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# DIFFUSION THERMO AND THERMAL DIFFUSION EFFECTS ON MHD FREE CONVECTION FLOW OF RIVLIN-ERICKSEN FLUID PAST A SEMI INFINITE VERTICAL PLATE

P. Chandra Reddy\*, M.C. Raju\*, G.S.S. Raju\*\*, C. Madhava Reddy\*\*\*

#### **Author Affiliation:**

- \*Department of Mathematics, Annamacharya Institute of Technology and Sciences (Autonomous), Rajampet-516126, A.P., India.
- \*\*Department of Mathematics, JNTUA College of Engineering, Pulivendula, A.P., India.
- \*\*\*Department of Mathematics, S.V. University, Tirupati 517502, A.P, India.

## **Corresponding Author:**

**P. Chandra Reddy,** Department of Mathematics, Annamacharya Institute of Technology and Sciences (Autonomous), Rajampet-516126, A.P., India.

E-mail: chandramsc01@gmail.com

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## **Abstract**

An attempt is made to study diffusion thermo and thermal diffusion effects on MHD free convection flow of Rivlin-Ericksen fluid past a semi infinite vertical plate with constant mass flux numerically. A uniform transverse magnetic field is applied perpendicular to the plate. The presence of viscous dissipation is also considered. The dimensionless governing equations are coupled and non linear system of partial differential equations which are solved by finite difference method. The effects of various physical parameters viz. Magnetic parameter (M), Viscoelastic parameter ( $\lambda$ ), Grashof number (G), modified Grashof number (Gc), Eckert number (E), Prandtl number (Pr), Schmidt numbere (Sc), Soret number (So), Dufour number (Df) on velocity, temperature and concentration are studied. Variations in skin friction, Nusselt number and Sherwood number are also discussed. The results are presented with the help of graphs. It is noticed that the velocity distribution increases for increasing values of G, Gc,  $\lambda$ , Df, So, E and decreasing values of M, Pr, Sc. The temperature rises with increasing values of Df, Sc, E where as the temperature falls down for increasing values of So and Pr. The concentration increases for increasing values of So and Pr, but a reverse effect is found in the case of Df, Sc and E.

**Keywords**: MHD, free convection flow; Rivlin–Ericksen fluid; constant mass flux;

## 1. INTRODUCTION

MHD free convection flows frequently occurs in petro-chemical industry, chemical vapor deposition on surface, cooling of nuclear reactors, heat exchanger design, forest fire dynamics and geophysics as well as magneto-hydrodynamic power generation systems. The unsteady magneto-hydrodynamic free convection flows past an infinite plate have received much attention because of non-linearity of the governing equations. The Rivlin-Ericksen elastic-viscous fluid has relevance and importance in geophysical fluid dynamics, industries and chemical technology. Anandareddy et al. [1] studied unsteady free convective MHD non Newtonian flow through a porous medium bounded by an infinite inclined porous plate. Naby et al. [2-3] presented finite difference solution of radiation effects on MHD unsteady free convection flow over vertical porous plate and also with variable surface temperature. Finite difference schemes are explained by Conte and Boor [4]. MHD Fluid flow problems – A study of magnetic field effects on some flows in channels and past infinite plates was employed by Raju et al. [5]. Network simulation method applied to radiation and viscous dissipation effects on MHD unsteady free convection over vertical porous plate was analyzed by Jordan [6]. Roy et al. [7] explained non-similar solution of an unsteady mixed convection flow over a vertical cone with suction or injection. Chamkha et al. [8] considered free convection flow over a truncated cone embedded in a porous medium saturated with pure or saline water at low temperatures. Mansour et al. [9] analyzed combined heat and mass transfer in natural convection flow on a vertical cylinder in a micropolar fluid. Elbashbeshy [10] studied free convection flow with variable viscosity and thermal diffusivity along a vertical plate in the presence of the magnetic field. Ganesan and Rani [11] discussed unsteady free convection MHD flow past a vertical cylinder heat and mass transfer. Seddeek and Abdelmeguid [12] analyzed the effects of radiation and thermal diffusivity on heat transfer over a stretching surface with variable heat flux. Sahoo et al. [13] studied magneto hydrodynamic unsteady free convection flow past an infinite vertical plate with constant suction and heat sink. Raptis [14] analyzed unsteady free convection flow through a porous medium. Kinyanjui et al. [15] employed finite difference analysis of free convection effects on MHD problem for a vertical plate in a dissipative rotating fluid system with constant heat flux and Hall current. Ravi kumar et al. [16] studied combined effects of heat absorption and MHD on convective Rivlin-Ericksen flow past a semi-infinite vertical porous plate with variable temperature and suction. Humera Noushima et al. [17] analyzed hydromagnetics free convective Rivlin-Ericksen flow through a porous medium with variable permeability. Thermal instability of compressible Rivlin-Ericksen rotating fluid permeated with suspended dust particles in porous medium have been studied by Rana [18]. Uwanta et al. [19] examined the effects of mass transfer on hydro magnetic free convective Rivlin-Ericksen flow through a porous medium with time dependent suction. Varshney et al. [20] the effects of rotatory Rivlin-Ericksen fluid on MHD free convective and mass transfer flow through porous medium with constants heat and mass flux across moving plate.

When heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and the driving potentials are of a more intricate nature. It has been observed that energy flux can be generated not only by temperature gradients but also by concentration gradients. The energy flux caused by a concentration gradient is termed the diffusion-thermo (Dufour) effect. On the other hand, mass flux can also be generated by temperature gradients and this embodies the thermal-diffusion (Soret) effect. In several studies related to heat and mass transfer process, Soret and Dufour effects are neglected on the basis that they are of a smaller order of magnitude than the effects described by Fourier's and Fick's laws. But these effects are considered as second order phenomena and may become significant in the areas such as hydrology, petrology, geosciences, etc. The Soret effect has been utilized for isotope separation and in mixture between gases with very light molecular weight and of medium molecular weight. The Dufour effect was found to be of order of considerable magnitude so that it cannot be neglected (Eckert and Drake [21]). Srinivasacharya and Ram Reddy [22] established Soret and Dufour effects on mixed convection in a non-Darcy porous medium. Unsteady MHD free convection and chemically reactive flow past an infinite vertical porous plate as well as MHD thermal diffusion natural convection flow between heated inclined plates in porous medium have been analysed by Raju et al. [23, 24]. Srinivasacharya and Upendar Mendu [25] discussed Soret and Dofour effects on MHD free convection in a micropolar Fluid. Soret and Dufour effects on mixed convection from a vertical plate in power-law fluid saturated porous medium as well as free convection in MHD micropolar fluid have presented by Srinivasacharya et al. [26,27]. Dursunkaya and Worek [28] presented diffusion-thermo and thermal-diffusion effects in transient and steady natural convection from a vertical surface, whereas Kafoussias and Williams [29] discussed the same effects on mixed convective and mass transfer steady laminar boundary layer flow over a vertical flat plate with temperature dependent viscosity. Postelnicu [30] studied numerically the influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media taking into account Soret and Dufour effects. Both free and forced convection boundary layer flows with Soret and Dufour effects have been discussed by Abreu et al. [31]. Alamet al. [32] have investigated Dufour and Soret effects on mixed convection flow past a vertical porous flat plate with variable suction. Lakshmi Narayana and Murthy [33] considered Soret and Dufour effects in a doubly stratified Darcy porous medium. Beget al.[34] analyzed numerical study of micropolar convective heat and mass transfer in a non-darcy porous regime with Soret and Dufour effects. Finite element study of natural convection heat and mass transfer in a micropolar fluid saturated porous regime with Soret/ Dufour effects was established by Rawat and Bhargava [35]. Afify [36] presented similarity solution in MHD effects of thermal diffusion and diffusion thermo on free convective heat and mass transfer over a stretching surface considering suction or injection. Beget al. [37] studied computational fluid dynamics modeling of buoyancy induced viscoelastic flow in a porous medium with magnetic field. Cheng [38] established the influence of lateral mass flux on free convection boundary layers in a saturated porous medium. Lai et al.[39] discussed the influence of lateral mass flux on mixed convection over inclined surfaces in saturated porous media. Mahdy [40] investigated Soret and Dufour effect on double diffusion mixed convection from a vertical surface in a porous medium saturated with a non-Newtonian fluid. Hayat et al. [41] discussed Soret and Dufour effects for three-dimensional flow in a viscoelastic fluid over a stretching surface. Alamet al. [42] discussed Dufour and Soret effects on MHD free convective heat and mass transfer flow past a vertical porous flat plate embedded in a porous medium. Seethamahalakshmi et al. [43] considered MHD free convective mass transfer flow past an infinite vertical porous plate with variable suction and Soret effect.

Motivated by the above studies, an investigation has been carried out to study the Soret and Dufour effects on MHD free convection flow of Rivlin-Ericksen fluid past an semi infinite vertical plate with constant mass flux. We have extended the work of Saravana et al. [44], who studied mass transfer effects on MHD viscous flow past an impulsively started infinite vertical plate with constant mass flux. Soret and Dufour effects are significant when density differences exist in the flow regime. For example, when species are introduced at a surface in a fluid domain, with a different (lower) density than the surrounding fluid, both Soret (thermal-diffusion) and Dufour (diffusion-thermo) effects can become influential. Soret and Dufour effects are important for intermediate molecular weight gases in coupled heat and mass transfer in fluid binary systems (see Kabier and Chamkha [45]). The novelty of this study is the consideration of simultaneous effects of Soret and Dufour numbers.

## 2. FORMULATION OF THE PROBLEM

Consider the flow of a viscous incompressible electrically conducting visco-elastic second order Rivlin-Ericksen fluidpast an impulsively started semi-infinite vertical plate in the presence of diffusion thermo and thermal diffusion effects with constant mass flux. Let  $\mathbf{x}^*$  axis is taken along the plate in the upward direction vertically along the plate and  $\mathbf{y}^*$  axis is chosen normal to it. It is assumed that initially the temperature of the plate  $T^*_w$  and the fluid  $T^*_\infty$ , the species concentration at the plate  $T^*_w$  and

in the fluid  $C_{\infty}^*$  to be the same. At time  $t^*>0$ , the plate temperature is changed to  $T_{w}^*$  causing convection currents to flow near the plate and mass is supplied at a constant rate to the plate and the plate starts moving upward due to impulsive motion, gaining a velocity of  $U_0$ . A uniform magnetic field of intensity  $B_0$  is applied in the y-direction. Hence, the velocity and the magnetic field are given

by q = (u, 0, 0) and  $B = (0, B_0, 0)$ . The flow beings lightly conducting the magnetic Reynolds number is very much less than unity and therefore the induced magnetic field can be neglected. Hence the governing equations of this flow are as follows.

$$\frac{\partial u^*}{\partial t^*} = g \beta \left( T^* - T_{\infty}^* \right) + g \beta^* \left( C^* - C_{\infty}^* \right) + v \frac{\partial^2 u^*}{\partial v^{*2}} + K_0^* \frac{\partial^3 u^*}{\partial v^{*2} \partial t^*} - \frac{\sigma \mu_e^2 B_0^2}{\rho} u^*$$
(1)

$$\rho C_{p} \frac{\partial T^{*}}{\partial t^{*}} = K \frac{\partial^{2} T^{*}}{\partial v^{*2}} + \mu \left( \frac{\partial u^{*}}{\partial v^{*}} \right)^{2} + \frac{D K_{T} \rho}{C_{s}} \frac{\partial^{2} C^{*}}{\partial v^{*2}}$$

$$(2)$$

$$\frac{\partial C^*}{\partial t^*} = D \frac{\partial^2 C^*}{\partial y^{*2}} + D_1 \frac{\partial^2 T^*}{\partial y^{*2}}$$
(3)

The corresponding initial and boundary conditions are

$$u^{*} = 0, T^{*} = T_{\infty}^{*}, C^{*} = C_{\infty}^{*} \qquad \text{for all } y^{*}, t^{*} \le 0$$

$$u^{*} = U_{0}, T^{*} = T_{w}^{*}, \frac{dC^{*}}{dy^{*}} = -\frac{j^{11}}{D} \quad \text{at} \quad y^{*} = 0$$

$$u^{*} = 0, T^{*} \to T_{\infty}^{*}, C^{*} \to C_{\infty}^{*} \qquad \text{as} \quad y^{*} \to \infty$$

$$(4)$$

Where  $\mathbf{u}^*$  is the fluid velocity along the plate in the  $\mathbf{x}^*$ -direction,  $\mathbf{t}^*$  is the time,  $\mathbf{g}$  is the acceleration due to gravity,  $\boldsymbol{\beta}$  is the coefficient of volume expansion,  $\boldsymbol{\beta}^*$  is the coefficient of thermal expansion with concentration,  $\mathbf{T}^*$  is the fluid temperature near the plate,  $T_{\infty}^*$  is the temperature of the fluid far away from the plate,  $T_{w}^*$  is the temperature of the fluid,  $\mathbf{C}^*$  is the species concentration in the fluid near the plate,  $C_{\infty}^*$  is the species concentration in the fluid far away from the plate,  $\mathbf{j}^{11}$  is the mass flux per unit area at the plate,  $\mathbf{v}$  is the kinematic viscosity,  $K_0^*$  is the coefficient of kinematic visco-elastic parameter,  $\mathbf{\sigma}$  is the electrical conductivity of the fluid,  $\mu_e$  is the magnetic permeability,  $\mathbf{B}_0$  is the strength of applied magnetic field,  $\mathbf{\rho}$  is the density of the fluid,  $C_p$  is the specific heat at constant pressure,  $\mathbf{K}$  is the thermal conductivity of the fluid,  $\mathbf{\mu}$  is the viscosity of the fluid,  $\mathbf{D}$  is the molecular diffusivity,  $\mathbf{U}_0$  is the velocity of the plate,  $\mathbf{K}_T$  is the thermal diffusion ratio,  $\mathbf{C}_s$  is the concentration susceptibility and  $\mathbf{D}_1$  is the thermal diffusion coefficient.

Now the following non-dimensional parameters are introduced:

$$u = \frac{u^*}{U_0}, \ t = \frac{t^* U_0^2}{v}, \ y = \frac{y^* U_0}{v}, \ \theta = \frac{T^* - T_{\infty}^*}{T_{w}^* - T_{\infty}^*}, \ C = \frac{C^* - C_{\infty}^*}{\left(j^{11} v / D U_0\right)}$$

$$G = \frac{vg\beta \left(T_w^* - T_\infty^*\right)}{U_0^3}, \text{ (Grash of number)}$$

$$Gc = \frac{vg\beta_1(j^{11}v/DU_0)}{U_0^3}$$
, (modified Grash of number)

$$\lambda = \frac{K_0^* U_0^2}{v^2}$$
, (Visco-elastic parameter)

$$M = \frac{\sigma \mu_e^2 B_0^2 v}{\rho U_0^2}, \text{ (Magnetic parameter)}$$

$$Pr = \frac{v\rho C_p}{K}, \text{ (Prandtl number)}$$

$$E = \frac{\mu U_0^2}{\nu \rho C_p \left(T_w^1 - T_\infty^1\right)}, \text{ (Eckert number)}$$

$$Df = \frac{K_T \dot{J}^{11}}{C_s C_p U_0 (T_w^* - T_\infty^*)}, (\text{Dufour number})$$

$$Sc = \frac{v}{D}$$
, (Schmidt number)

$$S_0 = \frac{DD_1 (T_w^* - T_\infty^*) U_0}{i^{11} v^2}$$
, (Soret number)

By using the above non-dimension quantities in set of Equations (1)-(3) reduces to

$$\frac{\partial u}{\partial t} = Gr \,\theta + Gc \,C + \frac{\partial^2 u}{\partial y^2} + \lambda \left(\frac{\partial^3 u}{\partial y^2 \partial t}\right) - M \,u \tag{5}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr} \frac{\partial^2 \theta}{\partial y^2} + E \left( \frac{\partial u}{\partial y} \right)^2 + D f \frac{\partial^2 C}{\partial y^2}$$
 (6)

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} + S_0 \frac{\partial^2 \theta}{\partial y^2} \tag{7}$$

The corresponding initial and boundary conditions are:

$$u = 0, \ \theta = 0, \ C = 0 \quad for \ all \ y, t \le 0$$

$$u = 1, \ \theta = 1, \ \frac{dC}{dy} = -1 \quad at \ y = 0$$

$$u \to 0, \ \theta \to 0, \ C \to 0 \quad as \quad y \to \infty$$

$$(8)$$

#### 3. METHOD OF SOLUTION

Equations (5)-(7) are coupled non-linear partial differential equations and are to be solved by using the initial and boundary conditions (8). However exact solution is not possible for this set of equations and hence we solve these equations by finite-difference method. The equivalent finite difference schemes of equations for (5)-(7) are as follows:

$$\frac{u_{i,j+1} - u_{i,j}}{\Delta t} = G \theta_{i,j} + G c C_{i,j} + \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{(\Delta y)^2} + \lambda \left( \frac{u_{i-1,j+1} - 2u_{i,j+1} + u_{i+1,j+1} - u_{i-1,j} + 2u_{i,j} - u_{i+1,j}}{\Delta t (\Delta y)^2} \right) - M u_{i,j}$$
(9)

$$\Pr\left(\frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta t}\right) = \left(\frac{\theta_{i-1,j} - 2\theta_{i,j} + \theta_{i+1,j}}{\left(\Delta y\right)^2}\right) + \Pr\left(\frac{u_{i+1,j} - u_{i,j}}{\Delta y}\right)^2 + \Pr\left(\frac{C_{i+1,j} - 2C_{i,j} + C_{i-1,j}}{\left(\Delta y\right)^2}\right)$$

$$\tag{10}$$

$$Sc\left(\frac{C_{i,j+1} - C_{i,j}}{\Delta t}\right) = \left(\frac{C_{i-1,j} - 2C_{i,j} + C_{i+1,j}}{\left(\Delta y\right)^2}\right) + Sc S_0\left(\frac{\theta_{i-1,j} - 2\theta_{i,j} + \theta_{i+1,j}}{\left(\Delta y\right)^2}\right)$$
(11)

Here, the index i refer to y and j to time. The mesh system is divided by taking  $\Delta y = 0.1$ . From equation (8), we have the following equivalent initial condition

$$u(i,0) = 0, \ \theta(i,0) = 0, C(i,0) = 0 \text{ for all } i$$
 (12)

The boundary conditions from (8) are expressed in finite-difference form as follows

$$u(0, j) = 1, \theta(0, j) = 1, C_{i-1, j} - C_{i+1, j} = -2.\Delta y \text{ for all } j$$

$$u(i_{\text{max}}, j) = 0, \theta(i_{\text{max}}, j) = 0, C(i_{\text{max}}, j) = 0 \text{ for all } j$$
(Here  $i_{\text{max}}$  was taken as 20)

First the velocity at the end of time step viz, u(i,j+1)(i=1,20) is calculated from (9) in terms of velocity, temperature and concentration at points on the earlier time-step. Then  $\theta$  (i, j +1) is computed from (10) and C (i, j +1) is computed from (11). The procedure is repeated until t = 0.5 (i.e. j = 500). During computation  $\Delta t$  was chosen as 0.001.

## **Skin-friction:**

The skin-friction in non-dimensional form is given by

$$\tau = -\left(\frac{du}{dy}\right)_{y=0}$$
, where  $\tau = \frac{\tau^1}{\rho U_0^2}$ 

#### Rate of heat transfer:

The dimensionless rate of heat transfer is given by

$$Nu = -\left(\frac{d\theta}{dy}\right)_{y=0}$$

## Rate of mass transfer:

The dimensionless rate of mass transfer is given by

$$Sh = -\left(\frac{dC}{dy}\right)_{y=0}$$

# 4. RESULTS AND DISCUSSION

In the present work a representative set of graphical results for the velocity, temperature, concentration, local skin-friction coefficient, local Nusselt number(rate of heat transfer) and the local Sherwood number(rate of mass transfer) is presented and discussed for various parameters encountered in the governing equations of the problem.

In order to assess the accuracy of our method, we have compared our results with accepted data sets for the effect of visco-elastic parameter on velocity distribution corresponding to the case computed by Saravana et al. [44]. The results of this comparison are found to be in very good agreement (see figure 1). The effects of various parameters on velocity profiles are depicted in figures 2-9. It is examined that the velocity starts from a higher value at the plate surface and decrease to the free stream value far away from the plate surface satisfying the far field boundary condition for all parameter values of the problem. Figure 2 exhibits the effect of magnetic parameter on the velocity of the fluid. It is noticed that the fluid velocity decreases with increasing values of magnetic parameter. This is because the transverse magnetic field sets in Lorentz force, which retards the fluid velocity. Figures 3& 4 display the velocity profiles with the effect of Grashof number for heat and mass transfer. It is observed that the fluid velocity increases and reaches its maximum over a short distance from the plate and then gradually reduce to zero under the increment of both the cases of Grashof number and modified Grashof number. This happens because of the presence of thermal and solutal buoyancy which has the tendency of increase in velocity. Figure 5 demonstrates effect of visco-elastic parameter on velocity. It

is clear that the velocity increases with the increasing values of visco-elastic parameter. The effect of Prandtl number on velocity is presented in figure 6. It shows that the velocity reduces for increasing values of Prandtl number. Since, fluid of low Prandtl number has high thermal diffusivity hence attains higher temperature in steady state, which in turn means more buoyancy force i.e. more fluid velocity with respect to comparatively high Prandtl fluid.

Figure 7 illustrates the effect of Schmidt number on velocity. The Schmidt number embodies the ratio of the momentum diffusivity to the species (mass) diffusivity. It physically relates the relative thickness of the hydrodynamic boundary layer and mass-transfer boundary layer. It is found that as Schmidt number increases the velocity field decreases. Figure 8 depicts the variation of the velocity boundary-layer with the Soret number. It is noticed that the velocity boundary layer thickness increases with an increase in the Soret number. Figure 9 exhibits the variation of the velocity boundary-layer with the Dufour number. It is shown that the velocity boundary layer thickness increases with an increase in the Dufour number. Figure 10 displays the influence of the Eckert number on the velocity. It is clearly revealed that the effect of Eckert number is to increase the velocity distribution in the flow region. This happens due to the fact that the heat energy is stored in liquid due to the frictional heating. Thus the effect of increasing Eckert number is to enhance the velocity of the fluid.

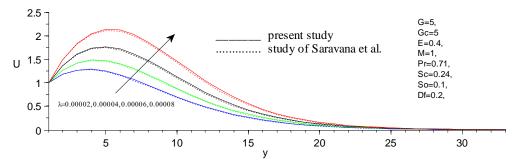


Fig 1:Comparision of the results: Effect of visco-elastic parameter on velocity distribution in the absence of Soret and Dufour effects

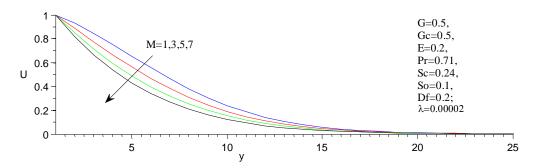


Fig 2:Effect of Magnetic parameter on velocity

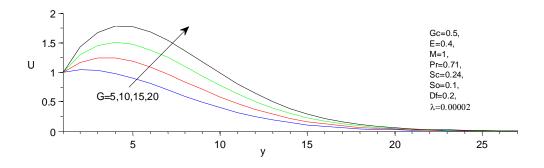


Fig 3: Effect of Grashof number for heat transfer on velocity

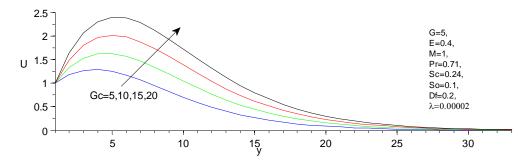


Fig 4: Effect of Grashof number for mass transfer on velocity

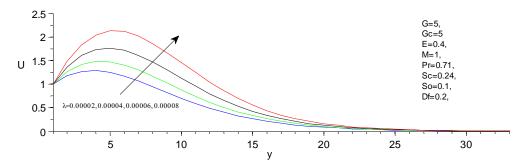


Fig 5: Effect of visco-elastic parameter on velocity

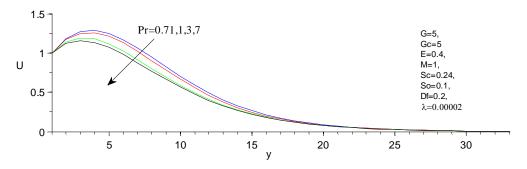


Fig. 6: Effect of Prandtl number on Velocity

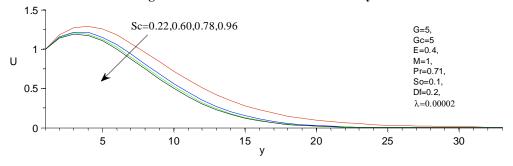


Fig 7: Effect of Schmidt number on Velocity

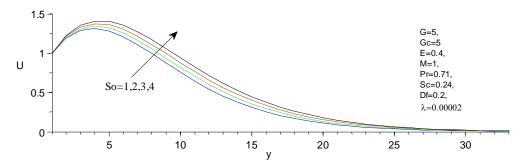


Fig 8: Effect of Soret number on Velocity

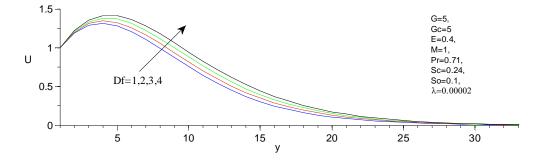


Fig 9: Effect of Dufour number on Velocity

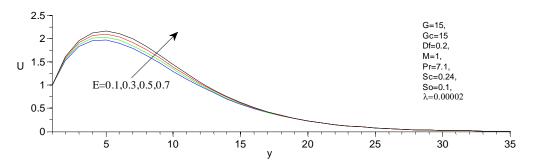


Fig 10: Effect of Eckert number on Velocity

As per the boundary conditions of the flow field under consideration, the temperature of the fluid attains its maximum value at the plate surface and decreases exponentially to the free stream zero value away from the plate. This is examined in Figures 11-15. The effect of Prandtl number on temperature is shown in figure 11. It is noticed with the increase in Prandtl number the surface temperature decreases. This would happen because reduced fluid velocity would mean heat is not convected readily and hence surface temperature decreases. The effect of Dufour number on temperature is shown in figure 12. It is evident that the temperature of the fluid rises for increasing values of Dufour number.

Figure 13 illustrates the effect of Schmidt number on the temperature. As the Schmidt number increases an increasing trend in the temperature field is noticed. This is because the fluid velocity is higher for low Schmidt number, as discussed earlier; the heat is convected readily from surface resulting in the relatively higher cooling of the surface, so surface temperature is lower for low Schmidt number. The increase in the order of reaction and reaction rate parameter practically does not vary surface temperature.

Figure 14 displays the variation of the thermal boundary-layer with the Soret number. It is noticed that the thermal boundary layer thickness decreases with an increase in the Soret number. The effect of the Eckert number on the temperature of the fluid is displayed in Figure 15. It is observed that increasing the value of the Eckert number causes increases in the fluid temperature. This increase in the fluid temperature increases the thermal buoyancy effects which induces more flow along the plate represented by increases in the linear velocity as discussed earlier.

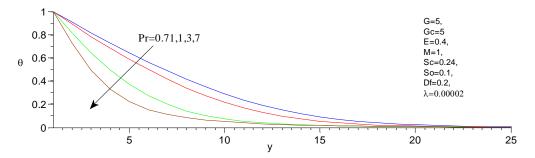


Fig 11: Effect of Prandtl number on Temperature

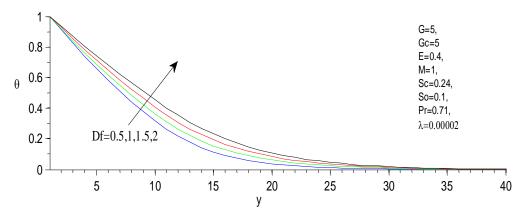


Fig 12: Effect of Dufour number on Temperature

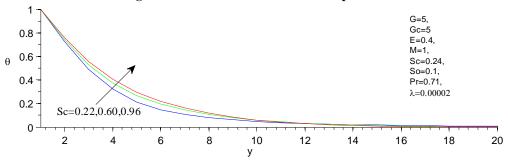


Fig 13: Effect of Schmidt number on Temperature

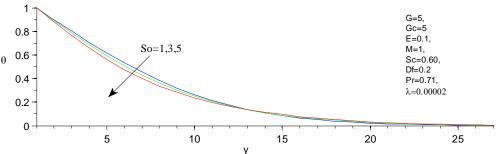


Fig 14: Effect of Soret number on Temperature

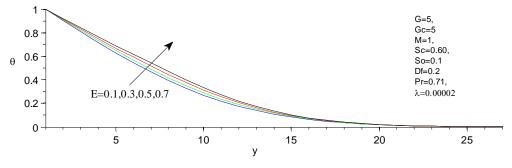


Fig 15: Effect of Eckert number on Temperature

Figure 16 shows the variation of the concentration boundary-layer with the Dufour number. It is noticed that the concentration boundary layer thickness decreases with an increase in the Dufour number. Figure 17 illustrates the effect of Schmidt number on the concentration. It is clear that as the Schmidt number increases, there is a decreasing trend in the concentration field. Not much of significant contribution of Schmidt number is found far away from the plate. Figure 18 depicts the variation of the concentration boundary-layer with the Soret number. It is observed that the concentration boundary layer thickness increases with an increase in the Soret number. Figure 19 presents the effect of Prandtl number on Concentration. It is clearly revealed that the concentration rises with increasing values of Prandtl number. Figure 20 demonstrates the effect of Eckert number on Concentration. We can observe that as the Eckert number values increases the concentration decreases.

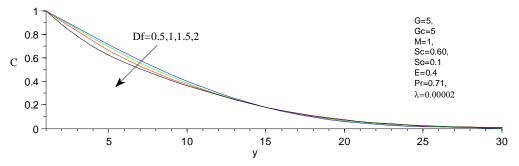


Fig 16: Effect of Dufour number on Concentration

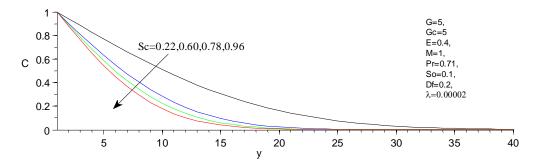


Fig 17: Effect of Schmidt number on Concentration

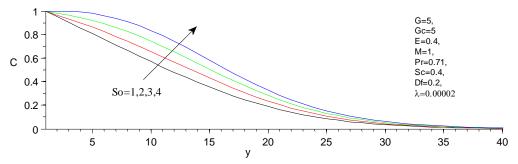


Fig 18: Effect of Soret number on Concentration

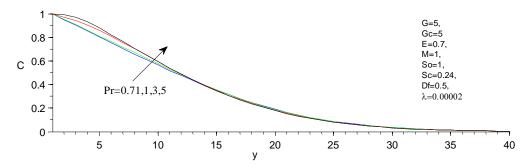


Fig 19: Effect of Prandtl number on Concentration

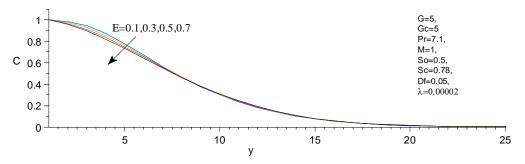


Fig 20: Effect of Eckert number on Concentration

We have also presented some graphs of the surface skin friction  $(\tau)$ , Nusselt number (Nu), and Sherwood number (Sh). The behavior of skin friction under the effect of magnetic parameter and viscoelastic parameter is presented in the figures 21-22. It is noticed that the skin friction coefficient rises with increasing values of magnetic parameter, but a reverse effect is found in the case of viscoelastic parameter. The local skin friction coefficient decreases under the influence of Dufour number as well as soret number. This is evident from figures 23-24. Figure 25 depicts theeffect of Prandtl number on rate of heat transfer. It is shown that the rate of heat transfer rises for increasing values of Prandtl number. The effect of Eckert number on rate of heat transfer is presented in figure 26. It is evident that the Nusselt number decreases with an increase in Eckert number. Figures 27-28 demonstrate the effect of Dufour number and Schmidt number on Rate of heat transfer. It is noticed that as Dufour number increases Nusselt number decreases, but a reverse effect is noticed in the case of Schmidt number. The effect of Schmidt number on Rate of mass transfer is displayed in figure 29. It is shown that as Schmidt number increases the Sherewood number increases. Figures 30-31 depict the behavior of sherwood number under the influence of Soret and Dufour numbers. It is observed that Sherwood number decreases with the increasing values of both the numbers.

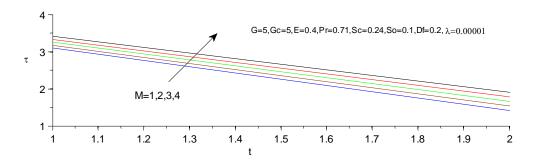


Fig 21: Effect of magnetic parameter on skin friction

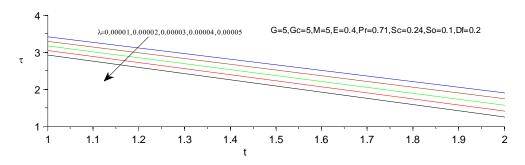


Fig 22: Effect of visco-elastic parameter on skin friction

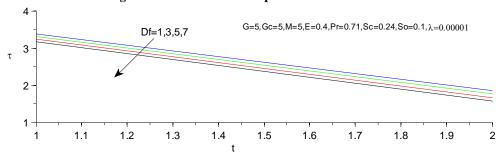


Fig 23: Effect of Dufour number on skin friction

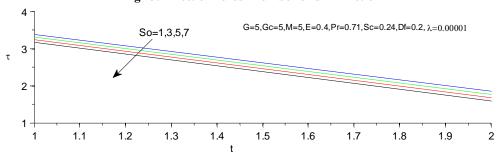


Fig 24: Effect of Soret number on skin friction

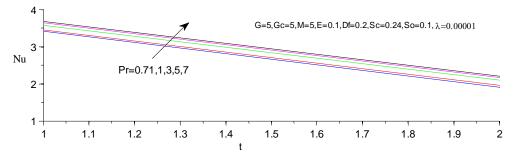


Fig 25: Effect of Prandtl number on Rate of heat transfer

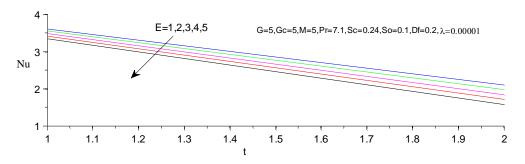


Fig 26: Effect of Eckert number on Rate of heat transfer

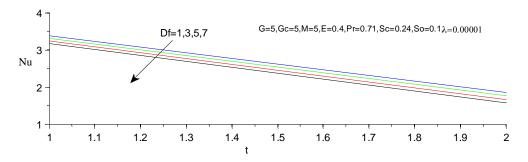


Fig 27: Effect of Dufour number on Rate of heat transfer

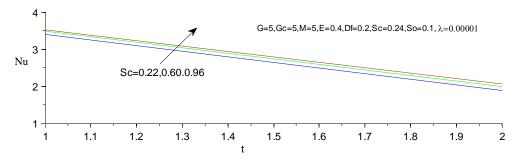


Fig 28: Effect of Schmidt number on Rate of heat transfer

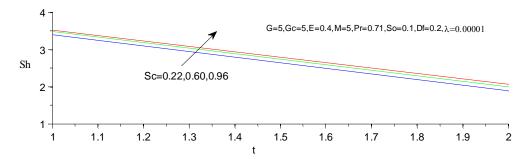


Fig 29: Effect of Schmidt number on Rate of mass transfer

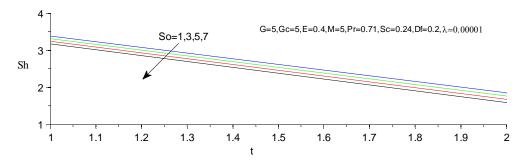


Fig 30: Effect of Soret number on Rate of mass transfer

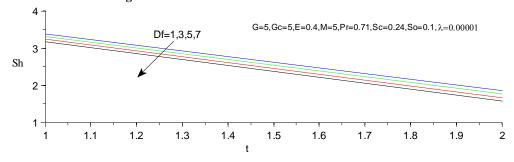


Fig 31: Effect of Dufour number on Rate of mass transfer

## 5. CONCLUSION

The governing equations for diffusion thermo and thermal diffusion effects on MHD free convection flow of Rivlin-Ericksen fluid past a semi infinite vertical plate with constant mass flux was formulated. The dimensionless governing equations were solved for velocity, temperature and concentration by using finite difference method. The effects of various parameters on velocity, temperature and concentration are studied through graphs. Comparison with previously published work was performed and the results are found to be in excellent agreement. It is found that the velocity increases for increasing values of G, Gc,  $\lambda$ , E and decreasing values of M, Pr, Sc. In the case of Soret and Dufour numbers it is observed that the velocity distribution increases with the increasing values of both the numbers. The temperature rises with increasing values of Df, Sc, E where as the temperature falls down for increasing values of So and Pr. We observed that the concentration increases for increasing values of So and Pr. but a reverse effect is found in the case of Df. Sc and E.

Numerical results for the local skin-friction coefficient, local Nusselt number, and local Sherwood number were also reported graphically. It is noticed that the skin friction coefficient increases with increasing values of magnetic parameter, but a reverse effect is found in the case of visco-elastic parameter. The local skin friction coefficient decreases under the influence of Dufour number as well as Soret number. It is shown that the rate of heat transfer rises for increasing values of Prandtl number. The Nusselt number decreases with an increase in Eckert number. It is noticed that as Dufour number increases Nusselt number decreases, but a reverse effect is noticed in the case of Schmidt number. It is noticed that as Schmidt number increases the Sherewood number increases. Also it is observed that Sherwood number decreases with the increasing values of both Soret and Dufour numbers.

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