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T-INTUITIONISTIC FUZZY H-IDEALS OF *BCK*-ALGEBRAS

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Abstract

Objectives: The aim of this paper is to introduce the concept of t-intuitionistic fuzzy H-ideals of *BCK*-algebras as a generalization of intuitionistic fuzzy H-ideals *BCK*-algebras and study the effect of some modal operators on t-intuitionistic fuzzy H-ideals of *BCK*–algebras.
Methods: The method adapted to study the objectives is analytic/logical method. Some examples and counter examples are provided in support of theorem and remarks etc. We observe that if t=1 the t-intuitionistic fuzzy H-ideal of BCK-algebra becomes a intuitionistic fuzzy H-ideal of BCK-algebra.
Findings: Here we define t-intuitionistic fuzzy H-ideals of *BCK*–algebras. Some noble properties of t-intuitionistic fuzzy H-ideals are stated and proved. We observe that t-intuitionistic fuzzy H-ideal of *BCK* –algebras is invariant under modal operators and homomorphic mappings. We proved that the intersection and cartesian product of two or more t-intuitionistic fuzzy H-ideals of *BCK*–algebras is again a t-intuitionistic fuzzy H-ideal.
Application/Improvements: t-intuitionistic fuzzy H-ideal of BCK-algebra can be used in medical diagnosis, image processing, artficial intelligency etc.

Keywords: BCK-algebra, t-intuitionistic fuzzy set, t-intuitionistic fuzzy H-ideal, Modal operator, Homomorphism.

1. INTRODUCTION

Extending the concept of fuzzy sets [1], Atanassov [2] introduced the notion of intuitionistic fuzzy subset (IFS) in 1983. Rosenfeld [3] was the first who consider the case of a groupoid in terms of fuzzy sets. Since then these ideas have been applied to other algebraic structures such as group, semigroup, ring, vector spaces etc. There are several authors who considered fuzzy sets and intuitionistic fuzzy sets in different algebraic structures, for instance, Jun et al. [4,5], Kuroki [6], Kim et al. [7], Barbhuiya et al. [8,9,10,11]. The study of BCK-algebras was initiated by Imai and Iseki [12] in 1966. In the same year, Iseki [13] introduced the notion of a BCI-algebra which is a generalization of a BCK-algebra. . For the general development of BCK/BCI-algebras, ideal theory plays an important role. In 1999, Khalid and Ahmad [14] introduced fuzzy H-ideals in *BCI*-algebras. In 2010, Satyanarayan et al. [15] introduced intuitionistic fuzzy H-ideals in BCK-algebras and also several interesting properties of these concepts were studied. The intuitionistic fuzzy modal operators $\Box$ and $\Diamond$ was introduced by Atanassov [2] in 1983. The extension on both the operators $\Box$ and $\Diamond$ is the new operator D_α which represents both of them. Further in [16] Atanassov extended the operators $\Box$, $\Diamond$, D_α to a new operator $F_{\alpha,\beta}$ called (α,β)-modal operator. The extended modal operator $F_{\alpha,\beta}$ is not the final generalised modal operator, the other generalisation are $\oplus,\otimes$, $\oplus_\alpha,\otimes_\alpha$, $\oplus_{\alpha,\beta},\otimes_{\alpha,\beta}$, $E_{\alpha,\beta},\oplus_{\alpha,\beta,\gamma}$, $\otimes_{\alpha,\beta,\gamma}$, $\boxdot_{\alpha,\beta,\gamma,\delta}$ and $\boxdot_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta}$ [see 17,18,19,20,21,11,22,23].

The effect of all the modal operators on IFSs is again an IFSs. Modal operators play an important rule in the study of IFSs. In [24, 25] Sharma introduced the idea of t-intuitionistic fuzzy sets in fuzzy subgroups and fuzzy subrings. Here in this paper, we introduced the notion of t-intuitionistic fuzzy H-ideals in BCK-algebras and some related properties are investigated. Also we studied the effect of some model operators on t-intuitionistic fuzzy H-ideals of *BCK*–algebras.

2. PRELIMINARIES

In this section, we will review some concept related to BCK-algebras.Throughout this paper X will denote a BCK-algebras.

Definition 2.1: [4,13] An algebra $(X,*,0)$ of type (2, 0) is called a BCK-algebra if it satisfies the following axioms:

(i) $((x*y)*(x*z))*(z*y)=0$

(ii) $(x*(x*y))*y=0$

(iii) $x*x=0$

(iv) $0*x=0$

(v) $x*y=0$ and $y*x=0 \Rightarrow x=y$ for all $x,y,z\in X$

We can define a partial ordering " $\leq$ " on X by $x\leq y$ iff $x*y=0$.

Definition 2.2: [4,13] A BCK-algebra X is said to be commutative if it satisfies the identity $x\wedge y=y\wedge x$ where $x\wedge y=y*(y*x)\ \forall x,y\in X$. In a commutative BCK-algebra, it is known that $x\wedge y$ is the greatest lower bound of x and y.

In a BCK-algebra X, the following hold:

(i) $x*0=x$

(ii) $(x*y)*z=(x*z)*y$

(iii) $x*y\leq x$

(iv) $(x*y)*z\leq(x*z)*(y*z)$

(v) $x\leq y$ implies $x*z\leq y*z$ and $z*y\leq z*x$.

A BCK-algebra X is said to be associative if it satisfies the identity $(x*y)*z=x*(y*z)\ \forall x,y,z\in X$. A non empty subset S of a BCK-algebra X is called a subalgebra X if $x * y \in X$, for all $x, y \in S$. A non empty subset I of a BCK algebra X is called an ideal of X if (i) $0 \in I$ (ii) $x * y \in I$ and $y \in I \Rightarrow x \in I$ for all $x, y \in X$. A nonempty subset I of a BCK -algebra X is said to be an H-ideal [5] of X if it satisfies (i) and (iii) $x*(y*z) \in I$ and $y \in I \Rightarrow x*z \in I$ for all $x, y, z \in X$.

A fuzzy subset μ of a BCK-algebra X is called a

(**A**) fuzzy subalgebra of X if $\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$, for all $x, y \in X$.

(B) doubt fuzzy subalgebra of X if $\mu(x * y) \leq \min\{\mu(x), \mu(y)\}$ for all $x, y \in X$.

(**C**) fuzzy ideal [26] of X if it satisfies the following axioms:

(i) $\mu(0) \geq \mu(x)$
(ii) $\mu(x) \geq \min\{\mu(x * y), \mu(y)\}$ for all $x, y \in X$.

(**D**) fuzzy H-ideal [27,28]of X if it satisfies the following axioms:
(i) $\mu(0) \geq \mu(x)$
(ii) $\mu(x*z) \geq \min\{\mu(x*(y*z)), \mu(y)\}$ for all $x, y, z \in X$.

Definition 2.3: [29] An intuitionistic fuzzy set (IFS) A of a non empty set X is an object of the form $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$, where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ with the condition $0 \leq \mu_A(x) + \nu_A(x) \leq 1, \forall x \in X$. The numbers $\mu_A(x)$ and $\nu_A(x)$ denote respectively the degree of membership and the degree of non-membership of the element x in set A. For the sake of simplicity, we shall use the symbol $A = (\mu_A, \nu_A)$ for the intuitionistic fuzzy set $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$. The function $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ for all $x \in X.$ is called the degree of uncertainty of $x \in A$. The class of IFSs on a universe X is denoted by $IFS(X)$.

Definition 2.4: **[**16,29,30] If $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ and
$B = \{< x, \mu_B(x), \nu_B(x) > | x \in X\}$ are any two IFSs of a set X , then
$A \subseteq B$ if and only if for all $x \in X, \mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$,
$A = B$ if and only if for all $x \in X, \mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$,
$A \cap B = \{< x, (\mu_A \cap \mu_B)(x), (\nu_A \cup \nu_B)(x) > | x \in X\}$,
where $(\mu_A \cap \mu_B)(x) = \min\{\mu_A(x), \mu_B(x)\}$ and $(\nu_A \cup \nu_B)(x) = \max\{\nu_A(x), \nu_B(x)\}$,
$A \cup B = \{< x, (\mu_A \cup \mu_B)(x), (\nu_A \cap \nu_B)(x) > | x \in X\}$,
where $(\mu_A \cup \mu_B)(x) = \max\{\mu_A(x), \mu_B(x)\}$ and $(\nu_A \cap \nu_B)(x) = \min\{\nu_A(x), \nu_B(x)\}$.

Definition 2.5: **[**30] If $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ and
$B = \{< x, \mu_B(x), \nu_B(x) > | x \in X\}$ are any two IFSs of a set X , then their cartesian product is defined by
$A \times B = \{< (x, y), (\mu_A \times \mu_B)(x, y), (\nu_A \times \nu_B)(x, y) > | x, y \in X\}$,
where $(\mu_A \times \mu_B)(x, y) = \min\{\mu_A(x), \mu_B(y)\}$ and $(\nu_A \times \nu_B)(x, y) = \max\{\nu_A(x), \nu_B(y)\}$.

Definition 2.6: [4]An IFS $A = (\mu_A, \nu_A)$ in X is called an intuitionistic fuzzy ideal of X, if it satisfies the following axioms:
(i) $\mu_A(0) \geq \mu_A(x)$

(ii) $\nu_A(0) \le \nu_A(x)$

(iii) $\mu_A(x) \ge \min\{\mu_A(x*y), \mu_A(y)\}$,

(iv) $\nu_A(x) \le \max\{\nu_A(x*y), \nu_A(y)\} \; \forall \; x, y \in X$.

Definition 2.7: [5] An IFS $A = (\mu_A, \nu_A)$ in X is called an intuitionistic fuzzy H-ideal of X, if it satisfies the following axioms:

(i) $\mu_A(0) \ge \mu_A(x)$

(ii) $\nu_A(0) \le \nu_A(x)$

(iii) $\mu_A(x*z) \ge \min\{\mu_A(x*(y*z)), \mu_A(y)\}$,

(iv) $\nu_A(x*z) \le \max\{\nu_A(x*(y*z)), \nu_A(y)\} \; \forall \; x, y, z \in X$.

Definition 2.8: [16,29,30] For any IFS $A = \{< x, \mu_A(x), \nu_A(x) > | \, x \in X\}$ of X and $\alpha \in [01]$, the operator $\Box : IFS(X) \to IFS(X), \Diamond : IFS(X) \to IFS(X), D_\alpha : IFS(X) \to IFS(X)$ are defined as

(i) $\Box\,(A) = \{< x, \mu_A(x), 1 - \mu_A(x) > | \, x \in X\}$ is called necessity operator

(ii) $\Diamond\,(A) = \{< x, 1 - \nu_A(x), \nu_A(x) > | \, x \in X\}$ is called possibility operator

(iii) $D_\alpha(A) = \{< x, \mu_A(x) + \alpha\pi_A(x), \nu_A(x) + (1-\alpha)\pi_A(x) > | \, x \in X\}$ is called α-modal operator.

Clearly $\Box\,(A) \subseteq A \subseteq \Diamond\,(A)$ and the equality hold, when A is a fuzzy set also $D_0(A) = \Box\,(A)$ and $D_1(A) = \Diamond(A)$. Therefore the α-Modal operator $D_\alpha(A)$ is an extension of necessity operator $\Box\,(A)$ and possibility operator $\Diamond\,(A)$.

Definition 2.9: [16,30] For any IFS $A = \{< x, \mu_A(x), \nu_A(x) > | \, x \in X\}$ of X and for any $\alpha, \beta \in [01]$ such that $\alpha + \beta \le 1$, the (α, β)-modal operator $F_{\alpha,\beta} : IFS(X) \to IFS(X)$ is defined as $F_{\alpha,\beta}(A) = \{< x, \mu_A(x) + \alpha\pi_A(x), \nu_A(x) + \beta\pi_A(x) > | \, x \in X\}$, where $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ for all $x \in X$. Therefore we can write

$F_{\alpha,\beta}(A)$ as $F_{\alpha,\beta}(A)(x) = (\mu_{F_{\alpha,\beta}(A)}(x), \nu_{F_{\alpha,\beta}(A)}(x))$

where $\mu_{F_{\alpha,\beta}}(x) = \mu_A(x) + \alpha\pi_A(x)$ and $\nu_{F_{\alpha,\beta}(A)}(x)) = \nu_A(x) + \beta\pi_A(x)$.

Clearly, $F_{0,1}(A) = \Box\,(A), F_{1,0}(A) = \Diamond\,(A)$ and $F_{\alpha,1-\alpha}(A) = D_\alpha(A)$

Definition 2.10: [16,29,30] *For any IFS* $A = \{< x, \mu_A(x), \nu_A(x) > | \, x \in X\}$ *of* X *and* $\alpha \in [01]$, *the operators* $\oplus, \otimes, \oplus_\alpha, \otimes_\alpha : IFS(X) \to IFS(X)$ *are defined as*

(i) $\oplus(A) = \{< x, \dfrac{\mu_A(x)}{2}, \dfrac{\nu_A(x)+1}{2} > | \, x \in X\}$

(ii) $\otimes(A) = \{< x, \dfrac{\mu_A(x)+1}{2}, \dfrac{\nu_A(x)}{2} > | \, x \in X\}$

(iii) $\oplus_\alpha A = \{< x, \alpha\mu_A(x), \alpha\nu_A(x) + 1 - \alpha > | \, x \in X\}$

(iv) $\otimes_\alpha A = \{< x, \alpha\mu_A(x) + 1 - \alpha, \alpha\nu_A(x) > | \, x \in X\}$

Therefore $\oplus_{0.5} A = \oplus A$ and $\otimes_{0.5} A = \otimes A$ Hence $\oplus_\alpha, \otimes_\alpha$ are the generalisation of $\oplus, \otimes$.

Definition 2.11: [26,27,28,30] *For any IFS* $A = \{< x, \mu_A(x), \nu_A(x) > | \, x \in X\}$ *of* X *and* $\alpha, \beta, \alpha + \beta \in [0\,1]$, *the operator* $\oplus_{\alpha,\beta}, \otimes_{\alpha,\beta}, E_{\alpha,\beta} : IFS(X) \to IFS(X)$ *are defined as*

(i) $\oplus_{\alpha,\beta}(A)=\{<x,\alpha\mu_A(x),\alpha\nu_A(x)+\beta>|\, x\in X\}$

(ii) $\otimes_{\alpha,\beta}(A)=\{<x,\alpha\mu_A(x)+\beta,\alpha\nu_A(x)>|\, x\in X\}$

(iii) $E_{\alpha,\beta}A=\{<x,\beta(\alpha\mu_A(x)+1-\alpha),\alpha(\beta\nu_A(x)+1-\beta)>|\, x\in X\}$

Therefore $\oplus_{0.5,0.5}A=\oplus A$ and $\otimes_{0.5,0.5}A=\otimes A, \oplus_{\alpha,1-\alpha}(A)=\oplus_{\alpha}(A), \otimes_{\alpha,1-\alpha}(A)=\otimes_{\alpha}(A)$ Hence $\oplus_{\alpha,\beta},\otimes_{\alpha,\beta}$ are the generalisation of $\oplus,\otimes$ and also $\oplus_{\alpha},\otimes_{\alpha}$. The operators $\oplus_{\alpha,\beta},\otimes_{\alpha,\beta}$ was introduced by Katerina Dencheva[27]. Which was further extended to operators as defined below

Definition 2.12: [26,27,28,30] *For any IFS* $A=\{<x,\mu_A(x),\nu_A(x)>|\, x\in X\}$ *of* X *and* $\alpha,\beta,\gamma\in[0\,1]$, *and* $max(\alpha,\beta)+\gamma\le 1$ *the operator* $\oplus_{\alpha,\beta,\gamma},\otimes_{\alpha,\beta,\gamma}: IFS(X)\to IFS(X)$ *are defined as*

(i) $\oplus_{\alpha,\beta,\gamma}(A)=\{<x,\alpha\mu_A(x),\beta\nu_A(x)+\gamma>|\, x\in X\}$

(ii) $\otimes_{\alpha,\beta,\gamma}(A)=\{<x,\alpha\mu_A(x)+\gamma,\beta\nu_A(x)>|\, x\in X\}$

Therefore $\oplus_{0.5,0.5,0.5}A=\oplus A$, $\otimes_{0.5,0.5,0.5}A=\otimes A,$ $\oplus_{\alpha,\alpha,1-\alpha}(A)=\oplus_{\alpha}(A),$ $\otimes_{\alpha,1-\alpha}(A)=\otimes_{\alpha}(A),\ \oplus_{\alpha,\alpha,\beta,}(A)=\oplus_{\alpha,\beta,}(A),\ \otimes_{\alpha,\alpha,\beta}(A)=\otimes_{\alpha,\beta}(A)$ Hence $\oplus_{\alpha,\beta,\gamma},\otimes_{\alpha,\beta,\gamma}$ are the generalisation of all operators $\oplus,\otimes,\oplus_{\alpha},\otimes_{\alpha},\oplus_{\alpha,\beta},\otimes_{\alpha,\beta}$. The extension of operators $\oplus_{\alpha,\beta,\gamma}$, and $\otimes_{\alpha,\beta,\gamma}$ is the operator $\boxdot_{\alpha,\beta,\gamma,\delta}$ and is defined below

Definition 2.13: [26,27,28,30] *For any IFS* $A=\{<x,\mu_A(x),\nu_A(x)>|\, x\in X\}$ *of* X *and* $\alpha,\beta,\gamma.\delta\in[0\,1]$, *and* $max(\alpha,\beta)+\gamma+\delta\le 1$ *the operator* $\boxdot_{\alpha,\beta,\gamma,\delta}: IFS(X)\to IFS(X)$ *are defined as*

$\boxdot_{\alpha,\beta,\gamma,\delta}(A)=\{<x,\alpha\mu_A(x)+\gamma,\beta\nu_A(x)+\delta>|\, x\in X\}$

Therefore $\boxdot_{0.5,0.5,0.0,0.5}A=\oplus A,\ \boxdot_{0.5,0.5,0.5,0.0}A=\otimes A$

$\boxdot_{\alpha,\alpha,0,1-\alpha}(A)=\oplus_{\alpha}(A), \boxdot_{\alpha,\alpha,1-\alpha,0}(A)=\otimes_{\alpha}(A)$

$\boxdot_{\alpha,\alpha,0,\beta}(A)=\oplus_{\alpha,\beta}(A),\ \boxdot_{\alpha,\alpha,\beta,0}(A)=\otimes_{\alpha,\beta,}(A)$

$\boxdot_{\alpha,\beta,0,\gamma,}(A)=\oplus_{\alpha,\beta,\gamma}(A),\ \boxdot_{\alpha,\beta,\gamma,0}(A)=\otimes_{\alpha,\beta,\gamma}(A)$

Also $E_{\alpha,\beta}(A)=\boxdot_{\alpha\beta,\alpha\beta,(1-\alpha)\beta,(1-\beta)\alpha}(A)$

Hence $\boxdot_{\alpha,\beta,\gamma,\delta}$ are the generalisation of all operators as dicussed above. Now final extension above operators are the operator $\boxcircle_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta}$, which is defined by

Definition 2.14: [26,27,28,30] For any IFS $A=\{<x,\mu_A(x),\nu_A(x)>|\, x\in X\}$ of X and $\alpha,\beta,\gamma.\delta,\varepsilon,\zeta\in[0\,1]$, and $max(\alpha-\zeta,\beta-\varepsilon)+\gamma+\delta\le 1, min(\alpha-\zeta,\beta-\varepsilon)+\gamma+\delta\ge 0$ the operator $\boxcircle_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta}: IFS(X)\to IFS(X)$ are defined as

$\boxcircle_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta}(A)=\{<x,\alpha\mu_A(x)-\varepsilon\nu_A(x)+\gamma,\beta\nu_A(x)-\zeta\mu_A(x)+\delta>|\, x\in X\}$

Therefore $\boxcircle_{0.5,0.5,0,0.5,0,0}A=\oplus A$ and $\boxcircle_{0.5,0.5,0.5,0,0,0}A=\otimes A$

$\boxcircle_{\alpha,\alpha,0,1-\alpha,0,0}(A)=\oplus_{\alpha}(A), \boxcircle_{\alpha,\alpha,1-\alpha,0,0,0}(A)=\otimes_{\alpha}(A)$

$\boxcircle_{\alpha,\alpha,0,\beta,0,0}(A)=\oplus_{\alpha,\beta,}(A), \boxcircle_{\alpha,\alpha,\beta,0,0,0}(A)=\otimes_{\alpha,\beta,}(A)$

$\boxed{\circ}_{\alpha,\beta,0,\gamma,0,0}(A) = \oplus_{\alpha,\beta,\gamma}(A), \boxed{\circ}_{\alpha,\beta,\gamma,0,0,0}(A) = \otimes_{\alpha,\beta,\gamma}(A)$

Also $\boxed{\circ}_{\alpha,\beta,\gamma,\delta,0,0}(A) = \boxed{\bullet}_{\alpha,\beta,\gamma,\delta}(A)$

Hence $\boxed{\circ}_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta}(A)$ are the final generalisation of all operators dicussed above.

Definition 2.15: Let X and Y be two non empty sets and $f : X \to Y$ be a mapping. Let A and B be IFS's of X and Y respectively . Then the image of A under the map f is denoted by f(A) and is defined by $f(A)(y) = (\mu_{f(A)}(y), \nu_{f(A)}(y))$, where $\mu_{f(A)}(y) = \begin{cases} \vee\{\mu_A(x) : x \in f^{-1}(y)\} \\ 0 \quad otherwise \end{cases}$ $\nu_{f(A)}(y) = \begin{cases} \wedge\{\nu_A(x) : x \in f^{-1}(y)\} \\ 1 \quad otherwise \end{cases}$ also pre image of B under f is denoted by $f^{-1}(B)$ and is defined as $f^{-1}(B)(x) = (\mu_{f^{-1}(B)}(x), \nu_{f^{-1}(B)(x)} = (\mu_B(f(x)), \nu_B(f(x))); \forall x \in X$.

Definition 2.16: Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy set of BCK-algebra X. Let $t \in [0,1]$ then the instuitionistic fuzzt set A^t of X called *t*- intuitionistic fuzzy subset (*t*-IFS) of X w.r.t. *A* and is denoted by $A^t = \{< x, \mu_{A^t}(x), \nu_{A^t}(x) > | x \in X\} = \langle \mu_{A^t}, \nu_{A^t} \rangle$ where $\mu_{A^t}(x) = \min\{\mu_A(x), t\}$ and $\nu_{A^t}(x) = \max\{\nu_A(x), 1-t\} \quad \forall x \in X$

Remark 2.17: [24,25] Let $A^t = \langle \mu_{A^t}, \nu_{A^t} \rangle$ and $B^t = \langle \mu_{B^t}, \nu_{B^t} \rangle$ be two *t*-intuitionistic fuzzy subsets of BCK-algebra X then $(A \cap B)^t = A^t \cap B^t$

Remark 2.18: [24,25] Let $f : X \to Y$ be a mapping. Let A and B be two IFS of X and Y respectively ,then $f^{-1}(B^t) = \left(f^{-1}(B)\right)^t$ and $f(A^t) = \left(f(A)\right)^t \quad ; \forall t \in [0,1]$

Definition 2.19: Let $A^t = \langle \mu_{A^t}, \nu_{A^t} \rangle$ and $B^t = \langle \mu_{B^t}, \nu_{B^t} \rangle$ be two t-intuitionistic fuzzy subsets of BCK- algebra X , then their cartesian product $A^t \times B^t = \langle \mu_{A^t \times B^t}, \nu_{A^t \times B^t} \rangle$ is defined by

$$\mu_{A^t \times B^t}(x, y) = \min\left\{\mu_{A^t}(x), \mu_{B^t}(y)\right\}$$
$$\nu_{A^t \times B^t}(x, y) = \max\left\{\nu_{A^t}(x), \nu_{B^t}(y)\right\} \qquad \forall\ x, y \in X.$$

Definition 2.20: Let $A = (\mu_A, \nu_A)$ be an t-intuitionistic fuzzy set in X and $\alpha, \beta \in [0\ \ 1]$ then the μ level set $\mu_A^{\ \alpha}$ and the ν level set $\nu_{A\beta}$ of A is defined by

$\mu_A^{\ \alpha} = \{\ x \in X \ / \ \mu_A(x) \geq \alpha\ \}$ and $\nu_{A\beta} = \{\ x \in X \ / \ \nu_A(x) \leq \beta\ \}$

3. T- INTUITIONISTIC FUZZY H-IDEALS OF *BCK*-ALGEBRAS

In this section, we define t- intuitionistic fuzzy H-ideals of *BCKI*-algebras and prove some interesting properties of these ideals.

Definition 3.1: An IFS $A = (\mu_A, \nu_A)$ in X is called an t- intuitionistic fuzzy subalgebra of X, if it satisfies the following axioms:

(i) $\mu_{A^t}(x * y) \geq \min\{\mu_{A^t}(x), \mu_{A^t}(y)\}$,

(ii) $\nu_{A^t}(x*y) \le \max\{\nu_{A^t}(x), \nu_{A^t}(y)\}\ \forall\ x, y \in X$.

Definition 3.2: An IFS $A = (\mu_A, \nu_A)$ in X is called an t- intuitionistic fuzzy ideal of X, if it satisfies the following axioms:

(i) $\mu_{A^t}(0) \ge \mu_{A^t}(x)$

(ii) $\nu_{A^t}(0) \le \nu_{A^t}(x)$

(iii) $\mu_{A^t}(x) \ge \min\{\mu_{A^t}(x*y), \mu_{A^t}(y)\}$,

(iv) $\nu_{A^t}(x) \le \max\{\nu_{A^t}(x*y), \nu_{A^t}(y)\}\ \forall\ x, y \in X$.

Definition 3.3: An IFS $A = (\mu_A, \nu_A)$ in X is called an t- intuitionistic fuzzy H-ideal of X, if it satisfies the following axioms:

(i) $\mu_{A^t}(0) \ge \mu_{A^t}(x)$

(ii) $\nu_{A^t}(0) \le \nu_{A^t}(x)$

(iii) $\mu_{A^t}(x*z) \ge \min\{\mu_{A^t}(x*(y*z)), \mu_{A^t}(y)\}$,

(iv) $\nu_{A^t}(x*z) \le \max\{\nu_{A^t}(x*(y*z)), \nu_{A^t}(y)\}\ \forall\ x, y, z \in X$.

Theorem 3.4: If $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy H-ideal of BCK- algebra X then A is also t-intuitionistic fuzzy H-ideal of X.

Proof: Since $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy H-ideal of X, then

(i) $\mu_A(0) \ge \mu_A(x)$

(ii) $\nu_A(0) \le \nu_A(x)$

(iii) $\mu_A(x*z) \ge \min\{\mu_A(x*(y*z)), \mu_A(y)\}$,

(iv) $\nu_A(x*z) \le \max\{\nu_A(x*(y*z)), \nu_A(y)\}\ \forall\ x, y, z \in X$.

Now, $\mu_{A^t}(0) = \min\{\mu_A(0), t\} \ge \min\{\mu_A(x), t\} = \mu_{A^t}(x)$

$\nu_{A^t}(0) = \max\{\nu_A(0), 1-t\} \le \max\{\nu_A(x), 1-t\} = \nu_{A^t}(x)$

$$\mu_{A^t}(x*z) = \min\{\mu_A(x*z), t\} \ge \min\left\{\min\left\{\mu_A\left(x*(y*z)\right), \mu_A(y)\right\}, t\right\}$$

$$= \min\left\{\min\left\{\mu_A(x*(y*z)), t\right\}, \min\left\{\mu_A(y), t\right\}\right\}$$

$$= \min\left\{\mu_{A^t}\left(x*(y*z)\right), \mu_{A^t}(y)\right\}$$

Similarly we have $\nu_{A^t}(x*z) \le \max\left\{\nu_{A^t}\left(x*(y*z)\right), \nu_{A^t}(y)\right\}\quad \forall\ x, y, z \in X$.

Hence A is an t-intuitionistic fuzzy H-ideal of X.

Remark 3.5: The converse of above Theorem need not be true as shown in Example below.

Example 3.6: Consider a BCK -algebra $X = \{0, 1, 2\}$ with the following cayley table 1.

Table1: Illustration of converse of Theorem 3.4

*	0	1	2
0	0	2	1
1	1	0	2
2	2	1	0

The IF subset $A=\{<x,\mu_A(x),\nu_A(x)>|\, x\in X\}$ given by $\mu_A(0)=0.6$, $\mu_A(1)=0.4$, $\mu_A(2)=0.8$, and $\nu_A(0)=0.4$, $\nu_A(1)=0.6$, $\nu_A(2)=0.7$, is not an IF H-ideal of X. Since $\mu_A(0)<\mu_A(2)$. Now take t=0.3 , then $\mu_A(x)>$ t =0.3 and $\nu_A(x)$ < 1-t=0.7 for all $x\in X$. Therefore $\mu_{A^t}(0)\geq\mu_{A^t}(x)$, $\nu_{A^t}(0)\leq\nu_{A^t}(x)$, $\mu_{A^t}(x*z)\geq\min\{\mu_{A^t}(x*(y*z)),\mu_{A^t}(y)\}$, and $\nu_{A^t}(x*z)\leq\max\{\nu_{A^t}(x*(y*z)),\nu_{A^t}(y)\}\ \forall\ x,y,z\in X$.
Hence A is an t-intuitionistic fuzzy H-ideal of X.

Theorem 3.7: If $A=(\mu_A,\nu_A)$ is an intuitionistic fuzzy set of BCK-algebra X and let $t<min\{p,1-q\}$, where $p=min\{\mu_A(x)\,|\,x\in X\}$ and $q=max\{\nu_A(x)\,|\,x\in X\}$ then A is also t-intuitionistic fuzzy H-ideal of BCK-algebra X.

Proof. Since $t<min\{p,1-q\}$

$$\begin{aligned}
&t<min\{p,1-q\}\\
&\Rightarrow p>t \quad\text{and}\quad 1-q>t\\
&\Rightarrow p>t \quad\text{and}\quad q<1-t\\
&\Rightarrow min\{\mu_A(x)\,|\,x\in X\}>t \quad\text{and}\quad max\{\nu_A(x)\,|\,x\in X\}<1-t\\
&\Rightarrow \mu_A(x)>t,\forall\quad x\in X \quad\text{and}\quad \nu_A(x)<1-t,\forall\quad x\in X
\end{aligned}$$

Therefore

$$\mu_{A^t}(0)\geq\mu_{A^t}(x),\quad \mu_{A^t}(x*z)\geq\min\{\mu_{A^t}(x*(y*z)),\mu_{A^t}(y)\},$$
$$\nu_{A^t}(0)\leq\nu_{A^t}(x)\ ,\quad \nu_{A^t}(x*z)\leq\max\{\nu_{A^t}(x*(y*z)),\nu_{A^t}(y)\}\ \forall\ x,y,z\in X,\ \text{hold}$$

Hence A is t-intuitionistic fuzzy H-ideal of X.

Theorem 3.8: The intersection of two t-intuitionistic fuzzy H-ideals of BCK-algebra X is also t-intuitionistic fuzzy H-ideal of BCK-algebra of X.

Proof. Let $A=(\mu_A,\nu_A)$ and $B=(\mu_B,\nu_B)$ are two t-intuitionistic fuzzy H-ideal of BCK-algebra X , then

$$(\mu_{A^t}(0)\geq\mu_{A^t}(x),\ \mu_{A^t}(x*z)\geq\min\{\mu_{A^t}(x*(y*z)),\mu_{A^t}(y)\},$$
$$\nu_{A^t}(0)\leq\nu_{A^t}(x),\ \nu_{A^t}(x*z)\leq\max\{\nu_{A^t}(x*(y*z)),\nu_{A^t}(y)\}\ \forall\ x,y,z\in X.$$

and

$$\mu_{B^t}(0)\geq\mu_{B^t}(x),\ \mu_{B^t}(x*z)\geq\min\{\mu_{B^t}(x*(y*z)),\mu_{B^t}(y)\},$$
$$\nu_{B^t}(0)\leq\nu_{B^t}(x),\ \nu_{B^t}(x*z)\leq\max\{\nu_{B^t}(x*(y*z)),\nu_{B^t}(y)\}\ \forall\ x,y,z\in X.$$

Now,

$$\begin{aligned}
(i)\ \ \mu_{(A\cap B)^t}(0)&=\min\{\mu_{A\cap B}(0),t\}=\min\left\{\min\left\{\mu_A(0),\mu_B(0)\right\},t\right\}\\
&=\min\left\{\min\left\{\mu_A(0),t\right\},\min\left\{\mu_B(0),t\right\}\right\}\\
&=\min\left\{\mu_{A^t}(0),\mu_{B^t}(0)\right\}\\
&\geq\min\left\{\mu_{A^t}(x),\mu_{B^t}(x)\right\}\\
&=\mu_{A^t\cap B^t}(x)=\mu_{(A\cap B)^t}(x)
\end{aligned}$$

$$\begin{aligned}
(ii)\ \ \nu_{(A\cap B)^t}(0)&=\max\{\nu_{A\cap B}(0),t\}=\max\left\{\max\left\{\nu_A(0),\nu_B(0)\right\},t\right\}\\
&=\max\left\{\max\left\{\nu_A(0),t\right\},\max\left\{\nu_B(0),t\right\}\right\}\\
&=\max\left\{\nu_{A^t}(0),\nu_{B^t}(0)\right\}
\end{aligned}$$

$$\leq \max\left\{\nu_{A^t}(x), \nu_{B^t}(x)\right\}$$
$$= \nu_{A^t \cap B^t}(x) = \nu_{(A\cap B)^t}(x)$$

(iii) $\mu_{B^t}(x*z) \geq \min\{\mu_{B^t}(x*(y*z)), \mu_{B^t}(y)\}$,

$$\mu_{(A\cap B)^t}(x*z) = min\{\mu_{(A\cap B)}(x*z), t\}$$
$$= \min\{min(\mu_{(A)}(x*z), \mu_{(B)}(x*z)), t\}$$
$$\geq min\{min\{\min(\mu_A(x*(y*z)), \mu_A(y)), \min(\mu_B(x*(y*z)), \mu_B(y))\}, t\}$$
$$\geq min\{min\{\min\{\mu_A(x*(y*z)), \mu_B(x*(y*z))\}, t), \min\{\mu_A(y), \mu_B(y)\}, t\}\}$$
$$= min\{min(\mu_{A\cap B}(x*(y*z)), t), min(\mu_{A\cap B}(y), t)\}$$
$$= min\{\mu_{A^t}(x*(y*z)), \mu_{B^t}(y)\}$$
$$\Rightarrow \mu_{(A\cap B)^t}(x*z) \geq min\{\mu_{(A\cap B)^t}(x*(y*z)), \mu_{(A\cap B)^t}(y)\}$$

Similarly we can show that

(iv) $\nu_{(A\cap B)^t}(x*z) \leq max\{\nu_{(A\cap B)^t}(x*(y*z)), \nu_{(A\cap B)^t}(y)\}$

Theorem 3.9: The intersection of any number of t-intuitionistic fuzzy H-ideals of BCK- algebra X is also t-intuitionistic fuzzy H-ideal of X.

Theorem 3.10: For every t-intuitionistic fuzzy H-ideal A^t of an associative BCK-algebra X, if the following inequality $x*a \leq$ b holds in X, then

(i) $\mu_{A^t}(x*a) \geq \mu_{A^t}(b)$

(ii) $\nu_{A^t}(x*a) \leq \nu_{A^t}(b), \forall x \in X.$

Proof: Let $x, a, b \in X$ such that $x*a \leq b$ then $(x*a)*b = 0$ and since A^t is t-intuitionistic fuzzy H-ideal of X ,so

We have $\mu_{A^t}(x*a) \geq min\left\{\mu_{A^t}(x*(b*a)), \mu_{A^t}(b)\right\}$

$$= min\left\{\mu_{A^t}((x*b)*a), \mu_{A^t}(b)\right\} \quad \text{since X is associative}$$
$$= min\left\{\mu_{A^t}((x*a)*b), \mu_{A^t}(b)\right\}$$
$$= min\left\{\mu_{A^t}(0), \mu_{A^t}(b)\right\}$$
$$= \mu_{A^t}(b) \quad \text{because } \mu_{A^t}(0) \geq \mu_{A^t}(x)$$

Therefore, $\mu_{A^t}(x*a) \geq \mu_{A^t}(b)$

Again $\nu_{A^t}(x*a) \leq \max\left\{\nu_{A^t}(x*(b*a)), \nu_{A^t}(b)\right\}$

$$= \max\left\{\nu_{A^t}((x*b)*a), \nu_{A^t}(b)\right\} \quad \text{since X is associative}$$
$$= \max\left\{\nu_{A^t}((x*a)*b), \nu_{A^t}(b)\right\}$$
$$= \max\left\{\nu_{A^t}(0), \nu_{A^t}(b)\right\}$$
$$= \nu_{A^t}(b) \quad \text{because } \nu_{A^t}(0) \leq \nu_{A^t}(x)$$

Hence $\nu_{A^t}(x*a) \le \nu_{A^t}(b)$

Theorem 3.11: If an intuitionistic fuzzy set $A=(\mu_A,\nu_A)$ be an intuitionistic fuzzy H-ideal of a BCK-algebra X and if $x \le y$ holds in X, then

(i) $\mu_{A^t}(x) \ge \mu_{A^t}(y)$

(ii) $\nu_{A^t}(x) \le \nu_{A^t}(y)$,

Proof Straightforward.

Theorem 3.12: Let $A=(\mu_A,\nu_A)$ be an t-intuitionistic fuzzy H-ideal in BCK-algebra X if and only if the level sets μ_A^{α} and $\nu_{A\beta}$ are H-ideals of X where $\alpha, \beta \in [0\ \ 1]$.

Proof **Let** $A=(\mu_A,\nu_A)$ be an t-intuitionistic fuzzy H-ideal in BCK-algebra X and let $\alpha, \beta \in [0\ \ 1]$, x ∈ μ_A^{α} then $\mu_A(x) \ge \alpha$ and also $\mu_{A^t}(0) \ge \mu_{A^t}(x)$ for all x ∈ X. Now

$\min\{\ \mu_A(0), t\} = \mu_{A^t}(0) \ge \mu_{A^t}(x) = \min\{\ \mu_A(x), t\}$ which implies $\mu_A(0) \ge \mu_A(x)$

i.e., $\mu_A(0) \ge \alpha$ i.e., 0 ∈ μ_A^{α}. Let x, y, z ∈ X be such that $x*(y*z)$, y ∈ μ_A^{α} then $\mu_A(x*(y*z)) \ge \alpha$, $\mu_A(y) \ge \alpha$ also

$$\mu_{(A\cap B)^t}(x*z) \ge min\{\mu_{(A\cap B)^t}(x*(y*z)), \mu_{(A\cap B)^t}(y)\} \text{ for all x, y, z } \in \text{ X.}$$

$$\begin{aligned} \min\{\ \mu_A(x*z), t\} = \mu_{(A\cap B)^t}(x*z) &\ge min\{\mu_{(A\cap B)^t}(x*(y*z)), \mu_{(A\cap B)^t}(y)\} \\ &= \min\{\ min\{\mu_{(A\cap B)}(x*(y*z)), t\}, min\{\mu_{(A\cap B)}(y), t\}\} \\ &= \min\{\ min\{\mu_{(A\cap B)}(x*(y*z)),\ \mu_{(A\cap B)}(y)\}, t\} \end{aligned}$$

$$\Rightarrow \mu_{(A\cap B)}(x*z) \ge min\{\mu_{(A\cap B)}(x*(y*z)), \mu_{(A\cap B)}(y)\} = min\{\alpha, \alpha\} = \alpha \ \Rightarrow x*z \in \mu_A^{\alpha}$$

Hence μ_A^{α} is an H-ideals of X. Similarly we can show $\nu_{A\beta}$ is an H-ideals of X.

Conversely, let level sets μ_A^{α} and $\nu_{A\beta}$ are H-ideals of X for $\alpha, \beta \in [0\ \ 1]$.Let x∈ X such that $\mu_A(x) = \alpha$ and $\nu_A(x) = \beta$ then $x \in \mu_A^{\alpha}$ and $x \in \nu_{A\beta}$ Since μ_A^{α} and $\nu_{A\beta}$ are H-ideals of X Therefore $0 \in \mu_A^{\alpha}$ and $0 \in \nu_{A\beta}$ hence $\mu_A(0) \ge \alpha = \mu_A(x)$ and $\nu_A(0) \le \beta = \nu_A(x)$. Again if $\mu_A(x*z) \ge \min\{\mu_A(x*(y*z)), \mu_A(y)\}$, does not holds for $x, y, z \in X$ then there exists some $x', y', z' \in X$ such that $\mu_A(x'*z') < \min\{\mu_A(x'*(y'*z')), \mu_A(y')\}$, take $t_0 = \{\mu_A(x'*z') + \min\{\mu_A(x'*(y'*z')), \mu_A(y')\}\}/2$, then $\mu_A(x'*z') \le t_0 \le \min\{\mu_A(x'*(y'*z')), \mu_A(y')\}$, i.e., $\mu_A(x'*(y'*z')) \ge t_0$, $\mu_A(y') \ge t_0$ and $\mu_A(x'*z') \le t_0$. Which implies $x'*(y'*z')$, y' ∈ μ_A^{α} but $x'*z' \notin \mu_A^{\alpha}$. Which contradicts the fact that μ_A^{α} is an H-ideals of X. Hence we must have $\mu_A(x*z) \ge \min\{\mu_A(x*(y*z)), \mu_A(y)\}$. Similarly we can show that $\nu_A(x*z) \le \max\{\nu_A(x*(y*z)), \nu_A(y)\}\ \forall\ x, y, z \in X$. Therefore $A=(\mu_A,\nu_A)$ is an intuitionistic fuzzy H-ideal of X and hence by Theorem 3.4 A is an t-intuitionistic fuzzy H-ideal of X.

Theorem 3.13: If intuitionistic fuzzy set $A=(\mu_A,\nu_A)$ is t-intuitionistic fuzzy H-ideal of BCK algebra X. Then

(i) $\mu_{A^t}(0*(0*x)) \ge \mu_{A^t}(x)$

(ii) $\nu_{A^t}\left(0*\left(0*x\right)\right)\le \nu_{A^t}(x)$

We have $\mu_{A^t}\left(0*\left(0*x\right)\right)\ge min\left\{\mu_{A^t}\left(0*\left(x\left(0*x\right)\right)\right),\mu_{A^t}(x)\right\}$

$=min\left\{\mu_{A^t}\left(0*\left(x*0\right)\right),\mu_{A^t}(x)\right\}$

$=min\left\{\mu_{A^t}\left(0*x\right),\mu_{A^t}(x)\right\}$

$=min\left\{\mu_{A^t}\left(0\right),\mu_{A^t}(x)\right\}$

$=\mu_{A^t}(x) \qquad \forall\ x\in X$

Therefore, $\mu_{A^t}\left(0*\left(0*x\right)\right)\ge \mu_{A^t}(x) \qquad \forall\ x\in X.$

Again, $\nu_{A^t}\left(0*\left(0*x\right)\right)\le \max\left\{\nu_{A^t}\left(0*\left(x\left(0*x\right)\right)\right),\nu_{A^t}(x)\right\}$

$=\max\left\{\nu_{A^t}\left(0*\left(x*0\right)\right),\nu_{A^t}(x)\right\}$

$=\max\left\{\nu_{A^t}\left(0*x\right),\nu_{A^t}(x)\right\}$

$=\max\left\{\nu_{A^t}\left(0\right),\nu_{A^t}(x)\right\}$

$=\nu_{A^t}(x) \qquad \forall\ x\in X$

Therefore, $\nu_{A^t}\left(0*\left(0*x\right)\right)\le \nu_{A^t}(x)\ , \qquad \forall\ x\in X.$

Theorem 3.14: If A be IF H-ideal of BCK-algebra of X, then $F_{\alpha,\beta}\left(A\right)$ are also t-intuitionistic fuzzy H-ideal of BCK-algebra X.

Proof: If $A=\left(\mu_A,\nu_A\right)$ be an intuitionistic fuzzy H-ideal of BCK-algebra X then A is also t-intuitionistic fuzzy H-ideal of X.

Therefore, (i) $\mu_{A^t}(0)\ge \mu_{A^t}(x)$

(ii) $\nu_{A^t}(0)\le \nu_{A^t}(x)$

(iii) $\mu_{A^t}(x*z)\ge \min\{\mu_{A^t}(x*(y*z)),\mu_{A^t}(y)\}$,

(iv) $\nu_{A^t}(x*z)\le \max\{\nu_{A^t}(x*(y*z)),\nu_{A^t}(y)\}\ \forall\ x,y,z\in X.$

Proof We have $F_{\alpha,\beta}(A^t)(x)=(\mu_{F_{\alpha,\beta}(A^t)}(x),\nu_{F_{\alpha,\beta}(A^t)}(x))$

Where $\mu_{F_{\alpha,\beta}(A^t)}(x)=\mu_{A^t}(x)+\alpha\pi_{A^t}(x)$ and $\nu_{F_{\alpha,\beta}(A^t)}(x)=\nu_{A^t}(x)+\beta\pi_{A^t}(x)$. Now

(i)

$$\begin{aligned}\mu_{F_{\alpha,\beta}(A^t)}(0)&=\mu_{A^t}(0)+\alpha\pi_{A^t}(0)\\&=\mu_{A^t}(0)+\alpha(1-\mu_{A^t}(0)-\nu_{A^t}(0))\\&=\alpha+(1-\alpha)\mu_{A^t}(0)-\alpha\nu_{A^t}(0)\\&\ge\alpha+(1-\alpha)\mu_{A^t}(x)-\alpha\nu_{A^t}(x)\\&=\mu_{A^t}(x)+\alpha(1-\mu_{A^t}(x)-\nu_{A^t}(x))\\&=\mu_{A^t}(x)+\alpha\pi_{A^t}(x)\end{aligned}$$

$$= \mu_{F_{\alpha,\beta}(A^t)}(x)$$

$$\therefore \mu_{F_{\alpha,\beta}(A^t)}(0) \geq \mu_{F_{\alpha,\beta}(A^t)}(x)$$

(ii)

$$\nu_{F_{\alpha,\beta}(A^t)}(0) = \nu_{A^t}(0) + \beta\pi_{A^t}(0)$$

$$= \nu_{A^t}(0) + \beta(1 - \mu_{A^t}(0) - \nu_{A^t}(0))$$

$$= \beta + (1-\beta)\nu_{A^t}(0) - \beta\mu_{A^t}(0)$$

$$\leq \beta + (1-\beta)\nu_{A^t}(x) - \beta\mu_{A^t}(x)$$

$$= \nu_{A^t}(x) + \beta(1 - \nu_{A^t}(x) - \mu_{A^t}(x))$$

$$= \nu_{A^t}(x) + \beta\pi_{A^t}(x)$$

$$= \mu_{F_{\alpha,\beta}(A^t)}(x)$$

$$\therefore \nu_{F_{\alpha,\beta}(A^t)}(0) \leq \nu_{F_{\alpha,\beta}(A^t)}(x)$$

(iii)

$$\mu_{F_{\alpha,\beta}(A^t)}(x*z)$$

$$= \mu_{A^t}(x*z) + \alpha\pi_{A^t}(x*z)$$

$$= \mu_{A^t}(x*z) + \alpha(1 - \mu_{A^t}(x*z) - \nu_{A^t}(x*z))$$

$$= \alpha + (1-\alpha)\mu_{A^t}(x*z) - \alpha\nu_{A^t}(x*z)$$

$$\geq \alpha + (1-\alpha)\min(\mu_{A^t}(x*(y*z)), \mu_{A}(y)) - \alpha\max(\nu_{A^t}(x*(y*z)), \nu_{A^t}(y))$$

$$= \alpha\{1 - \max(\nu_{A^t}(x*(y*z)), \nu_{A^t}(y))\} + (1-\alpha)\min(\mu_{A^t}(x*(y*z)), \mu_{A^t}(y))$$

$$= \alpha\min(1 - \nu_{A^t}(x*(y*z)), 1 - \nu_{A^t}(y))\} + (1-\alpha)\min(\mu_{A^t}(x*(y*z)), \mu_{A^t}(y))$$

$$= \min\{\alpha(1 - \nu_{A^t}(x*(y*z))) + (1-\alpha)\mu_{A^t}(x*(y*z)), \alpha(1 - \nu_{A^t}(y)) + (1-\alpha)\mu_{A^t}(y)\}$$

$$= \min\{\mu_{A^t}(x*(y*z)) + \alpha(1 - \mu_{A^t}(x*(y*z)) - \nu_{A^t}(x*(y*z))), \mu_{A^t}(y) + \alpha(1 - \mu_{A^t}(y) - \nu_{A^t}(y))\}$$

$$= \min\{\mu_{F_{\alpha,\beta}(A^t)}(x*(y*z)), \mu_{F_{\alpha,\beta}(A^t)}(y)\}$$

$$\therefore \mu_{F_{\alpha,\beta}(A^t)}(x*z) \geq \min\{\mu_{F_{\alpha,\beta}(A^t)}(x*(y*z)), \mu_{F_{\alpha,\beta}(A^t)}(y)\}$$

Similarly we can prove

$$\nu_{F_{\alpha,\beta}(A^t)}(x*z) \leq \max\{\nu_{F_{\alpha,\beta}(A^t)}(x*(y*z)), \nu_{F_{\alpha,\beta}(A^t)}(y)\}$$

Hence $F_{\alpha,\beta}(A^t)$ is a t-intuitionistic fuzzy H-ideal of X.

Remark 3.15: The converse of above Theorem need not be true as shown in Example below.

Example 3.16: Consider a BCK-algebra $X = \{0,1,2,3,4\}$ with the following cayley table 2.

Table 2: Illustration of converse of Theorem 3.14.

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	1	1
2	2	2	0	2	2
3	3	3	3	0	3
4	4	4	4	4	0

The IF subset $A=\{<x,\mu_A(x),\nu_A(x)>| x\in X\}$ given by $\mu_A(0)=0.66$, $\mu_A(1)=0.6$, $\mu_A(2)=0.5$, $\mu_A(3)=0.5$, $\mu_A(4)=0.4$ and $\nu_A(0)=0.34$, $\nu_A(1)=0.3$, $\nu_A(2)=0.5$, $\nu_A(3)=0.5$, $\nu_A(4)=0.6$ is not an IF H-ideal of X. Since $\nu_A(0)\not\leq\nu_A(1)$. Now take $\alpha=0.2,\beta=0.7,\alpha+\beta\leq 1$, then $F_{\alpha,\beta}(A)=\{<x,\mu_{F_{\alpha,\beta}(A)}(x),\nu_{F_{\alpha,\beta}(A)}(x)>| x\in X\}$ is $\mu_{F_{0.2,0.7}(A)}(0)$, $=0.66$, $\mu_{F_{0.2,0.7}(A)}(1)=0.62$, $\mu_{F_{0.2,0.7}(A)}(2)=0.5$, $\mu_{F_{0.2,0.7}(A)}(3)=0.5$, $\mu_{F_{0.2,0.7}(A)}(4)=0.4$ and $\nu_{F_{0.2,0.7}(A)}(0)=0.34$, $\nu_{F_{0.2,0.7}(A)}(1)=0.37$, $\nu_{F_{0.2,0.7}(A)}(2)=0.5$, $\nu_{F_{0.2,0.7}(A)}(3)=0.5$, $\nu_{F_{0.2,0.7}(A)}(4)=0.6$. It can easily verified that $F_{0.2,0.7}(A)$ is a IF H-ideal of X. Again choose any $t_0\in(0,0.4)$, then $F_{0.2,0.7}(A)$ is a t_0-IF H-ideal of X.

Corollary 3.17: If A be IF H-ideal of BCK-algebra of X then $\Box A$ *and* $\Diamond A$ are also t-intuitionistic fuzzy H-ideal of BCK-algebra X.

Theorem 3.18: If A be IF H-ideal of BCK-algebra of X then $\boxed{\circ}_{\alpha,\beta,\gamma,\delta,\varepsilon,\xi}(A)$ are also t-intuitionistic fuzzy H-ideal of BCK-algebra X.

Proof: If $A=(\mu_A,\nu_A)$ be an intuitionistic fuzzy H-ideal of BCK-algebra X then A is also t-intuitionistic fuzzy H-ideal of X.

Therefore, (i) $\mu_{A^t}(0)\geq\mu_{A^t}(x)$

(ii) $\nu_{A^t}(0)\leq\nu_{A^t}(x)$

(iii) $\mu_{A^t}(x*z)\geq\min\{\mu_{A^t}(x*(y*z)),\mu_{A^t}(y)\}$,

(iv) $\nu_{A^t}(x*z)\leq\max\{\nu_{A^t}(x*(y*z)),\nu_{A^t}(y)\}\ \forall\, x,y,z\in X$.

.

Proof We have $\boxed{\circ}_{\alpha,\beta,\gamma,\delta,\varepsilon,\varsigma}(A^t)(x)=(\mu_{\boxed{\circ}_{\alpha,\beta,\gamma,\delta,\varepsilon,\varsigma}(A^t)}(x),\nu_{\boxed{\circ}_{\alpha,\beta,\gamma,\delta,\varepsilon,\varsigma}(A^t)}(x))$

Where $\mu_{\boxed{\circ}_{\alpha,\beta,\gamma,\delta,\varepsilon,\varsigma}(A^t)}(x)=\alpha\mu_{A^t}(x)-\varepsilon\nu_{A^t}(x)+\gamma$ and

$\nu_{\boxed{\circ}_{\alpha,\beta,\gamma,\delta,\varepsilon,\varsigma}(A^t)}(x)=\beta\nu_{A^t}(x)-\varsigma\mu_{A^t}(x)+\delta$. Now

(i)

$$\mu_{\boxed{\circ}_{\alpha,\beta,\gamma,\delta,\varepsilon,\varsigma}(A^t)}(0)=\alpha\mu_{A^t}(0)-\varepsilon\nu_{A^t}(0)+\gamma$$

$$\geq\alpha\mu_{A^t}(x)-\varepsilon\nu_{A^t}(x)+\gamma$$
$$=\mu_{\boxed{\circ}_{\alpha,\beta,\gamma,\delta,\varepsilon,\varsigma}(A^t)}(x)$$

$$\therefore\mu_{\boxed{\circ}_{\alpha,\beta,\gamma,\delta,\varepsilon,\varsigma}(A^t)}(0)\geq\mu_{\boxed{\circ}_{\alpha,\beta,\gamma,\delta,\varepsilon,\varsigma}(A^t)}(x)$$

(ii)

$$\nu_{\boxed{\circ}_{\alpha,\beta,\gamma,\delta,\varepsilon,\varsigma}(A^t)}(0)=\beta\nu_{A^t}(0)-\varsigma\mu_{A^t}(0)+\delta$$

$$\leq\beta\nu_{A^t}(x)-\varsigma\mu_{A^t}(x)+\delta$$
$$=\nu_{\boxed{\circ}_{\alpha,\beta,\gamma,\delta,\varepsilon,\varsigma}(A^t)}(x)$$

$$\therefore \nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\varepsilon,\varsigma}(A^t)}(0) \le \nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\varepsilon,\varsigma}(A^t)}(x)$$

(iii)

$$\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\varepsilon,\varsigma}(A^t)}(x*z) = \alpha\mu_{A^t}(x*z) - \varepsilon\nu_{A^t}(x*z) + \gamma$$

$$\ge \alpha \min\{\mu_{A^t}(x*(y*z)), \mu_{A^t}(y)\} - \varepsilon \max\{\nu_{A^t}(x*(y*z)), \nu_{A^t}(y)\} + \gamma$$
$$\ge \min\{\alpha\mu_{A^t}(x*(y*z)), \mu_{A^t}(y)\} + \varepsilon \min\{1-\nu_{A^t}(x*(y*z)), 1-\nu_{A^t}(y)\} + \gamma - \varepsilon$$
$$= \min\{\alpha\mu_{A^t}(x*(y*z)) + \varepsilon(1-\nu_{A^t}(x*(y*z)), \alpha\mu_{A^t}(y) + \varepsilon(1-\nu_{A^t}(y))\} + \gamma - \varepsilon$$
$$= \min\{\alpha\mu_{A^t}(x*(y*z)) + \varepsilon(1-\nu_{A^t}(x*(y*z)) + \gamma - \varepsilon, \alpha\mu_{A^t}(y) + \varepsilon(1-\nu_{A^t}(y)) + \gamma - \varepsilon\}$$
$$= \min\{\alpha\mu_{A^t}(x*(y*z)) + \varepsilon(1-\nu_{A^t}(x*(y*z)) + \gamma - \varepsilon, \alpha\mu_{A^t}(y) + \varepsilon(1-\nu_{A^t}(y)) + \gamma - \varepsilon\}$$
$$= \min\{\alpha\mu_{A^t}(x*(y*z)) - \varepsilon\nu_{A^t}(x*(y*z)) + \gamma, \alpha\mu_{A^t}(y) - \varepsilon\nu_{A^t}(y)) + \gamma\}$$
$$= \min\{\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\varepsilon,\varsigma}(A^t)}(x*(y*z)), \mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\varepsilon,\varsigma}(A^t)}(y)\}$$

Which implies

$$\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\varepsilon,\varsigma}(A^t)}(x*z) \ge \min\{\mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\varepsilon,\varsigma}(A^t)}(x*(y*z)), \mu_{\boxdot_{\alpha,\beta,\gamma,\delta,\varepsilon,\varsigma}(A^t)}(y)\}$$

Similarly we can prove

(iv) $\nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\varepsilon,\varsigma}(A^t)}(x*z) \le \max\{\nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\varepsilon,\varsigma}(A^t)}(x*(y*z)), \nu_{\boxdot_{\alpha,\beta,\gamma,\delta,\varepsilon,\varsigma}(A^t)}(y)\}$

Remark 3.19: The converse of above Theorem is also not true.

Corollary 3.20: If A be IF H-ideal of BCK-algebra of X, then

(i) $\oplus(A)$ is also an t-intuitionistic fuzzy H-ideal of BCK-algebra X.

(ii) $\otimes(A)$ is also an t-intuitionistic fuzzy H-ideal of BCK-algebra X.

(iii) $\oplus_\alpha(A)$ is also a t-intuitionistic fuzzy H-ideal of BCK-algebra X.

(iv) $\otimes_\alpha(A)$ is also an t-intuitionistic fuzzy H-ideal of BCK-algebra X..

(v) $\oplus_{\alpha,\beta}(A)$ is also an t-intuitionistic fuzzy H-ideal of BCK-algebra X.

(vi) $\otimes_{\alpha,\beta}(A)$ is also an t-intuitionistic fuzzy H-ideal of BCK-algebra X.

(vii) $\oplus_{\alpha,\beta,\lambda}(A)$ is also an t-intuitionistic fuzzy H-ideal of BCK-algebra X.

(viii) $\otimes_{\alpha,\beta,\gamma}(A)$ is also an t-intuitionistic fuzzy H-ideal of BCK-algebra X..

(viii) $\boxdot_{\alpha,\beta,\gamma,\delta}(A)$ is also an t-intuitionistic fuzzy H-ideal of BCK-algebra X.

Remark 3.21: The converse of above Corollary(s) is also not true.

Definition 3.22: Let X and Y be two BCK-algebras, then a mapping $f : X \to Y$ is said to be homomorphism if $f(x*y) = f(x)*f(y), \forall x, y \in X$.

Theorem 3.23: Let $f : X \to Y$ be a homomorphism of BCK –algebras then f(0)=0.

Proof. Straightforward.

Theorem 3.24: Let $f : X \to Y$ be a homomorphism of BCK-algebras, If A be a t-intuitionistic fuzzy H-ideal of Y , then $f^{-1}(A))$ is t-intuitionistic fuzzy H-ideal of X.

Proof. A be a t-intuitionistic fuzzy fuzzy H-ideal of Y. Let $x, y \in X$ be any elements, then
$f^{-1}(A^t)(x) = (\mu_{f^{-1}(A^t)}(x), \nu_{f^{-1}(A^t)}(x))$, Now

(i) $\mu_{f^{-1}(A^t)}(0) = \mu_{A^t} f(0) \geq \mu_{A^t}[f(x)] = \mu_{f^{-1}(A^t)}(x)$ [Since A is a t-intuitionistic fuzzy H-idealof Y]

(ii) $\nu_{f^{-1}(A^t)}(0) = \nu_{A^t} f(0) \leq \nu_{A^t}[f(x)] = \nu_{f^{-1}(A^t)}(x)$

(iii) we have $f^{-1}(A^t)(x*z) = (\mu_{f^{-1}(A^t)}(x*z), \nu_{f^{-1}(A^t)}(x*z))$

$$\begin{aligned} \text{Now, } \mu_{f^{-1}(A^t)}(x*z) &= \mu_{A^t} f(x*z) \\ &= \mu_{A^t}[f(x)*f(z)] \\ &\geq min\{\mu_{A^t}(f(x*(y*z))), \mu_{A^t}(f(y))\} \\ &= min\{\mu_{f^{-1}(A^t)}(x*(y*z)), \mu_{f^{-1}(A^t}) (y)\} \end{aligned}$$

Therefore $\mu_{f^{-1}(A^t)}(x*z) \geq min\{\mu_{f^{-1}(A^t)}(x*(y*z)), \mu_{f^{-1}(A^t)}(y)\}$

Similarly we can show that

(iv) $$\nu_{f^{-1}(A^t)}(x*z) \leq max\{\nu_{f^{-1}(A^t)}(x*(y*z)), \nu_{f^{-1}(A^t)}(y)\}$$

Hence, $f^{-1}(A^t) = (f^{-1}(A))^t$ is t-intuitionistic fuzzy H-ideal of X.

Theorem 3.25: Cartesian product of two t-intuitionistic fuzzy H-ideals of X is again a t-intuitionistic fuzzy H-ideal of $X \times X$.

Proof. Let $A^t =< \mu_{A^t}, \nu_{A^t} >$ and $B^t =< \mu_{B^t}, \nu_{B^t} >$ be two t-intuitionistic fuzzy H-ideals of BCK/BCI-algebra X. Then their cartesian product $A^t \times B^t =< \mu_{A^t \times B^t}, \mu_{A^t \times B^t} >$, where

$$\mu_{A^t \times B^t}(x, y) = min\{\mu_{A^t}(x), \mu_{B^t}(y)\}$$
$$\nu_{A^t \times B^t}(x, y) = max\{\nu_{A^t}(x), \nu_{B^t}(y)\} \quad \forall x, y \in X.$$

Now

(i)
$$\begin{aligned} \mu_{A^t \times B^t}(0,0) &= \mu_{A^t \times B^t}(0,0) \\ &= min\{\mu_{A^t}(0), \mu_{B^t}(0)\} \\ &\geq min\{\mu_{A^t}(x), \mu_{A^t}(y)\} \\ &= \mu_{A^t \times B^t}(x, y) \\ \Rightarrow \mu_{A^t \times B^t}(0,0) &\geq \mu_{A^t \times B^t}(x, y) \end{aligned}$$

(ii)
$$\nu_{A^t \times B^t}(0,0) = \nu_{A^t \times B^t}(0,0)$$

$$= \max\{\nu_{A^t}(0), \nu_{B^t}(0)\}$$
$$\leq \max\{\nu_{A^t}(x), \nu_{A^t}(y)\}$$
$$= \nu_{A^t \times B^t}(x, y)$$
$$\Rightarrow \nu_{A^t \times B^t}(0,0) \leq \nu_{A^t \times B^t}(x, y)$$

(iii)

$$\mu_{A^t \times B^t}((x_1, y_1) * (x_3, y_3))$$
$$= \mu_{A^t \times B^t}(x_1 * x_3, y_1 * y_3)$$
$$= min\{\mu_{A^t}(x_1 * x_3), \mu_{B^t}(y_1 * y_3)\}$$
$$\geq min\{min\{\mu_{A^t}(x_1 * (x_2 * x_3)), \mu_{A^t}(x_2)\}, min\{\mu_{B^t}(y_1 * (y_2 * y_3)), \mu_{B^t}(y_2)\}\}$$
$$= min\{min\{\mu_{A^t}(x_1 * (x_2 * x_3)), \mu_{B^t}(y_1 * (y_2 * y_3))\}, min\{\mu_{A^t}(x_2), \mu_{B^t}(y_2)\}\}$$
$$= min\{\mu_{A^t \times B^t}((x_1 * (x_2 * x_3), y_1 * (y_2 * y_3)), \mu_{A^t \times B^t}((x_2, y_2)\}$$
$$= min\{\mu_{A^t \times B^t}((x_1, y_1) * (x_2 * x_3, y_2 * y_3)), \mu_{A^t \times B^t}((x_2, y_2)\}$$
$$= min\{\mu_{A^t \times B^t}((x_1, y_1) * ((x_2, y_2) * (x_3, y_3))), \mu_{A^t \times B^t}((x_2, y_2)\}$$

$$\Rightarrow \mu_{A^t \times B^t}((x_1, y_1) * (x_3, y_3)) \geq min\{\mu_{A^t \times B^t}((x_1, y_1) * ((x_2, y_2) * (x_3, y_3))), \mu_{A^t \times B^t}((x_2, y_2)\}$$

Similarly we can show that

(iv)

$$\nu_{A^t \times B^t}((x_1, y_1) * (x_3, y_3)) \leq \max\{\nu_{A^t \times B^t}((x_1, y_1) * ((x_2, y_2) * (x_3, y_3))), \nu_{A^t \times B^t}((x_2, y_2)\}$$

Hence $A^t \times B^t =< \mu_{A^t \times B^t}, \mu_{A^t \times B^t} >$ is a t-intuitionistic fuzzy H-ideal of $X \times X$.

Corollary 3.26: If $A^t =< \mu_{A^t}, \nu_{A^t} >$ and $B^t =< \mu_{B^t}, \nu_{B^t} >$ be two t-intuitionistic fuzzy H-ideals of BCK-algebra X. Then $\Box(A^t \times B^t)\, \Diamond(A^t \times B^t), F_{\alpha,\beta}(A^t \times B^t)$ are also t-intuitionistic fuzzy H-ideals of $X \times X$.

4. CONCLUSION

We introduced the notion of t-intuitionistic fuzzy H-ideal of BCK-algebra and we studied several properties. We observe that If t=1 the t-intuitionistic fuzzy H-ideal of BCK-algebra becomes a intuitionistic fuzzy H-ideal of BCK-algebra. Also we studied the effect of some Modal operators on intuitionistic fuzzy H-ideal. In our opinion, these definitions and results can be extended to other algebraic systems also.

5. REFERENCES

1. Zadeh L. A. Fuzzy sets. *Information and Control*. 1965; 8, 338-353.
2. Atanassov K. T. Intuitionistic Fuzzy Sets. *VII ITKR's Session, Sofia,(Deposed in Central Sci. Techn. Library of Bulg. Acad. of Sci.*1983 June, 1697/84.
3. Rosenfeld A. Fuzzy subgroups. *J Math Anal Appl*. 1971; 35, 512–517.
4. Jun Y. B. and Kim K. H. Intuitionistic fuzzy ideals of BCK-algebras. *Int. J. Math. Math.Sci*. 2000; 24, 839-849.

5. Jun Y.B., Davvaz B. and Khan A. Filters of ordered semigroups based on the fuzzy points. *Journal of Intelligent and Fuzzy Systems*. 2013; 24(3), 619–630.
6. Kuroki N. Fuzzy bi-ideals in semigroups. *Comment Math Univ St Pauli*. 1979; 28, 17–21.
7. Kim C. B. and Kim H. S. On BG-algebras. *Demonstratio Mathematica*. 2008; 41(3), 497-505.
8. Barbhuiya S. R. Intuitionistic Q-fuzzy Ideals of BG-Algebra. *International Journal of Computer Applications*. 2014; 94(16), 8-14.
9. Barbhuiya S. R. Doubt fuzzy ideals of BF-algebra. *IOSR Journal of Mathematics (IOSR- JM)*. 2014; Ver. VII (Mar-April) ; 10(2), pp. 65-70.
10. Barbhuiya S. R. $(\in,\in\vee q)$-Intuitionistic Fuzzy Ideals of BG-algebra. *Fuzzy Information and Engineering (Elsevier)*. 2015; 7(1), 31-48.
11. Barbhuiya S. R. Modal Operator $\square_{\alpha,\beta,\gamma,\delta,\varepsilon,\zeta}$ On Intuitionistic Fuzzy BG -algebras. *Annals of Fuzzy Mathematics and Informatics*. 2016 February; 11(2) , 241-257.
12. Imai Y. and Iseki K. On Axiom systems of Propositional calculi XIV. *Proc, Japan Academy*. 1966; 42, 19-22.
13. Iseki K. On some ideals in BCK-algebras. *Math.Seminar Notes*. 1975; 3, 65-70.
14. Khalid H. M. and Ahmad B. Fuzzy H-ideals in BCI-algebras. *Fuzzy Sets and Systems*. 1999; 101, 153-158.
15. Satyanarayan B., Madhavi U. B. and Prasad R. D. On intuitionistic fuzzy H-ideals in BCK-algebras. *Int. J. Algebra*. 2010; 4(13-16), 743-749.
16. Atanassov K. T. More on intuitionistic Fuzzy Sets. *Fuzzy sets and systems*. 1989; 33(1), 37- 45.
17. Atanassov K. T. Two operators on intuitionistic fuzzy sets. *Comptes Rendus de Academie bulgare des Sciences*. 1988; 41 (5), 35-38.
18. Atanassov K. T. On the Model Operators defined over the intuitionistic Fuzzy Sets. *Notes on intuitionistic Fuzzy Sets*. 2004; 10(1), 7-12.
19. Atanassov K. T. On the new intuitionistic fuzzy operator $X_{a,b,c,d,e,f}$. *Notes on intuitionistic Fuzzy Sets*. 2010; 16(2), 35-38.
20. Atanassov K. T. A new intuitionistic fuzzy Model operator. Notes on intuitionistic Fuzzy Sets. 2003; 9(2), 56-60.
21. Barbhuiya S. R. t-Intuitionistic fuzzy subalgebras of BG-Algebra. *Advanced Trends in Mathematics*. 2015; 2(2), 1-13.
22. Harlenderova M. and Olomouc J. R. Model Operators on MV-algebra. *Mathematica Bohemica*. 2006; 131(1), 39-48.
23. Sharma P. K. Model Operator $F_{\alpha,\beta}$ in Intuitionistic Fuzzy Groups. *Annals of Pure and Applied Mathematics*. 2014; 7(1), 19-28.
24. Sharma P. K. t-Intuitionistic Fuzzy Quotient Group. *Advances in Fuzzy Mathematics*. 2012; (1), 1-9.
25. Sharma P. K. t -Intuitionistic Fuzzy Subrings. *IJMS*. 2012; 11(3-4), 265-275.
26. Cuvalcioglu G. The extension of modal operators' diagram with last operators. *Notes on Intuitionistic Fuzzy Sets*. 2013; 19(3), 56-61.
27. Dencheva K. Extension of intuitionistic fuzzy modal operators $\oplus$ and $\otimes$. *Proc. of the Second Int. IEEE Symp. On Intelligent Systems, Varna,*. 2004 June 22-24; 3, 21-22.
28. Cuvalcioglu G. On the diagram of one type modal operators on intuitionistic fuzzy sets: last expanding with $Z_{\alpha,\beta}^{\omega,\theta}$. *Iranian J. of Fuzzy Systems*. 2013; 10(1), 89-106.
29. Atanassov K. T. Intutionistic fuzzy sets. *Fuzzy sets and Systems*. 1986; 1, 87-96.
30. Atanassov K. T. On Intuitionistic Fuzzy Sets Theory. Published by Springer-Verlag Berlin: Heidelberg, 2012.