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A STOCHASTIC PRODUCTION INVENTORY MODEL FOR DETERIORATING ITEMS WITH BAYESIAN ESTIMATION

N.S. Indhumathy*, Dr. P.R. Jayashree**

Author Affiliation:
* Research Scholar, Department of Statistics, Presidency College, Chennai, Tamil Nadu 600005, India
E-mail: indhu.stats@gmail.com
** Assistant Professor, Department of Statistics, Presidency College, Chennai, Tamil Nadu 600005, India
E-mail: vprjaya@gmail.com

Corresponding Author:
N.S. Indhumathy, Research Scholar, Department of Statistics, Presidency College, Chennai, Tamil Nadu 600005, India

E-mail: indhu.stats@gmail.com

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Abstract

A stochastic inventory periodic review production system with inter demand time as exponential distribution is considered in this paper. The model is developed on the basis of constant production rate for deteriorating items. The rate of deterioration follows two parameters Weibull distribution. The shape and scale parameters of Weibull distribution are estimated through MLE. Shortages are not allowed and the unit production cost is inversely proportional to the demand rate. The model contains the exponential parameter which is unknown and is estimated through MLE and Baye's under a squared error loss function. The conjugate Gamma prior is used as the prior distribution of exponential distribution. Finally, a numerical MCMC simulation is used to compare the estimators obtained with Expected risk and are shown graphically. The objective of the paper is to develop an optimum policy that minimizes the total average cost by using the above estimates of the parameter. The sensitivity analysis is also carried out for the model with percentage change in the parameters.

Keywords: Baye's estimation, Constant production rate, Exponential distribution, Gamma prior, Maximum likelihood function, Optimal time periods, Squared loss function, Stochastic demand, Weibull deterioration.

1. INTRODUCTION

Inventory is an important ingredient of any business. It is a process and place by "proper and in time" utilization of which an enterprise can save a certain amount of production cost and the inventory cost. A periodic inventory system is an inventory system that updates inventory at the end of a specific period of time which updates inventory records at the end of each month, quarter, or year. Maximum items deteriorate over time.

Electronic products may become obsolete as technology changes; fashion trends depreciate the value of cloths over time; batteries die out as they become old. The outcome of time is even more serious for consumable goods such as food stuff and drugs. In many inventory models, authors have considered economic order quantity models for deteriorating items by taking demand as a key factor. There are mainly two categories of demand in the present studies, one is deterministic and the other is stochastic demand. In stochastic demand the parameters are unknown and can be estimated through MLE, method of moments and Baye's. Bayesian estimation approach has received a lot of attention for estimating the unknown parameters. It makes use of one's prior knowledge about the parameters and also takes into consideration the data available. If one's prior knowledge about the parameter is available, it is suitable to make use of an informative prior but in a situation where one does not have any prior knowledge about the parameter and cannot obtain vital information from experts to this regard, and then a non-informative prior will be a suitable alternative to use.

In a recent research, M.A. Hargia and L. Benkherouf (1994) have considered optimal and heuristic inventory replenishment models for deteriorating items with exponential time-varying demand. S.K. Ghosh, K.S. Chaudhuri (2004) focus on an order level inventory model for a deteriorating item with Weibull distribution deterioration, time-quadratic demand and shortages. Roy, T. and Chaudhary, K. S., (2009) was given a production inventory model under stock dependent demand, Weibull distribution deterioration and shortages. Sridevi. G., et al., (2010) have discussed demand dependent selling price inventory model for deteriorating items with Weibull rate of replenishment. Vidyadhar Kulkarni, Keqi yan (2012) presented production- inventory systems in stochastic environment and stochastic lead times. Amutha R., and Chandrasekaran E.,(2013) discussed an EOQ model for deteriorating item with Quadratic demand and Tie dependent holding cost Lianwu Yang., et. al., (2013) is presented the Baye's estimation of parameter of exponential distribution under a bounded loss function. S. Sarkar., et al., (2013) was given an EPQ model having Weibull distribution deterioration with shortages under permissible delay in payments. Gothi U. B., et. al., (2017) was given an inventory model of repairable items with exponential deterioration and linear demand rate.

In the classical inventory model depletion of inventory is caused by a constant rate of demand. The demand rate at each instant is determined by an underlying stochastic process is stochastic demand. Due to limited shelf-life and market demand, the inventory level continuously decreases. But subsequently, it was noticed that depletion of inventory may take place due to deterioration also. In this paper, a stochastic inventory periodic review production system with inter demand time as exponential distribution and shortages are not allowed the rate of deterioration is assumed to be Weibull distribution with two parameters. The shape parameter and scale parameters are estimated through MLE. The model contains the exponential parameter which is unknown and is estimated through MLE and Baye's under a squared error loss function. The conjugate Gamma prior is used as the prior distribution of exponential distribution. Finally, a numerical MCMC simulation is used to compare the estimators obtained with Expected risk and are shown graphically. The objective of the paper is to develop an optimum policy that minimizes the total average cost by using the above estimates of the parameter. The sensitivity analysis is also carried out for the model with percentage change in the parameters.

2. ASSUMPTIONS AND NOTATIONS

The following assumptions are used to develop mathematical model:

1. A single item is considered in the inventory system.
2. The inter demand time is assumed to be exponential distribution with pdf $D = R(t) = \lambda e^{-\lambda t}$ at any time $t \geq 0$
3. The production rate, say K= r R(t), where r >1.
4. The unit production cost is inversely proportional to the time dependent demand rate.
5. The rate of deterioration is time dependent, which is Weibull distribution deterioration with two parameters.

$$\theta(t)=\frac{f(t)}{p(t)}=\frac{\frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1}\exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right]}{\exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right]}=\frac{\beta}{\eta}\left(\frac{1}{\eta}\right)^{\beta-1}$$

$$\theta(t)=\frac{\beta}{\eta}\left(\frac{1}{\eta}\right)^{\beta-1} \quad where\ \alpha=\frac{1}{\eta^{\beta}}$$

and is denoted by $\theta(t)=\alpha\beta t^{\beta-1}$ $where\ 0\le\alpha\le1,\ \beta\ge1\ and\ t>0$

6. The deteriorated units can neither be repaired nor replaced during the cycle time.
7. Shortages are not allowed.

The following are notations used in this model:

r : Rate of production

$\frac{1}{\lambda}$: Expected rate of demand

c_1 : Production cost per unit time

c_2 : Holding cost per unit time

c_3 : Deterioration cost per unit time

TC : The total average cost

The unit production cost v is inversely related to the demand rate as $v=\alpha_1 D^{-\gamma}$, $where\ \ \alpha_1>0,\ \gamma>0\ and\ \ \gamma=1,\ \gamma\neq2.$ α_1 is positive as v and D are both non-negative; also higher demands result in lower unit cost of production. Therefore, v and D are inversely related and γ must be positive. $\frac{dv}{dD}=-\alpha_1\gamma D^{-(\gamma+1)}<0\ and\ \frac{d^2v}{dD^2}=\alpha_1\gamma(\gamma+1)D^{-(\gamma+2)}>0.$ Therefore marginal unit cost of production is an increasing function of D. Thus these results imply that, as the demand rate increases at an increasing time, the unit cost of production decreases. For this reason the manufacturer is encouraged to produce more as the demand for the item increases. The necessity of the restriction $\gamma=1$, $\gamma\neq2$ arises the nature of solution of problem. The following figure represents the instantaneous state of inventory level.

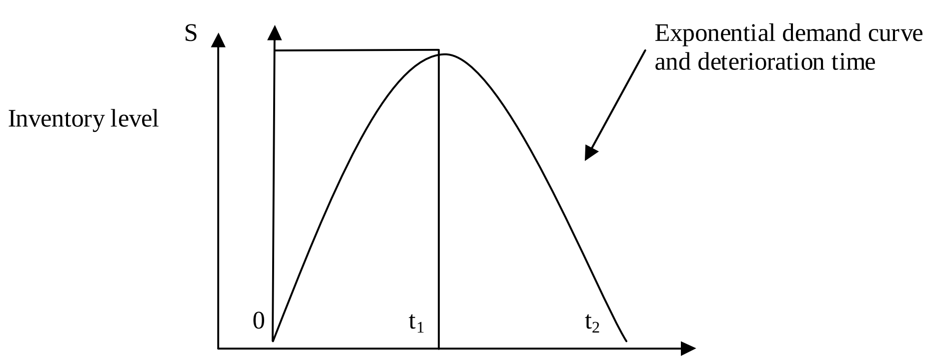

Figure 1: Schematic diagram represents the inventory level

3. DESCRIPTION OF THE MODEL

The stock level is initially zero. Production begins just after t=0, continues up to t= t_1 and stops as soon as the stock level becomes S. Then the inventory level decreases both due to demand and deterioration, till it becomes zero at t= t_2. Then the cycle repeats itself. Let $I(t)$ be the inventory level of the system at any time t ($0 \le t \le t_2$). Therefore, the inventory is described by the system of the differential equations:

$$\frac{d\,I(t)}{dt}+\theta(t)\,I(t)= K-R(t), \quad 0\le t\le t_1 \;------(1)$$

$$\frac{d\,I(t)}{dt}+\theta(t)\,I(t)=-R(t), \quad t_1\le t\le t_2 \;-------(2)$$

$$where\;\; \theta(t)=\alpha\beta\, t^{\beta-1}\; and\; R(t)=\lambda\, e^{-\lambda t}.$$

Using the values of θ(t) and R (t), equations (1) and (2) becomes respectively

$$\frac{d\,I(t)}{dt}+\;\alpha\beta t^{\beta-1}\,I(t)=(r-1)\,(\,\lambda e^{-\lambda t}), \quad 0\le t\le t_1 \;------(3)$$

with the conditions I(0) =0 and I(t_1) =S and

$$\frac{d\,I(t)}{dt}+\;\alpha\beta t^{\beta-1}\,I(t)=-\,\lambda e^{-\lambda t}\;,\; t_1\le t\le t_2 \;-------(4)$$

with the boundary conditions I(t_1) = S and I(t_2) = 0

The solution of (3) using the initial condition I (0) =0 is

$$I(t)\,=\,(1-\alpha t^{\beta})\;(r-1)\left[\lambda t+\frac{\alpha\lambda t^{\beta+1}}{\beta+1}\right]+\,c(1-\alpha t^{\beta}) \quad 0\le t\le t_1 \;----(5)$$

Neglecting powers of α higher than 1. This approximation is followed throughout the subsequent calculations. The solution of (4) using the condition $I(t_1) = S$ is

$$I(t)\,=\,S[1+\alpha[t_1^{\beta}-t^{\beta}]]\,+\,\lambda[t_1-t]+\frac{\alpha\lambda}{\beta+1}[t_1^{\beta+1}-t^{\beta+1}]-\alpha\lambda[t^{\beta+1}-t_1t^{\beta}] \quad t_1\le t\le t_2 \;----(6)$$

For finding value of S, using the boundary condition $I(t_2)=0$, equation (6) gives,

$$S[1+\alpha[t_1^{\beta}-t_2{}^{\beta}]]=\lambda[t_2-t_1]+\frac{\alpha\lambda}{\beta+1}[t_2^{\beta+1}-t_1{}^{\beta+1}]-\alpha\lambda[t_2t_1{}^{\beta}-t_1{}^{\beta+1}]$$

For a first-order approximation over α, this relation gives

$$S\,=\lambda[t_2-t_1]+\frac{\alpha\lambda}{\beta+1}[t_2^{\beta+1}-t_1{}^{\beta+1}]+\alpha\lambda[t_2t_1{}^{\beta}-t_1{}^{\beta+1}] \;-----(7)$$

Therefore,

$$I(t) = \begin{cases} (1-\alpha t^{\beta})\ (r-1)\left[\lambda t + \dfrac{\alpha\lambda\, t^{\beta+1}}{\beta+1}\right] + c(1-\alpha t^{\beta}) & if\quad 0 \le t \le t_1 \\ \left. \begin{array}{l} S\,[1+\alpha[t_1^{\beta} - t^{\beta}]] + \lambda[t_1 - t] + \dfrac{\alpha\lambda}{\beta+1}[t_1^{\beta+1} - t^{\beta+1}] \\ -\alpha\lambda\ [t^{\beta+1} - t_1 t^{\beta}] \end{array} \right] & if\quad t_1 \le t \le t_2 \end{cases} \quad ---(8)$$

Inventory during t = 0 to t_1 is

$$I_1 = \int_0^{t_1} I(t)\ dt$$

$$I_1 = \frac{\lambda t_1^2}{2}[r-1] - \frac{\alpha\beta\lambda t_1^{\beta+2}}{(\beta+1)(\beta+2)}[r-1] ----(9)$$

Inventory during t = t_1 to t_2 is

$$I_2 = \int_{t_1}^{t_2} I(t)\ dt$$

$$I_2 = \lambda t_2^2 + 2(\alpha\lambda\, t_2^2\, t_1^{\beta}) - 2(\alpha\lambda\, t_1^{\beta+1}\, t_2) - \lambda t_1 t_2 - \frac{\alpha\lambda\, t_2^{\beta+1}\, t_1}{\beta+1} - 2(\alpha\lambda\, t_2\, t_1^{\beta+1}) + 2(\alpha\lambda\, t_1^{\beta+2})$$
$$+\frac{\alpha\lambda\, t_2\, t_1^{\beta+1}}{\beta+1} - \frac{\lambda}{2}[t_2^2 - t_1^2] + \frac{\alpha\beta\lambda\, t_2^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{\alpha\beta\lambda\, t_1^{\beta+2}}{(\beta+1)(\beta+2)} ------(10)$$

for a first-order approximation over α. The total number of deteriorated items [0, t_2] is given by = production in [0, t_1] – demand in [0, t_2]

$$\int_0^{t_1} r\lambda e^{-\lambda t}\, dt - \int_0^{t_2} \lambda e^{-\lambda t}\, dt = r\lambda t_1 - \lambda t_2 ----(11)$$

Production cost in [0, t_1] is

$$\int_0^{t_1} r\lambda t\, dt = r\lambda \frac{t_1^2}{2} ------(12)$$

The total average total cost (TC) is given by

TC = Production cost + Inventory cost + Deteriorating cost

$$TC = \frac{1}{t_2}\Bigg\{ c_1\left[\frac{r\lambda t_1^2}{2}\right] + c_2\Bigg[(r-1)\frac{\lambda t_1^2}{2} - \frac{\alpha\beta\lambda t_1^{\beta+2}}{(\beta+1)(\beta+2)} + \lambda t_2^2 + 2\left(\alpha\lambda t_2^2 t_1^{\beta}\right)$$
$$-2\left(\alpha\lambda t_1^{\beta+1} t_2\right) - \lambda t_1 t_2 - \frac{\alpha\lambda t_2^{\beta+1} t_1}{\beta+1} - 2\left(\alpha\lambda t_2 t_1^{\beta+1}\right) + 2\left(\alpha\lambda t_1^{\beta+2}\right) + \frac{\alpha\lambda t_2 t_1^{\beta+1}}{\beta+1}$$
$$-\frac{\lambda}{2}\left(t_2^2 - t_1^2\right) + \frac{\alpha\beta\lambda}{(\beta+1)(\beta+2)}\left(t_2^{\beta+2} - t_1^{\beta+2}\right)\Bigg] + c_3\left[r\lambda t_1 - \lambda t_2 \right]\Bigg\} ----(13)$$

Using calculus to minimize the Average total cost for finding the optimum values of t_1 and t_2 are the solutions of the equation(13) gives

$$\frac{\partial TC}{\partial t_1} = 0 \;\; and \;\; \frac{\partial TC}{\partial t_2} = 0 ------(14)$$

Provided that they satisfy the sufficient conditions

$$\frac{\partial^2 TC}{\partial t_1^{\,2}} > 0, \; \frac{\partial^2 TC}{\partial t_2^{\,2}} > 0 \;\; and \left(\frac{\partial^2 TC}{\partial t_1^{\,2}}\right)\left(\frac{\partial^2 TC}{\partial t_2^{\,2}}\right) - \left(\frac{\partial^2 TC}{\partial t_1 \partial t_2}\right)^2 > 0$$

Equations (14) can be written as

$$c_1\left[\frac{r\lambda t_1}{t_2}\right] + c_2\Bigg[\frac{r\lambda t_1}{t_2} - \frac{r\alpha\beta\lambda t_1^{\beta+1}}{t_2(\beta+1)} - \frac{\lambda t_1}{t_2} + \frac{\alpha\beta\lambda t_1^{\beta+1}}{t_2(\beta+1)} + 2\left(\alpha\beta\lambda t_2 t_1^{\beta-1}\right) - 2\left(\alpha\lambda(\beta+1)t_1^{\beta}\right) - \lambda$$
$$-\frac{\alpha\lambda t_2^{\beta}}{\beta+1} - 2\left(\alpha\lambda(\beta+1)t_1^{\beta}\right) + 2\left(\frac{\alpha\lambda(\beta+2)t_1^{\beta+1}}{t_2}\right) + \alpha\lambda t_1^{\beta} + \frac{\lambda t_1}{t_2} - \frac{\alpha\beta\lambda t_1^{\beta+1}}{t_2(\beta+1)}\Bigg] + c_3\left[\frac{r\lambda}{t_2}\right] = 0 -----(15)$$

$$-c_1\left[\frac{r\lambda t_1^2}{2t_2^2}\right] + c_2\Bigg[\left[-\frac{r\lambda t_1^2}{2t_2^2}\right] + \frac{r\alpha\beta\lambda t_1^{\beta+2}}{t_2^2(\beta+1)(\beta+2)} + \frac{\lambda t_1^2}{2t_2^2} - \frac{\alpha\beta\lambda t_1^{\beta+2}}{t_2^2(\beta+1)(\beta+2)} + \lambda + 2\left(\alpha\lambda t_1^{\beta}\right)$$
$$-\frac{\alpha\beta\lambda t_2^{\beta-1} t_1}{\beta+1} - 2\left[\frac{\alpha\lambda t_1^{\beta+2}}{t_2^2}\right] - \frac{\lambda}{2} - \frac{\lambda t_1^2}{2t_2^2} + \frac{\alpha\beta\lambda t_2^{\beta}}{\beta+2} + \frac{\alpha\beta\lambda t_1^{\beta+2}}{t_2^2(\beta+1)(\beta+2)}\Bigg] - c_3\left[\frac{r\lambda t_1}{t_2^2}\right] = 0 -----(16)$$

The above equations are non-linear simultaneous equations (15) and (16) which cannot be solved directly. Moreover, the above equations also contain the unknown parameters and are found using MLE and Baye's in the subsequent sections of the paper, Also using R-Software to solve the non-lienar equations the optimal time periods t_1^* and t_2^* are obtained. Then the maximum inventory level and optimal total cost are also obtained for the optimal time periods t_1^* and t_2^* from equations (7) and (13) respectively.

4. ESTIMATING PROCEDURES

4.1 Maximum Liklihood Estimation for Weibull Distribution

Suppose $x_1, x_2, ..., x_n$ is a sample of size n units which follows the Weibull model.
The density function for two-parameter Weibull distribution is

$$f(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} \exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right] ---(17)$$

Where $f(t)$: Probability density function
β : The shape parameter
η : The scale parameter

The likelihood function of the pdf from the equation (17) is given as

$$L(x_i,\eta,\beta)=\prod_{i=1}^{n}\left\{\left(\frac{\beta}{\eta}\right)\left(\frac{x_i}{\eta}\right)^{\beta-1}\exp\left[-\left(\frac{x_i}{\eta}\right)^{\beta}\right]\right\}---(18)$$

Where β presents the shape parameter and η is the scale parameter. The log-likelihood function may be written as

$$\ell=n\ln(\beta)-n\beta\ln(\eta)+(\beta-1)\sum_{i=1}^{n}\ln(x_i)-\sum_{i=1}^{n}\left(\frac{x_i}{\eta}\right)^{\beta}\ ---(19)$$

From equation (19), differentiating the log-likelihood equation for the parameters η and β are given as,

$$\frac{\partial\ell}{\partial\eta}=-n\left(\frac{\beta}{\eta}\right)+\left(\frac{\beta}{\eta}\right)\sum_{i=1}^{n}\left(\frac{x_i}{\eta}\right)=0----(20)$$

$$\frac{\partial\ell}{\partial\beta}=\frac{n}{\beta}+\sum_{i=1}^{n}\left(\frac{x_i}{\eta}\right)-\sum_{i=1}^{n}\left(\frac{x_i}{\eta}\right)^{\beta}\ln\left(\frac{x_i}{\eta}\right)=0----(21)$$

From the equation (20) $\hat{\eta}$ is obtained in-terms of $\hat{\beta}$ in the form

$$\eta=\left[\frac{1}{n}\sum_{i=1}^{n}(x_i)^{\beta}\right]^{1/\beta}----(22)$$

4.2 Estimation of Parameter of Weibull Deterioration

Using the R-software, the above equations for estimating the parameters are solved and the output is given by

Parameter	Type	Estimate	S.E.
Shape	Shape	4.823793	0.9333107
Scale	Scale	1.994079	0.4066205

The Maximum likelihood estimate for η is $\hat{\eta}$=1.9940 and so $\alpha=\frac{1}{\eta^{\beta}}$

The Maximum likelihood estimate for $\hat{\beta}$ = 4.8237. So the value $\hat{\alpha}$ = 0.035

4.3 Maximum Likelihood Estimation for Exponential Distribution

The probability density function of the exponential distribution is given by,

$$f(x;\lambda)=\begin{cases}\lambda e^{-\lambda x} & if \quad x\geq 0\\ 0 & if \quad x<0\end{cases}\ ---------(1)$$

Suppose $X_1,X_2,\ldots,X_n$ is a random sample from exponential distribution (1). Let $(x_1,x_2,\ldots,x_n)$ be the observed values of $(X_1,X_2,\ldots,X_n)$, then the likelihood function based on $(x_1,x_2,\ldots,x_n)$ is given by,

$$L(\lambda/X)=\prod_{i=1}^{n}f(x_i,\lambda)=\prod_{i=1}^{n}\lambda e^{-\lambda x}=\lambda^n e^{-\lambda\sum_{i=1}^{n}x_i}$$

To calculate the maximum likelihood estimator, the natural logarithm of likelihood function is maximised i.e. differentiating with respect to λ and equating each result to zero.

$$\frac{d}{d\lambda}\left(\ln L(\lambda / X)\right) = n \ln \lambda - \sum_{i=1}^{n} x_i = 0$$

The MLE of λ given by $\dfrac{n}{\sum_{i=1}^{n} x_i}$

4.4 Baye's Estimation for Exponential Distribution

In this section, we consider the Baye's estimation for the parameter λ assuming the conjugate of prior distribution for λ as two parameter Gamma distribution given as

$$f(\lambda/\alpha,\beta) = \begin{cases} \dfrac{\beta^{\alpha}}{\gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta x} & ,\lambda \geq 0 \\ 0 & ,\lambda < 0 \end{cases} \qquad \alpha > 0,\ \beta > 0$$

The likelihood function is assumed as L (λ/x) and the posterior distribution is,

$$p(\lambda / x) \propto L(\lambda / x)\ \ f(\alpha,\beta)$$

$$p(\lambda / X) \propto (\lambda^{n}\, e^{-\lambda \sum_{i=1}^{n} x_i})(\lambda^{\alpha-1} e^{-\beta})$$

$$p(\lambda / X) \propto \lambda^{n+\alpha-1}\ e^{-\lambda[\beta + \sum_{i=1}^{n} x_i]}$$

This follows Gamma distribution with parameter $\gamma(n+\tilde{\alpha}\ \ ,\tilde{\beta} + \sum_{i=1}^{n} x_i)$

The mean and variance are given by

Mean= $\dfrac{\alpha}{\beta} = \dfrac{n+\tilde{\alpha}}{\tilde{\beta} + \sum_{i=1}^{n} x_i}$

Variance= $\dfrac{\tilde{\alpha}}{\tilde{\beta}^{2}}$

5. NUMERICAL SIMULATION

To compare the different estimators of the parameter λ of the exponential distribution, the risks under squared error loss of the estimates are considered. These estimators are obtained by maximum likelihood and Baye's methods under Expected risk. The MCMC procedure for Baye's estimation is as follows

(i) A sample of size n is then generated from the density of the exponential distribution, which is considered to be the informative sample.

(ii) The MLE and Baye's estimators are calculated with $\alpha = n+\tilde{\alpha}\ \ ,\beta = \tilde{\beta} + \sum_{i=1}^{n} x_i$

(iii) Steps (i) to (ii) are repeated N = 2000 times for different sample sizes and the risks under squared error loss of the estimates are computed by using:

Expected Risk ($\hat{\lambda}$)= $\dfrac{1}{N}\sum_{i=1}^{n}\left(\lambda_i - \hat{\lambda}\right)$ Where, $\hat{\lambda}_i$ is the estimate at the i^{th} i^{th} run

Assuming the value of λ = 0.03, the estimated value of $\hat{\lambda}$ using MLE and Baye's along with Expected risk are given in Table 1.

Table 1: Parameter Estimation and Expected Risk

n	Criteria	(i) λ= 0.03, $\alpha = n+\tilde{\alpha}$, $\beta = \tilde{\beta} + \sum_{i=1}^{n} x_i$			
		MLE	**α=0.5, β=0**	**α=1, β=0.5**	**α=1.5, β=1**
10	Estimated value ER	0.0318 0.00009	0.0317 0.00008	0.0316 0.00006	0.0316 0.00007
25	Estimated value ER	0.0316 0.00008	0.0315 0.00006	0.0314 0.00004	0.0312 0.00005
50	Estimated value ER	0.0311 0.000007	0.0310 0.000008	0.0308 0.000006	0.0308 0.000006
75	Estimated value ER	0.0298 0.000007	0.0297 0.000005	0.0297 0.000005	0.0297 0.000004
100	Estimated value ER	0.0297 0.000001	0.0296 0.0000006	0.0296 0.0000006	0.0296 0.0000003
125	Estimated value ER	0.0296 0.0000003	0.0296 0.0000002	0.0296 0.0000002	0.0296 0.0000002
150	Estimated value ER	0.0296 0.0000002	0.0296 0.0000002	0.0296 0.0000002	0.0296 0.0000002

It is seen that for small sample sizes the estimators under the Expected Loss function have smaller ER when choosing proper parameters α and β. But for larger sample sizes (n>50), all the estimators have approximately same ER. The obtained results are demonstrated in Table 1 and shown graphically in Figure 2 and 3. The estimated value of $\hat{\lambda} = 0.0296$

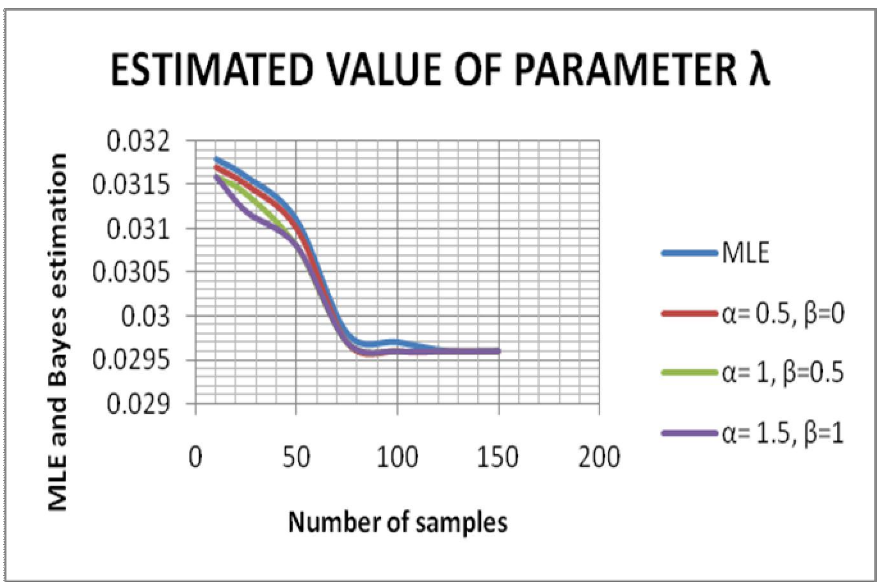

Figure: 2 MLE and Baye's Estimation

The above figure 2 shows the estimated value of parameter through MLE and Baye's.

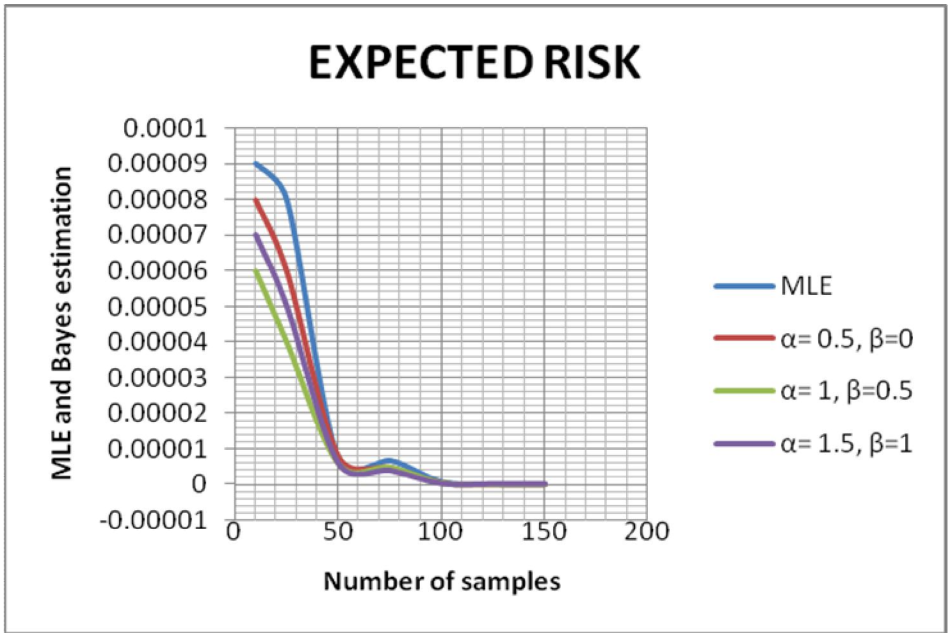

Figure 3: Expected Risk under Loss Function

The above figure 3 shows the expected risk under loss function for different α and β values through MLE and Baye's.

6. OPTIMAL SOLUTIONS USING ESTIMATED PARAMETERS

From the above MLE and Baye's estimates it is found that $\alpha = 0.0283$, $\beta = 5.0879$, $\lambda = 0.0296$ and the rate of deterioration is 0.1728. For finding the optimum values we assume c1 = 10, $c_2 = 4$, $c_3 = 0.05$ and r = 5 in appropriate units. Solving the non-linear equations (15) and (16) of the above model using R-software, the optimal time periods of t_1 and t_2 are obtained as $t_1^* = 1.2343$ and $t_2^* = 13.0407$ and the optimal average total cost is TC* = 2114.344 , the maximum inventory level is $S^* = 848.4855$,

7. SENSITIVITY ANALYSIS

The effects of changes in the system parameters α, β, λ on the optimum values of t_1^*, t_2^* and TC^* are studied in the model. The sensitivity analysis is performed by changing each of the parameter by +50%, +25%, +10%, -10%, -25%, -50%. The results are shown in Table-2.

On the basis of the results in Table, as the changes of parameter decreases, the cycle time periods and total cost also decreases.

Table 2: Variation of the Total Cost for Different Time Periods

Parameter changing	Change (%)	t_1^*	t_2^*	TC^*
α	+50	1.6347	18.5953	12947.22
	+25	1.4521	16.2301	5709.913
	+10	1.2290	14.8727	3418.581
	-10	1.0966	11.3685	667.3834
	-25	0.8862	8.6471	128.3848
	-50	0.7333	6.2478	18.1692
β	+50	1.74675	24.7337	13680.65
	+25	1.4898	16.3205	5886.059
	+10	1.3357	14.9321	3463.227
	-10	1.3025	10.9502	513.1999
	-25	0.9761	8.3652	102.8874
	-50	0.7192	4.8242	19.3492
λ	+50	1.8495	19.2969	15961.46
	+25	1.5412	16.0807	5346.012
	+10	1.3563	14.1510	2483.114
	-10	1.1097	11.5781	745.3759
	-25	0.9247	9.6484	249.9991
	-50	0.6165	6.4323	22.3108

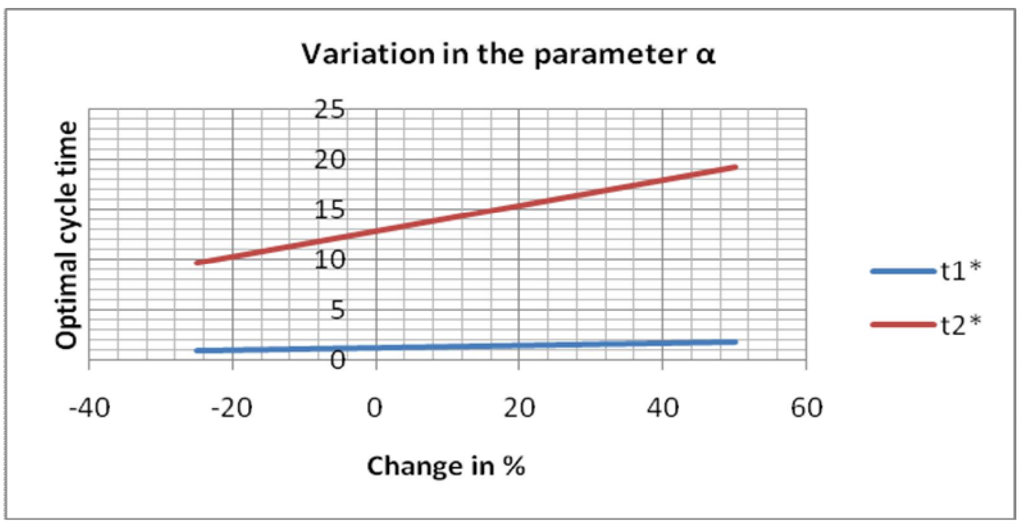

Figure 4: Percentage Change in the Parameter α

The above figure 4 shows changes in the parameter α decreases, t_1^* and t_2^* also decreases.

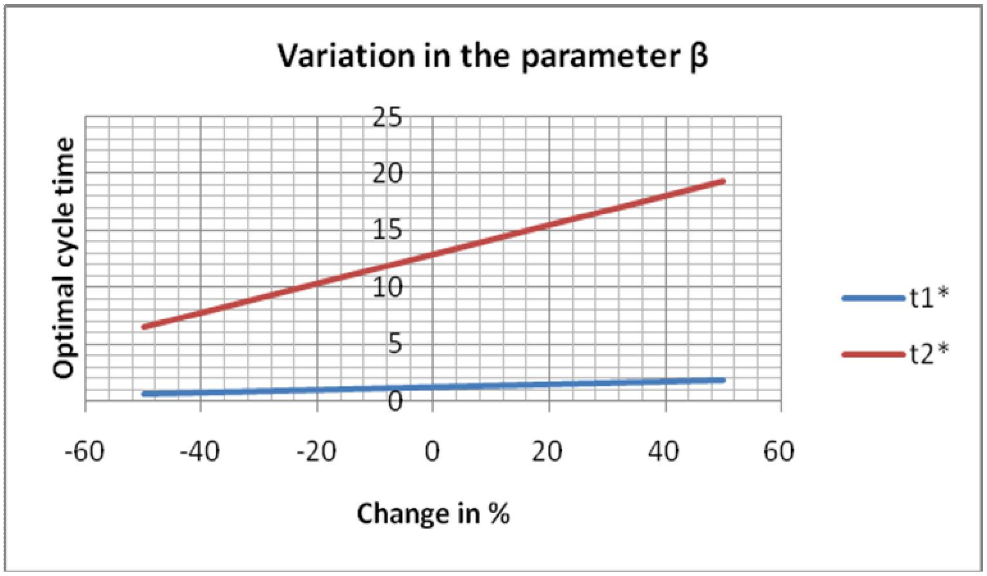

Figure 5: Percentage Change in The Parameter β

The above figure 5 shows changes in the parameter β decreases, t_1^* and t_2^* also decreases.

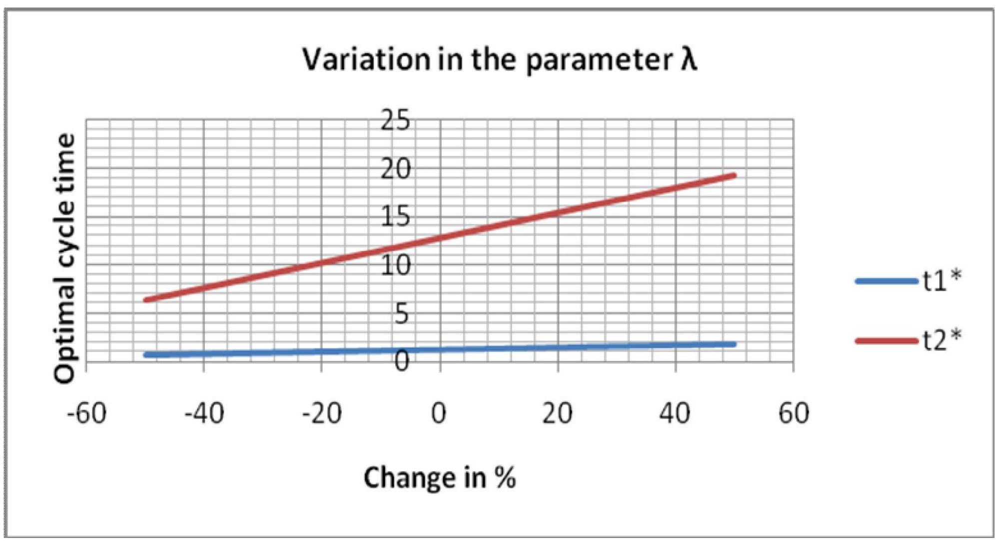

Figure 6: Percentage Change in the Parameter λ

The above figure 6 shows changes in the parameter λ decreases, t_1^* and t_2^* also decreases.

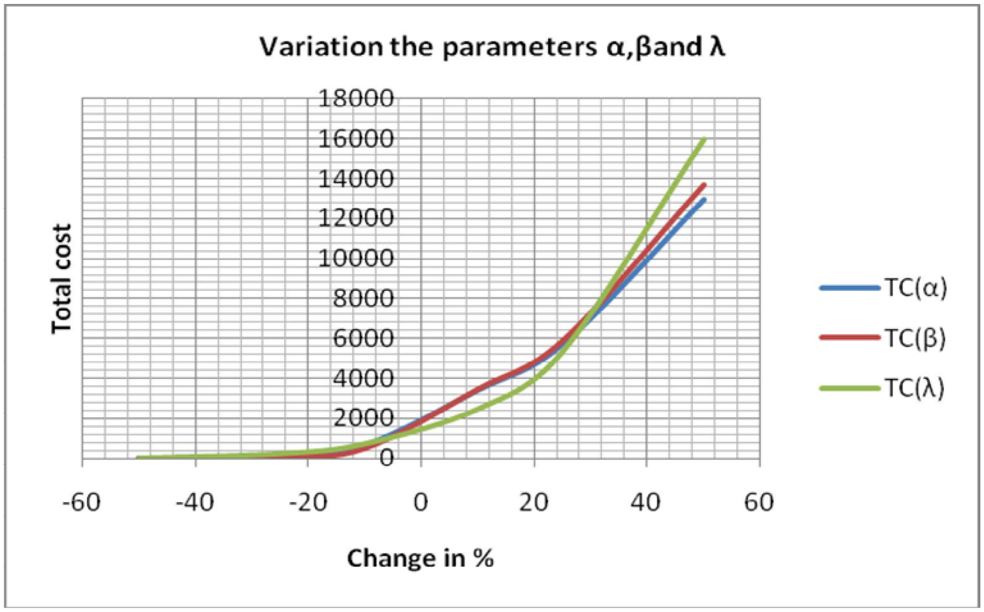

Figure 7: Percentage Change in Total Cost

The above figure 7 shows changes in the parameter α, β and λ decreases, total cost also decreases.

8. CONCLUSION

A stochastic Inventory periodic review production system with inter demand time as exponential distribution with constant production rate for deteriorating items is considered. The rate of deterioration is said to follow Weibull distribution with two parameters is also considered in this paper. The unit production cost is inversely proportional to the demand rate is considered in this study. The model parameters are estimated though MLE and Baye's and R- Software has been developed for the non-linear equations involved in the model. The shape parameter and scale parameters of Weibull distribution are estimated through MLE. The exponential parameter is estimated through MLE and Baye's under a squared error loss function. The conjugate Gamma prior is used as the prior distribution of exponential distribution. Finally, a numerical MCMC simulation is used to compare the estimators obtained with Expected risk and are shown graphically. By using the estimates of the parameters involved in the model, the optimal time periods and total cost have been found as $t_1^* = 1.2343$, $t_2^* = 13.0407$, $TC^* = 2114.344$, and the maximum inventory level is $S^* = 848.4855$. Sensitivity analysis is also carried out with percentage change in the parameters.

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