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ESTIMATION OF AVAILABILITY AND COST FOR COMPLEX SYSTEM WORKING IN DIFFERENT CLIMATES

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Abstract

In the present study, the authors try to evaluate the availability and cost function for a complex system which works under different climatic conditions. The complete system is made of two distinct parallel redundant components. The system works in reduced efficiency on failure of any of the redundant unit. The repair arrangement is immediate available for this failed unit. In case of failure of both units, there is no output and system has to put on waiting in availing the repair facilities. The failure and repair rates will be different for working of system in different climates. The mathematical expressions for profit function, various transition-states probabilities and availability for the complete system have been computed. To enhance practical utilization of the present model, time independent behavior of the system and a particular case (when all repair rates follow exponential distribution of time) have also been obtained. A numerical computation and its graphical representation is highlighted at the end to showcase important results of the study.

Keywords: Parallel redundancy, Markovian process, supplementary variables, steady-state behavior, availability, cost function.

1. INTRODUCTION

The considered system is non-Markovian, therefore the authors introduce a few supplementary variables to make this system Markovian. By using a limiting procedure and continuity argument, difference-differential equations for the various transition-states depicted in fig-1, have been obtained. The so obtained mathematical formulation is being solved with the help of Laplace transform. All the failure rates are distributed exponentially while all the repair rates are distributed generally. The complete system may fail because of human error.

The results of this study are of much importance for various systems of practical utility and can be used as it is for the similar configurations. For example, it is the time of computer and we may found several parts of it with similar configuration that ability differs for different climate conditions. If we do not care for these parts, it causes a big loss not only of important data saved inside it but also of money and time to remove such problems.
Also we can consider the example of airplane as it consists various parts within it of similar configuration and their ability differs with different climate conditions. If we do not care for this, it may cause a big loss not only of money but also of many lives that travel in it. Thus, we can use the results of present study to those sensitive parts to prevent a big loss.

2. ASSOCIATED ASSUMPTIONS:

The assumptions have been taken into account in this study are as follows:

(i) Initially all the units of system are working with full efficiency and are good.
(ii) Immediate repair can be provided to system in case of single unit failure while system put to wait for repair purpose when both units are in failed condition.
(iii) All repair rates are distributed generally.
(iv) All waiting and failure rates are distributed exponentially.
(v) The system works as new after availing repair facilities.
(vi) The complete system can also fail because of human generated errors.

3. NOTATIONS USED:

$\lambda_i(i=1,2--n)$		Rate of failure of first unit when it works in i^{th} climate condition.
$\mu_i(i=1,2--n)$		Rate of failure of second unit when it works in i^{th} climate condition.
w		Rate of wait to repair in case of failure of both units.
h		Rate of Human generated error.
$\gamma_i(y)\Delta / \beta_i(x)\Delta$ $(i=1,2,--n)$		While the system is working in i^{th} climate condition, the first order probability that one /two units of the system will be repaired in the time interval $(y, y+\Delta)$/ $(x, x+\Delta)$ under the condition that it was unrepaired up to the time y/x,.
$\alpha_h(z)\Delta$		The first order probability that the human generated error will be repaired in the time interval $(z, z+\Delta)$, under the condition that it was unrepaired up to the time z.
$P_0(\mathrm{t})$	:	Pr {system is all OK and operable at time t}.
$P_i^1(x,t)\Delta$	:	Pr {system is in reduced efficiency due to failure of one unit at time t}. Repair time consumed for this failure in i^{th} climate condition, lies in interval $(x, x+\Delta)$.

$P_i^2(t)$	:	Pr {system is fail at time t due to failure of both units of the system}. It is working in ith climate condition and it put on wait for repair.
$P_i^{2R}(y,t)\Delta$	:	Pr {the system is ready to avail repair of two units at time t}. Repair time consumed for this in ith climate condition lies in interval $(y, y+\Delta)$.
$P_h(z,t)\Delta$	:	Pr {the system is fail due to human generated error at time t}. Repair time consumed lies in interval $(z, z+\Delta)$.

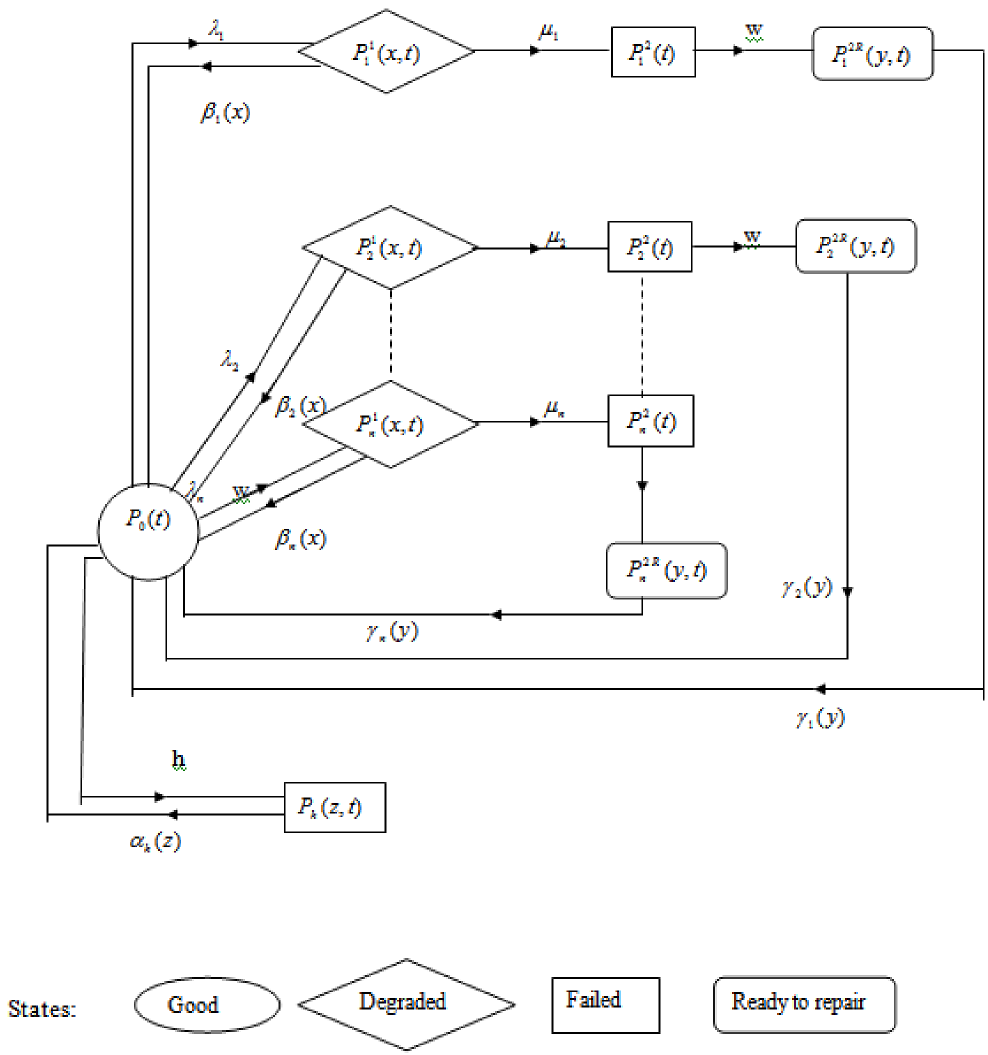

Fig-1 Transition of States-Diagram

4. MATHEMATICAL MODEL FORMULATION

By using limiting procedure and continuity argument, we formulate the following set of difference-differential equations, which is continuous in time and discrete in space, further governing of the behavior of the states is depicted in fig-1 of the considered system:

$$\left[\frac{d}{dt}+\sum_{i=1}^{n}\lambda_i+h\right]P_0(t)=\sum_{i=1}^{n}\int_0^\infty P_i^1(x,t)\beta_i(x)dx+\sum_{i=1}^{n}\int_0^\infty P_i^{2R}(y,t)\gamma_i(y)dy+\int_0^\infty P_h(z,t)\alpha_i(z)dz \quad (1)$$

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\mu_i+\beta_i(x)\right]P_i^1(x,t)=0,\quad \forall i=1,2,-----n \quad (2)$$

$$\left[\frac{d}{dt}+w\right]P_i^2(t)=\int_0^\infty P_i^1(x,t)\mu_i dx,\quad \forall i=1,2,-----n \quad (3)$$

$$\left[\frac{\partial}{\partial y}+\frac{\partial}{\partial t}+\gamma_i(y)\right]P_i^{2R}(y,t)=0,\quad \forall i=1,2,-----n \quad (4)$$

$$\left[\frac{\partial}{\partial z}+\frac{\partial}{\partial t}+\alpha_h(z)\right]P_h(z,t)=0 \quad (5)$$

Boundary conditions are:

$$P_i^1(0,t)=\lambda_i P_0(t),\quad \forall i=1,2,-----n \quad (6)$$

$$P_i^{2R}(0,t)=wP_i^2(t),\quad \forall i=1,2,-----n \quad (7)$$

$$P_h(0,t)=hP_0(t) \quad (8)$$

Initial conditions are:

$$P_0(t)=1,\ \text{otherwise zero} \quad (9)$$

5. SOLUTION OF THE MODEL

We shall solve the above set of equations for various states of fig-1 with the aid of Laplace transform to compute probabilities of various transition states. Therefore, by taking Laplace transform of equations (1) through (8) by the use of initial conditions (9), we obtained:

$$\left[s+\sum_{i=1}^{n}\lambda_i+h\right]\overline{P}_0(s)=1+\sum_{i=1}^{n}\int_0^\infty \overline{P}_i^1(x,s)\beta_i(x)dx+\sum_{i=1}^{n}\int_0^\infty \overline{P}_i^{2R}(y,s)\gamma_i(y)dy+\int_0^\infty \overline{P}_h(z,s)\alpha_h(z)dz \quad (10)$$

$$\left[\frac{\partial}{\partial x}+s+\mu_i+\beta_i(x)\right]\overline{P}_i^1(x,s)=0,\quad \forall i=1,2,-----n \quad (11)$$

$$[s+w]\overline{P}_i^2(s)=\mu_i\overline{P}_i^1(s),\quad \forall i=1,2,-----n \quad (12)$$

$$\left[\frac{\partial}{\partial y}+s+\gamma_i(y)\right]\overline{P}_i^{2R}(y,s)=0,\quad \forall i=1,2,-----n \quad (13)$$

$$\left[\frac{\partial}{\partial z}+s+\alpha_h(z)\right]\overline{P}_h(z,s)=0 \quad (14)$$

$$\overline{P}_i^1(0,s)=\lambda_i\overline{P}_0(s),\quad \forall i=1,2,-----n \quad (15)$$

$$\overline{P}_i^{2R}(0,s) = w\overline{P}_i^{2}(s), \quad \forall i = 1,2,-----n \tag{16}$$

and $\overline{P}_h(0,s) = h\overline{P}_0(s)$ (17)

Now, we get on integration of equation (11) by using equation (15) as $\forall i = 1,2,-----n$:

$$\overline{P}_i^{1}(x,s) = \overline{P}_i^{1}(0,s)e^{-(s+\mu_i)x-\int \beta_i(x)dx}$$

$$= \lambda_i \overline{P}_0(s)e^{-(s+\mu_i)x-\int \beta_i(x)dx}$$

$$\Rightarrow \overline{P}_i^{1}(s) = \lambda_i \overline{P}_0(s)\frac{1-\overline{S}_{\beta_i}(s+\mu_i)}{s+\mu_i}$$

or $\overline{P}_i^{1}(s) = \lambda_i \overline{P}_0(s)D_{\beta_i}(s+\mu_i)$ say

$$\forall i = 1,2,-----n \tag{18}$$

Equation (12) using (18) gives on simplification and:

$$\overline{P}_i^{2}(s) = \frac{\lambda_i\mu_i}{(s+w)}\overline{P}_0(s)D_{\beta_i}(s+\mu_i) \tag{19}$$

$$\forall i = 1,2,-----n$$

Next, integration of equation (13) by using (16) and (19), gives:

$$\overline{P}_i^{2R}(y,s) = \overline{P}_i^{2R}(0,s)e^{-sy-\int \gamma_i(y)dy}$$

$$= w\overline{P}_i^{2}(s)e^{-sy-\int \gamma_i(y)dy}$$

$$\Rightarrow \overline{P}_i^{2R}(s) = w\overline{P}_i^{2}(s)\frac{1-\overline{S}_{\gamma_i}(s)}{s}$$

$$= w\overline{P}_i^{2}(s)D_{\gamma_i}(s) \text{ say}$$

or, $$\overline{P}_i^{2R}(s) = \frac{\lambda_i\mu_i w}{(s+w)}\overline{P}_0(s)D_{\beta_i}(s+\mu_i)D_{\gamma_i}(s) \tag{20}$$

$\forall i = 1,2,-----n$

Again integration of equation (14) together with (17) gives:

$$\overline{P}_h(z,s) = \overline{P}_h(0,s)e^{-sz-\int \alpha_h(z)dz}$$

$$= h\overline{P}_0(s)e^{-sz-\int \alpha_h(z)dz}$$

$$\Rightarrow \overline{P}_h(s) = h\overline{P}_0(s)\frac{1-\overline{S}_h(s)}{s}$$

or, $$\overline{P}_h(s) = h\overline{P}_0(s)D_h(s) \text{ say} \tag{21}$$

Finally, on simplifying equation (10) using relevant results, generates:

$$\overline{P}_0(s) = \frac{1}{A(s)}$$

where $$A(s) = s + \sum_{i=1}^{n}\lambda_i + h - \sum_{i=1}^{n}\lambda_i\overline{S}_{\beta_i}(s+\mu_i) - h\overline{S}_h(s)$$

$$-\sum_{i=1}^{n}\frac{\lambda_i\mu_i w}{(s+w)}D_{\beta_i}(s+\mu_i)\overline{S}_{\gamma_i}(s) \tag{22}$$

Thus, we obtained the following Laplace transforms of various transition-states (depicted in fig-1) probabilities:

$$\overline{P}_0(s) = \frac{1}{A(s)} \tag{23}$$

$$\overline{P}_i^1(s) = \frac{\lambda_i}{A(s)} D_{\beta_i}(s+\mu_i), \quad \forall i = 1,2,-----n \tag{24}$$

$$\overline{P}_i^2(s) = \frac{\lambda_i \mu_i}{A(s)(s+w)} D_{\beta_i}(s+\mu_i) \quad \forall i = 1,2,-----n \tag{25}$$

$$\overline{P}_i^{2R}(s) = \frac{\lambda_i \mu_i w}{A(s)(s+w)} D_{\beta_i}(s+\mu_i) D_{\gamma_i}(s) \quad \forall i = 1,2,-----n \tag{26}$$

$$\overline{P}_h(s) = \frac{h}{A(s)} D_h(s) \tag{27}$$

Where, $D_i(j) = \dfrac{i}{i+j}, \forall\ i \ \ and \ \ j$

and $A(s)$ has mentioned in equation (22).

It is of much importance to noticing that

$$\overline{P}_0(s) + \sum_{i=1}^{n}\left[\overline{P}_i^1(s) + \overline{P}_i^2(s) + \overline{P}_i^{2R}(s)\right] + \overline{P}_h(s) = \frac{1}{s} \tag{28}$$

6. TIME INDEPENDENT BEHAVIOR OF THE SYSTEM

By using final value theorem, viz., $\underset{t\to\infty}{Lim}\, F(t) = \underset{s\to 0}{Lim}\, s\overline{F}(s) = F(say)$, provided the limit on right exists, in equations (23) through (27), we obtain the following time independent probabilities:

$$P_0 = \frac{1}{A'(0)} \tag{29}$$

$$P_i^1 = \frac{\lambda_i}{A'(0)} D_{\beta_i}(\mu_i), \quad \forall i = 1,2,-----n \tag{30}$$

$$P_i^2 = \frac{\lambda_i \mu_i}{wA'(0)} D_{\beta_i}(\mu_i) \quad \forall i = 1,2,-----n \tag{31}$$

$$P_i^{2R} = \frac{\lambda_i \mu_i}{A'(0)} D_{\beta_i}(\mu_i) M_{\gamma_i} \quad \forall i = 1,2,-----n \tag{32}$$

$$and \quad P_h = \frac{h}{A'(0)} M_h \tag{33}$$

Where, $A'(0) = \left[\dfrac{d}{ds} A(s)\right]_{s=0}$

and $M_a = -\overline{S}'_a(0) =$ mean time to repair a^{th} component.

7. PARTICULAR CASE

When all repair rates are distributed exponentially:

In this case, setting $\overline{S}_a(b) = \dfrac{a}{b+a}$ for all a and b, in equations (23-27), we can have the following transition – states probabilities:

$$\overline{P}_0(s) = \frac{1}{B(s)} \tag{34}$$

$$\overline{P}_i^1(s) = \frac{\lambda_i}{B(s)} \cdot \frac{1}{s+\mu_i+\beta_i}, \quad \forall i = 1,2,-----n \tag{35}$$

$$\overline{P}_i^2(s) = \frac{\lambda_i \mu_i}{B(s)(s+w)} \cdot \frac{1}{s+\mu_i+\beta_i} \quad \forall i = 1,2,-----n \tag{36}$$

$$\overline{P}_i^{2R}(s) = \frac{\lambda_i \mu_i w}{B(s)(s+w)} \cdot \frac{1}{s+\mu_i+\beta_i} \frac{1}{s+\gamma_i}, \quad \forall i = 1,2,-----n \tag{37}$$

and $$\overline{P}_h(s) = \frac{h}{B(s)} \cdot \frac{1}{s+\alpha_h} \tag{38}$$

Where,

$$B(s) = s + \sum_{i=1}^{n} \lambda_i + h - \sum_{i=1}^{n} \frac{\lambda_i \beta_i}{s+\mu_i+\beta_i} - \frac{h\alpha_h}{s+\alpha_h} - \sum_{i=1}^{n} \frac{\lambda_i \mu_i w \gamma_i}{(s+w)(s+\mu_i+\beta_i)(s+\gamma_i)}$$

8. AVAILABILITY OF SYSTEM

We have

$$P_{up}(s) = \overline{P}_0(s) + \sum_{i=1}^{\eta} \overline{P}_i^1(s)$$

Putting the values on R.H.S and on applying inverse Laplace transform, we may obtain the up state probability of the complete system and is given by:

$$P_{up}(t) = (1+E)e^{-(\lambda+h)t} - Ee^{-\mu t} \tag{40}$$

where, $\sum_{i=1}^{n} \lambda_i = \lambda, \sum_{i=1}^{n} \mu_i = \mu$

and $$E = \frac{\lambda}{\mu - \lambda - h} \tag{41}$$

Also, the down state probability is

$$P_{down}(t) = 1 - P_{up}(t) \tag{42}$$

It is interesting to note here that $P_{up}(0) = 1$

9. COST FUNCTION FOR THE SYSTEM

The formula to compute cost function for the system is

$$G(t) = C_1 \int_0^t P_{up}(t)dt - C_2 t \tag{43}$$

where, C_1 is the total revenue per unit time and C_2 is the repair cost per unit time. Using (40), equation (43) gives:

$$G(t) = C_1 \left\{ \frac{(1+E)}{(\lambda+h)} \left[1 - e^{-(\lambda+h)t}\right] - \frac{E}{\mu}\left[1 - e^{-\mu t}\right] \right\} - C_2 t \tag{44}$$

where, E, λ and μ are explained earlier.

10. NUMERICAL ILLUSTRATION

To observe the variations in the values of profit function and availability w.r.t. the time 't', let us choose the numerical computation as:
$\lambda = 0.01, \mu = 0.02, h = 0.03, C_1 = Rs7.00$ / unit time, $C_2 = Rs2.00$ / unit time and t=0,1,2----------.
Using these values in equation (40), one can compute the table-1 and the corresponding graph has shown in fig-2. By using this numerical illustration in equation (44), we compute the table -2 and the corresponding graph has shown through fig-3.

11. RESULTS AND DISCUSSION:

In present study, author has computed important parameters of reliability measures for a system which is working in different climates. "Inclusion of supplementary variables" technique for formulation of mathematical equations and Laplace transform for solution of equations have used to get requisite results. Time independent behavior of the system and a special case has also calculated to enhance the practical importance of the model. A numerical computation has considered highlighting important results.

Fig-2 reveals that the availability of system is decreasing smoothly and in uniform manner as we make increase in time *t*. A critical examination of Fig-3 yields that cost function increases approximately in constant manner with time *t*.

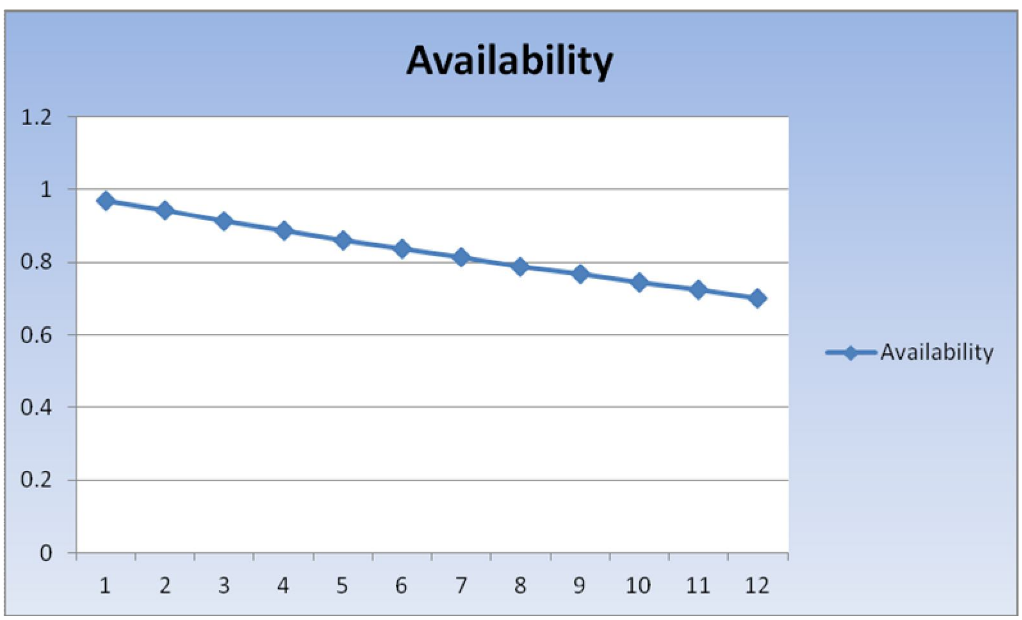

Fig. 2: Showing Availability Vs time

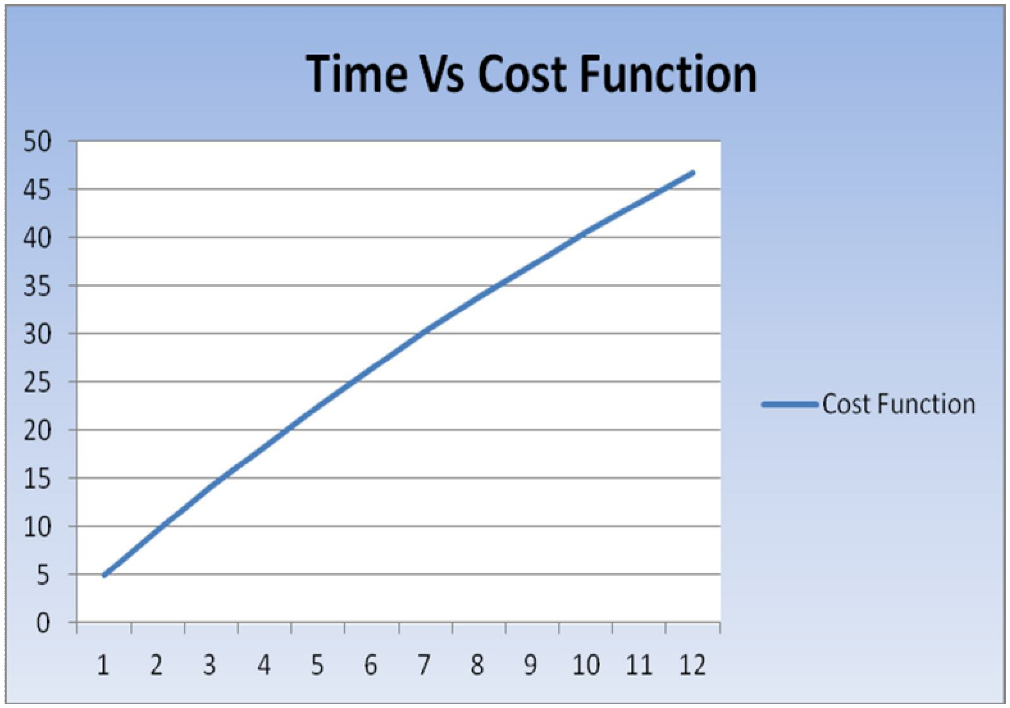

Fig 3: Cost Function Vs Time

REFERENCES

1. Cluzeau, T.; Keller, J.; Schneeweiss, W. (2008): "An Efficient Algorithm for Computing the Reliability of Consecutive-k-Out-Of-n: F Systems", IEEE TR. on Reliability, Vol.57 (1), 84-87.
2. Gupta P.P., Agarwal S.C. (1983): "A Boolean Algebra Method for Reliability Calculations", Microelectron. Reliability, Vol.23, 863-865.
3. Lai C.D., Xie M., Murthy D.N.P. (2005): "On Some Recent Modifications of Weibull Distribution", IEEE TR. on Reliability, Vol.54 (4), 563-569.
4. Tian, Z.; Yam, R. C. M.; Zuo, M. J.; Huang, H.Z. (2008): "Reliability Bounds for Multi-State k-out-of- n Systems", IEEE TR. on Reliability, Vol.57 (1), 53-58.
5. Zhimin He., Han T.L., Eng H.O. (2005): "A Probabilistic Approach to Evaluate the Reliability of Piezoelectric Micro-Actuators", IEEE TR. on Reliability, Vol.54 (1), 44-49.