a}$ and $\tilde{b}$ in F and any $\mathrm{k} \in \mathrm{R}$.

Rouben's ranking function:

The ranking function suggested by F. Roubens is defined by

$$R(\tilde{\mathrm{a}}) = \frac{1}{2}\int_0^1 (\inf \;\; \tilde{\mathrm{a}}_\alpha + \sup \;\; \tilde{\mathrm{a}}_\alpha) d\alpha$$

This reduces to

$$R(\tilde{\mathrm{a}}) = \frac{1}{2}(\mathrm{a}^L + \mathrm{a}^U + \tfrac{1}{2}(\beta - \alpha))$$

For a trapezoidal number

$$\tilde{a} = (\mathrm{a}^L - \mathrm{a}, \mathrm{a}^L, \mathrm{a}^U, \mathrm{a}^U + \beta)$$

3. SOLVING FUZZY MULTI OBJECTIVE NON LINEAR PROGRAMMING (FMONLP)

A fuzzy multi objective non linear programming problem is defined as followed

$$\text{Max } \tilde{\mathrm{Z}}_\mathrm{p} = \sum\nolimits_j \tilde{c}_{pj} x_j \qquad \text{p=1, 2……q}$$

s.t. $\sum_j a_{ij} x_j \le \tilde{b}$ $\quad i = 1, 2......m$ $\quad$ .... (3.2)

where $x_j \ge 0$

$\tilde{a}_{ij}$ and $\tilde{c}_{pj}$ are in the above relation are in trapezoidal form as

$\tilde{a}_{ij} = (a_{ij}^1, a_{ij}^2, a_{ij}^3, a_{ij}^4)$

$\tilde{c}_{pj} = (c_{ij}^1, c_{ij}^2, c_{ij}^3, c_{ij}^4)$

Definition 3.2:
$x \in X$ is said to be feasible solution to the FMONLP problem (3.2) if it satisfies constraints of (3.2).

Definition 3.3:
$x^* \in X$ is said to be an optimal solution to this FMONLP problem (3.2) if there does not exist another x ϵ X such that $\tilde{z}_i(x) \ge \tilde{z}_i(x^*)$ for all i =1, 2…q. Now the FMONLP can be transformed to a classic form of a MONLP by applying ranking fuction *R* as follows.

$Max\ R(\tilde{z}_p) = \sum_j R(\tilde{c}_{pj}) x_j$ $\quad$ p = 1, 2…q

s.t. $\sum_j R(\tilde{a}_{ij}) x_j \le R(\tilde{b}_i)$ $\quad$ I = 1, 2…m

$x_j \ge 0$

So we have

$Max\ z_p' = \sum_j c_{pj}' x_j$ $\quad$ p=1, 2…q

s.t. $\sum_j a_{ij}' x_j \le b_i'$ $\quad$ i=1, 2…m
(3.3)

$x_j \ge 0$

Where a_{ij}' , b_i' , c_j' are real numbers corresponding to the fuzzy numbers $\tilde{a}_{ij}$, $\tilde{b}_i$, $\tilde{c}_j$ respectively which are obtained by applying the ranking fuction *R*.

Lemma 3.4:

The optimum solution of (3.2) and (3.3) are equivalent.

Proof:

Let M_1, M_2 be set of all feasible solutions of (3.2) and (3.3) respectively.

Then x ϵ M_1 iff $\sum_j (\tilde{a}_{ij}) x_j \le (\tilde{b}_i)$ $\quad$ i=1, 2…m

By applying ranking function we have

$\sum_j R(\tilde{a}_{ij}) x_j \le R(\tilde{b}_i)$ $\quad$ i=1, 2…m

$$\Rightarrow \sum_j a'_{ij} x_j \leq b'_i$$

Hence x ϵ M_1

Thus $M_1 = M_2$

Let x* ϵ X be the complete optimal solution of (3.2)

Then $\tilde{z}_p(x^*) \geq \tilde{z}_p(x)$ for all x ϵ X

Where 'X' is a feasible set of solutions.
Thus

$$R\left(\tilde{z}_p(x^*)\right) \geq R\left(\tilde{z}_p(x)\right)$$

$$\Rightarrow R\left(\sum \tilde{c}_{pj} x_j^*\right) \geq R\left(\sum \tilde{c}_{pj} x_j\right)$$

$$\Rightarrow \sum R(\tilde{c}_{pj}) x_j^* \geq \sum R(\tilde{c}_{pj}) x_j \qquad \forall\ j=1, 2..q$$

$$\Rightarrow \sum c'_{pj} x_j^* \geq \sum c'_{pj} x_j \qquad \forall\ j=1, 2..q$$

$$\Rightarrow z'_p(x^*) \geq z'_p(x) \qquad \forall\ x$$

4. FUZZY PROGRAMMING TECHNIQUE

To solve MONLLP

$$Max\ z'_p = \sum_j c'_{pj} x_j \qquad p=1, 2,\ldots l$$

$$\text{s.t. } \sum_j a'_{ij} x_j \leq b'_i$$

$$x_j \geq 0 \qquad i=1, 2,\ldots n$$

We use fuzzy programming technique suggested by Zimmermann. This method is presented briefly in the following steps.

Step-1:
Solve the multi objective non linear programming problem by considering one objective at a time and ignoring all others. Repeat the process 'q' times for 'q' different objective functions.
Let $X_1, X_2, \ldots, X_q$ be the ideal situation for respective functions.

Step-2:
Using all the above q ideal solutions in the step-1 construct a pay-off matrix of size q by q. Then from pay-off matrix find the lower bound (L_p) and upper bound (U_p) for the objective function.

$$z'_p \text{ as } : L_p \leq z'_p \leq U_p \qquad p=1, 2,\ldots q$$

Step-3:
If we use the exponential membership function as defined (3.1) then an equivalent crisp model for the fuzzy model can be formulated as follows:

Min λ

$$\lambda \leq \frac{e^{-s\psi_p(x)} - e^{-s}}{1 - e^{-s}}, \quad p = 1,2,\ldots q$$

s.t. $\sum c'_{pj}x_j + (U_p - L_p)\lambda \geq U_p$ p=1, 2…q

$\sum a'_{ij}x_j \leq b'_i$ i=1, 2...m

$\lambda \geq 0$, $x_j \geq 0$, j=1, 2…n

The above problem can be further simplified as:
Min x_4
s.t
$s\{1 - \psi_p(x)\} \geq x_4$ p=1, 2…q
$\sum c'_{pj}x_j + (U_p - L_p)x_4 \geq U_p$ p=1, 2…q
$\sum a'_{ij}x_j \leq b'_i$ i=1, 2...m
$x_4 \geq 0$, $x_j \geq 0$, j=1, 2…n

Step-4:
Solve crisp model to find the optimal compromise solutions. Evaluate the values of objective functions at the obtained compromise solution.

5. NUMERICAL EXAMPLE

$$Max : \tilde{z}_1(x) = \tilde{2}x_1 + \tilde{3}x_2 - \tilde{2}x_1^2$$
$$Max : \tilde{z}_2(x) = \tilde{3}x_1 + \tilde{4}x_2 - \tilde{5}x_1^2 \quad \dots (5.1)$$

S.t
$$\tilde{1}x_1 + \tilde{4}x_2 \leq \tilde{4}$$
$$\tilde{1}x_1 + \tilde{1}x_2 \leq \tilde{2}$$
$$x_1,\ x_2 \geq 0$$

Where
$\tilde{2} = (1.9, 2.1, 2.2, 2.6)$
$\tilde{3} = (2.2, 2.3, 3.3, 3.8)$
$\tilde{2} = (1.2, 1.3, 2.3, 2.8)$
$\tilde{3} = (2.3, 2.5, 3.3, 3.5)$
$\tilde{4} = (3.2, 3.4, 4.2, 4.8)$
$\tilde{5} = (4.3, 4.4, 5.2, 5.7)$
$\tilde{1} = (0.8, 0.9, 1.1, 1.5)$
$\tilde{4} = (3.2, 4.0, 4.4)$
$\tilde{1} = (0.7, 0.9, 1.1, 1.3)$
$\tilde{1} = (0.6, 0.8, 1.3, 1.7)$
$\tilde{4} = (3.3, 3.4, 4.1, 4.4)$
$\tilde{2} = (1.8, 1.9, 2.2, 2.5)$

Using ranking function suggested by Ruben [7] the problem reduces to

$$max : z'_1(x) = R(\tilde{2})x_1 + R(\tilde{3})x_2 - R(\tilde{2})x_1^2$$
$$max : z'_2(x) = R(\tilde{3})x_1 + R(4)x_2 - R(\tilde{5})x_1^2$$

s.t
$$R(\tilde{1})x_1 + R(\tilde{4})x_2 \leq R(\tilde{4})$$
$$R(\tilde{1})x_1 + R(\tilde{1})x_2 \leq R(\tilde{2})$$
$$x_1, x_2 \geq 0$$

$$\Rightarrow max : z'_1(x) = 2.2x_1 + 2.9x_2 - 1.9x_1^2 \quad (5.2)$$

$$max: \; z_2'(x) = 2.9x_1 + 3.9x_2 - 4.9x_1^2 \quad (5.3)$$

s.t

$$1.1x_1 + 3.9x_2 \leq 3.8$$
$$1.1x_1 + 1.1x_2 \leq 2.1 \quad (5.4)$$

Solving (5.2) and (5.4) by Wolf's method we get

$$x_1 = \frac{539}{1482} = 0.3637, \; x_2 = \frac{50387}{257798} = 0.8718$$

Solving (5.3) and (5.4) by Wolf's method we get

$$x_1 = \frac{9}{49} = 0.1837, \; x_2 = \frac{1736}{1911} = 0.9226$$

The lower bound (L.B.) and upper bound (U.B.) of objective functions z_1' and z_2' have been computed as follows

Function	LB	UB
z_1'	3.0655	3.00770
z_2'	3.8065	3.9653

If we use exponential membership function with the parameter s=1, an equation crisp model can be formulated as

Min x_4

Subject to

$$S[z_1(x)] + x_4(U_1 - L_1) \geq S(U_1)$$
$$S[z_2(x)] + x_4(U_2 - L_2) \geq S(U_2)$$

$$\sum a_{ij}\, x_j \leq b_i' \qquad i = 1, 2...$$

As per Step-4, let us solve the problem which is used by exponential function and the problem reduced to

Min X_4

$$2.2x_1 + 2.9x_2 - 1.9x_1^2 + 0.115x_3 \geq 3.0770$$
$$2.9x_1 + 3.9x_2 - 4.9x_1^2 + 0.1588x_3 \geq 3.953$$
$$1.1x_1 + 3.9x_2 \leq 3.8 \quad \ldots\ldots. (5.5)$$
$$1.0x_1 + 1.1x_2 \leq 2.1$$
$$x_1, x_2 \geq 0$$

Due to presence of non-linear term x_1^2 in the constraint (5.5), the problem is too complex to solve. To avoid the situation taking advantage of

$$0.1837 \leq x_1 \leq 0.3637$$

We have $x_1^2 = a_{10}x_{10}^2 + a_{11}x_{11}^2 + a_{12}x_{12}^2 + a_{13}x_{13}^2$

Where

$$x_{10} = 0.1837$$
$$x_{11} = 0.2437$$

$$x_{12} = 0.3037$$

$$x_{13} = 0.3637$$

Then the problem reduces to

$$\begin{aligned}
&2.2x_1 + 2.9x_2 - 1.9(a_{10}x_{10}^2 + a_{11}x_{11}^2 + a_{12}x_{12}^2 + a_{13}x_{13}^2) + 0.115x_3 \ge 3.0770 \\
&2.9x_1 + 3.9x_2 - 4.9(a_{10}x_{10}^2 + a_{11}x_{11}^2 + a_{12}x_{12}^2 + a_{13}x_{13}^2) + 0.1588x_3 \ge 3.953 \\
&1.1x_1 + 3.9x_2 \le 3.8 \\
&1.0x_1 + 1.1x_2 \le 2.1 \\
&a_{10} + a_{11} + a_{12} + a_{13} = 1 \\
&x_1, x_2, a_{10}, a_{11}, a_{12}, a_{13} \ge 0
\end{aligned} \quad \text{……. (5.6)}$$

Solving (5.6) the optimal solution of the problem is obtained as:

$$x_1^* = 0.4518$$

$$x_2^* = 0.8469$$

Now the optimal value of the objective functions of FMONLPP (5.6) becomes

$$\begin{aligned}
z_1^* &= \tilde{2}x_1^* + \tilde{3}x_2^* - \tilde{2}{x_1^*}^2 \\
&= (1.9, 2.1, 2.2, 2.6)x_1^* + (2.2, 2.3, 3.3, 3.8)x_2^* - (1.2, 1.3, 2.3, 2.8){x_1^*}^2 \\
&= (2.4767, 2.6313, 3.31925, 3.8214)
\end{aligned}$$

$$\begin{aligned}
z_2^* &= \tilde{3}x_1^* + \tilde{4}x_2^* - \tilde{5}{x_1^*}^2 \\
&= (2.3, 2.5, 3.3, 3.5)x_1^* + (3.2, 3.4, 4.2, 4.8)x_2^* - (4.3, 4.4, 5.2, 5.7){x_1^*}^2 \\
&= (2.8715, 3.1108, 3.9865, 4.4829)
\end{aligned}$$

The membership functions corresponding to the fuzzy objective function are as follows.

$$\mu_{Z_1}^{E}(\mathrm{x}) = \begin{cases} 0 & x \le 2.4767 \\ \dfrac{x - 2.4767}{0.1546} & 2.4767 < x \le 2.6313 \\ 1 & 2.6313 < x \le 3.3192 \\ \dfrac{3.8214 - x}{0.5022} & 3.3192 < x \le 3.8214 \\ 0 & x > 3.8214 \end{cases}$$

$$\mu_{Z_2}^{E}(x) = \begin{cases} 0 & x \le 2.8715 \\ \dfrac{x-2.8715}{0.2393} & 2.8715 < x \le 3.1108 \\ 1 & 3.1108 < x \le 3.9865 \\ \dfrac{4.4829-x}{0.4964} & 3.9865 < x \le 4.4829 \\ 0 & x \ge 4.4829 \end{cases}$$

CONCLUSION

On comparison we see that the result obtained in this paper is very close to that obtained in [13] using trapezoidal membership function in Zimmerman's algoritm.Thus one can use exponential in place of trapezoidal membership function in the Zimmerman's algorithm.

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