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VERTEX PRIME LABELLING (VPL) OF SHEL RELATED GRAPHS

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Abstract

We discuss vertex prime labeling of graphs obtained by using Shel graph. We have obtained vpl of Shel graphs, antena of Shel, Path union of Sn (n =4), snake of Shel and one point union of Shel, star of shel and a few other.

Keywords: Shel, Path, Union, Vertex prime, Labelling, Graph, Double Path Union.

Subject Classification: 05C78

1. INTRODUCTION

Deretsky, Lee, and Mitchem [3] has introduced vertex prime labelling. Let G be a (p,q) graph f: E(G)→{1,2,..q} is a bijective function which introduces vertex label such that for each vertex of degree at least 2 the greatest common divisor of the labels on its incident edges is 1.The graph which has vertex prime labelling is known as vertex prime graph. For terminology and definitions we consider Dynamic survey of graph labelling [6][5]

2. PRELIMINARIES

When a set of non-zero integers contains:
1) The number 1
OR
2) a prime number and none of its multiples.
OR
3) a pair of consecutive natural numbers

Then the gcd of numbers is 1.This remains the base for further development.

A few definitions are given below.

2.1 Shel graph: Let Cn be the cycle given by $(v_1,v_2,v_3,\ldots v_nv_1)$.Draw (n-3) edges from single vertex say v_1 to $v_3,v_4..v_{n-1}$. Note that $|V(G)|=n,|E(G)|=2n-3$.This graph is denoted by S_n.

2.2 Path Pm: It is sequence of vertices and edges given by $(v_1,e_1,v_2,e_2,\ldots e_{m-1},v_m)$.It's length is m-1 and number vertices are m.

2.3 antena of G: Consider a G=(p,q)graph.At each of it's vertex attach a path of length m, then we get a antena graph antena(G,m).If we attach K paths of different length at each vertex of G then it is **k-antena(G).**

2.4 Path union Pm(G) It has a path Pm and at each vertex on it a copy of graph G is attached at a fixed point on G.

2.5 one point union $G=(G_1)^k$: Take k copies of graph G_1 .Fix a specific vertex on it.Fuse all copies of G_1 at this fixed point .The resultant graph is one point union. Note $|V(G)|=k|VG_1)|-k+1$ and $|E(G)| = k.|E(G_1)|$

2.6 Snake: S(G,k) It is a connected graph with k blocks whose
block-cut point graph is a path and each of the k blocks is isomorphic to G.

2.7 Double Path union. G=Pm(G1,G2): It has a path of length m and at each vertex there are two graphs G1 and G2 attached to it. $|V(G)|=m|V(G)|$and $|E(G)|=m+m(E(G)-1$

2.8 Flag of G:. It is obtained by attaching a pendent edge at suitable vertex of G where G is (p,q) graph. It is denoted by FL(G).$|V(FL(G))| = p+1$and $|E(FL(G)|= q+1$**..**

2.9 Fusion of vertices: Let $v \in V(G_1)$, $v' \in V(G_2)$ where G_1 and G_2 are two graphs. We fuse v and v' by replacing them with a single vertex say w and all edges incident with v in G_1 and that With v' in G_2 are incident with u in the new graph $G=G_1FG_2$.$Deg_Gu=deg_{G1}(v)_+ deg_{G2}(v')$ and $|V(G)| =|V(G_1)|+|V(G_{2.})|-1,|E(G)|=|E(G_1)|+|E(G_2)|$

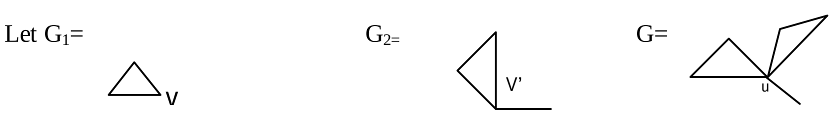

Fig 1: fusion of vertex v and v' to give vertex u

The fusion of two vertices in the same graph is described in [5].

Definition2.10: **Fusion Graph:** Let G_1 and G_2 be two graphs with $|V(G_1)|=P$. Take P copies of G_2. Choose a same fixed vertex v' in each copy of G_2.Toeach vertex in G1, fuse v' from one copy each of G_2. The resultant graph is fusion graph of G_1 and G_2 denoted by G_1 F G_2.Note that only if G_1 is isomorphic to G_2 then G_1FG_2 is same as G_2FG_1.
Note that $|V (G_1 F G_2)|=|V (G_1)| + |V(G_2)|-1$.
1. $|E (G_1 F G_2)|=P|E(G_2)| + E(G_1)$

3. RESULTS PROVED

3.1 Theorem: The shel graph Sn is vertex prime. Further the graph given by
i) FL(Sn))
ii) $SnFK_{1,m}$
iii) antena(Sn,m)
iv) k-antena(Sn)
are vertex prime graphs.

Proof: Let the cycle forming shel be given by $(v_1,e_1,v_2,e_1,v_3,e_3,..v_n,e_n,v_1)$.The chords at vertex v_1 and end at $v_3,v_4,..v_{n-1}$.are denoted by $e^1,e^2,..e^{n-3}$.Define a function f as follows $f:E(Sn)\rightarrow\{1,2,..,2n-3\}$ given by$f(e_i)$ =i for i= 1..n further $f(e^i)= n+i,i = 1,2,..,(n-1)$Obviously f is vertex prime.

To answer the further parts of theorem we assume the labeled copy of Sn as above is in our hands.

i)The pendent edge that forms flag of Sn be given label 2n-2.This will give vpl of FL(Sn)

ii)The m- pendent edges attached at vertex i be given by e^i_j j = 1,2,..m and i = 1,2,..n
We extend the function f above as follows:
$f(e^i_j) = n-3+(i-1)m+j$ where j = 1,2,..m,i = 1,2,,..n

iii)We identify the path (of length m) P_{m+1}at vertex i (i = 1,2,..n) $asp^i_m=(vi= v^i_1,e^i_1,e^i_2,..e^i_{m-1},v^i_m e^i_{m,}, v^i_{m+1})$
$f(e^i_j)= n-3 +(i-1)m+j$ j = 1,2..m, i = 1,2,..n

iv)The total number of edges at any vertex i on Sn that are on k- antenas attached at that vertex be ti..with edges on different antena at vertex i be being xi1,xi2,..xik such that $x_1+x_2+..x_K=t^i_k$ and let us assume that $xi_1 \le xi_2 \le xi_3 \ldots \le xi_k$.

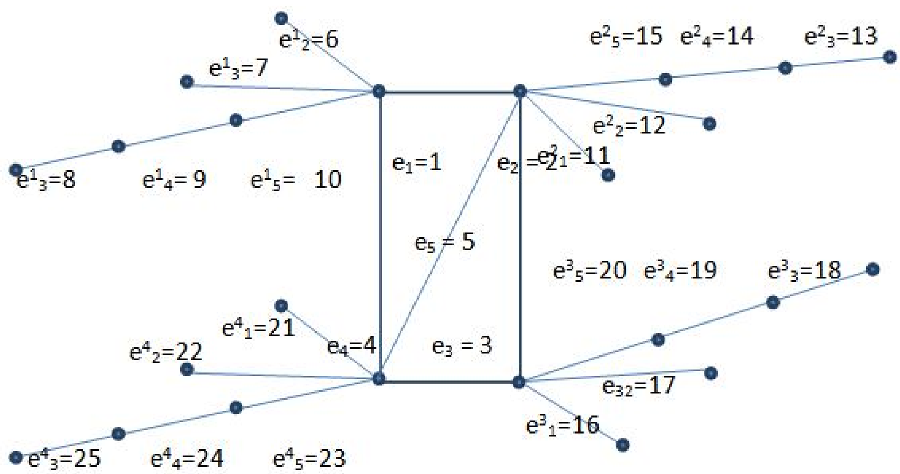

Fig 2: Vertex prime labeling of k-antena(Sn) with ordinary names and labels

These edges are named as $c^i_1,c^i_2,..c^i_{x1},..c^i_q$(q = ti) This is shown in fig 3.1 (The edges on new path at any vertex of Sn are counted from pendent vertex of path or Shel-end of path.It is not to be started counting at middle of path)We extend the same f given above as follows:

$f(c^i_j)= j+t1+t2+..,+t(i-1)$, i=1,2,..n.

The resultant graph is vertex prime. #

3.2 Theorem: Path union of Sn i.e. Pm(Sn) is vertex prime.(n>3)

Proof: Let G =Pm(Sn) .Let it be a (p,q) graph.The path be given by $(v_1,e_1,v_2,e_2,\ldots e_{m-1},v_m)$.The copy of shel attached (at apex on shel)at vertex i on path Pm be given by $\{v_i=v^i_1,e^i_1,v^i_2,e^i_2,\ldots.v^i_n,e^i_n,v_i\}.U\{e^i_{n+j} = (v^iv^i_j)/i = 1,2,\ldots,m.and\ j = 3,\ldots,n-1\ .\}$
Define a function $f:E(G)\rightarrow\{1,2,\ldots q)$
$f(e_j)=j$,j = 1,2,…,m-1.
$f(e^i_j) = j+ m-1 +(i-1)q'$ where $q'=|E(S_n)|=2n-3$ for j =1,2,3,..n.
$f(e^i_{n+j})= f(e^i_n)+j$,j=1 2 ..,n-1.
Thus the f is vertex prime function.

3.3 Theorem: S(Sn,m) is vertex prime.(n>3)

Proof: The path Pm is given by $(v_1,e_1,v_2,e_2,\ldots e_{m-1},v_m)$.The $u^i_1,u^i_2,\ldots,u^i$n-2 vertices between v_i and v_{i+1} (for i = 1,2,,,,m-1)that forms the i^{th} block of the snake .This is done by adding the edges $p^i_1=(v_iu^i_1),p^i_{j+1}=(u^i_ju^i_{j+1})$,j=1,2…,,(n-3) and $p^i_{n-1}=(u^i_{n-2}v_i)$ andxj$_=$(viu^i_j).j= 2,3,..,(n-2), i = 2,3,..m

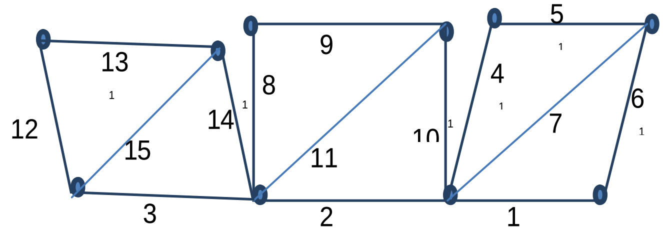

Fig. 3: $S(S_4,4)$ labeled copy

Define a function f as follows:
f:E(G)→{1,2,..q}where q is the number of edges on snake S(Sn,m).Define f(ei)=i for i = 1.2,..,m-1
Let k =(m-1)+(i-1)(2n-3)
$f(p^i_j)=j+k$,j= 1,2,3,..,(n-1)
$f(x^i_j)=f(p^i_{(n-3)})+j$.
Observe that the graph is vertex prime.#

3.4 Theorem: mixed-double path union of shel $Pm(Sn,S_k)$ is vertex prime.

Proof: Let G =$Pm(Sn,S_k)$ .Let it be a (p,q) graph.The path be given by $(v_1,e_1,v_2,e_2,\ldots e_{m-1},v_m)$.The copy of shelSn (at apex on shel) attached at vertex i on path Pm be given by $\{v_i=v^i_1,e^i_1,v^i_2,e^i_2,\ldots.v^i_n,e^i_n,v_i\}.U\{e^i_{n+j}=(v^i_1v^i_j)/i=1,2,\ldots,m.\text{and } j=3,\ldots,n-2\}$ and that Sk is given by $\{v_i=u^i_1,c^i_1,u^i_2,c^i_2,\ldots.u^i_k,c^i_k,u^i_1\}.U\{c^i_{k+j}=(u^i_1u^i_j)/i=1,2,\ldots,m.\text{and } j=3,\ldots,k-2\}$
Define a function f:E(G)→{1,2,…q)
$f(e_j)=j$,j = 1,2,…,m-1.
$f(e^i_j)$ = j+ m-1 +(i-1)q' where q'=$|E(S_n)|$-1,j = 1,2,–,n-1
$f(e^i_{n+j})=f(e^i_n)+j-2$,j=3 ..,n-2. ,
For the copy of S_k we have,$f(c^i_j)=f(e^m_{2n-3})$.+j+(i-1)(2k-3)
$f(c^i_{k+j})=f(c^i_j)+j-2$ where j= 3,4,k-1
Thus the f is vertex prime function.

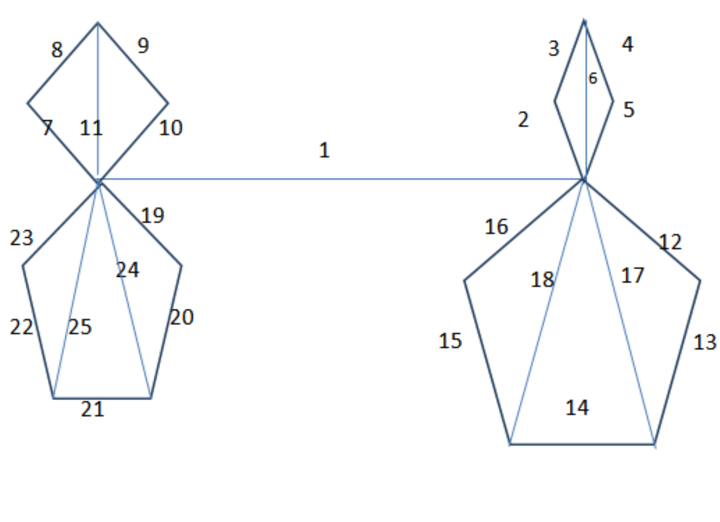

Fig. 4: A labeled copy of mix (double) path- union P_2S_4,S_5)—A vertex prime graph

3.5 Theorem : One point union of k copies of shel i.e. $G=(Sn)^k$ is vertex prime.(the point common to all shells is the apex of shel)

Proof: The copies of ShelSn be $S^1, S^2, \ldots S^k$.The vertices and edges on the i^{th} copy are given by $V(S^i)=\{v^i_1, v^i_2, ..v^i_n / i = 1,2..k\}$ .Note that $v^i_1 = v$ is the apex of any shels.$E(S^i_n)= \{e^i_j=(v^i_j v^i_{j+1})/i=1,2,..k$ and $j = 1,2..n$ with $j+1$ is taken (mod n)$\}U\{ci=(v^i_1 v^i_j)/i = 1,2,..k$ and $j = 3,..,n-1\}$

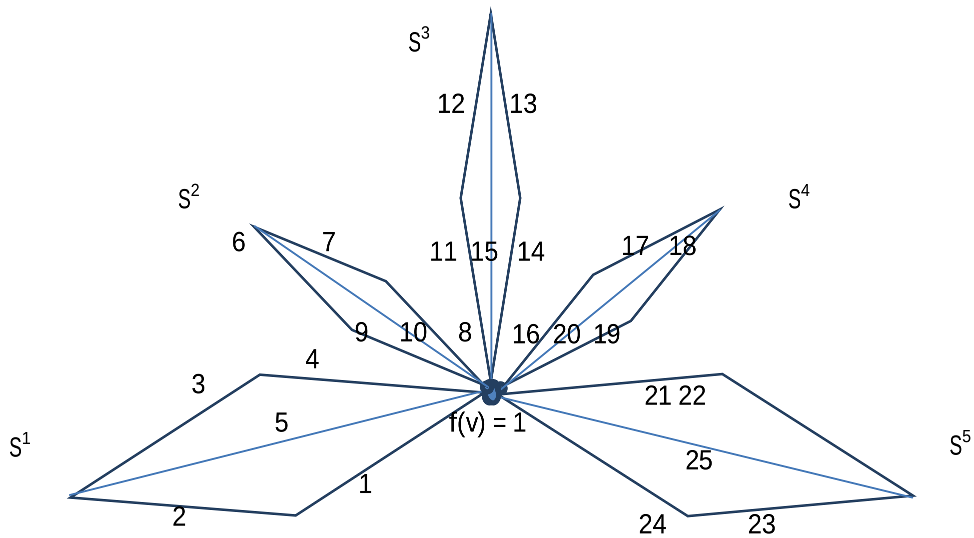

Fig. 5: VPL of $(S_4)^5$

Define f:E(G)$\rightarrow\{1,2,\ldots,|E(G)|$ As follows:
$f(e^i_j) = (k-1)|E(G)|+j$,$j = 1,2,..n$;$i=1,2,..k$
$f(c^i_j)= f(e^i_n)+j-2, j = 3,..,n-1$
Thus the graph is vertex prime.#

3.6 Theorem: Star of Sn is vertex prime.

Proof: It is obtained by attaching a copyofSn (at apex) each at every vertex of Sn.
It is actually fusion graph SnFSn.

We start with a copy of Sn given by $(v_1e_1,v_2,..v_{n,}e_n,v_1)$and chords $\{(v_1v_i)/i= 3,4,..n-1\}$

The copy of Snattached at i^{th} vertex of Sn be given by $\{v_1=v^i_1,v^i_2,..v^i n\}$ and $E=\{(v^i_j v^i_{j+1})/j=2,3,.n$ where $n+1$ is taken (mod n)$\}$ $U\{(v_i v^i_j)/j= 2,3,..n-1)\}$ at the ith vertex.

Define a function f as follows
f:E(G)$\rightarrow\{1,2,..|E|\}$by
$f(ei) = i$,$i= 1,2,..,n-1$.
$f(v_1v_j)=n+j-2$, $j = 3,4,..n-1$.
$f(v^j_j v_{j+1})=2n-3+(i-1)(2n-3)+j$ $j= 1,2,..n$
$F(v^i_1 v_j)=f(v^j_j v_{j+1})+j-2$, $j = 3,4,..,n-1$.That completes the proof.

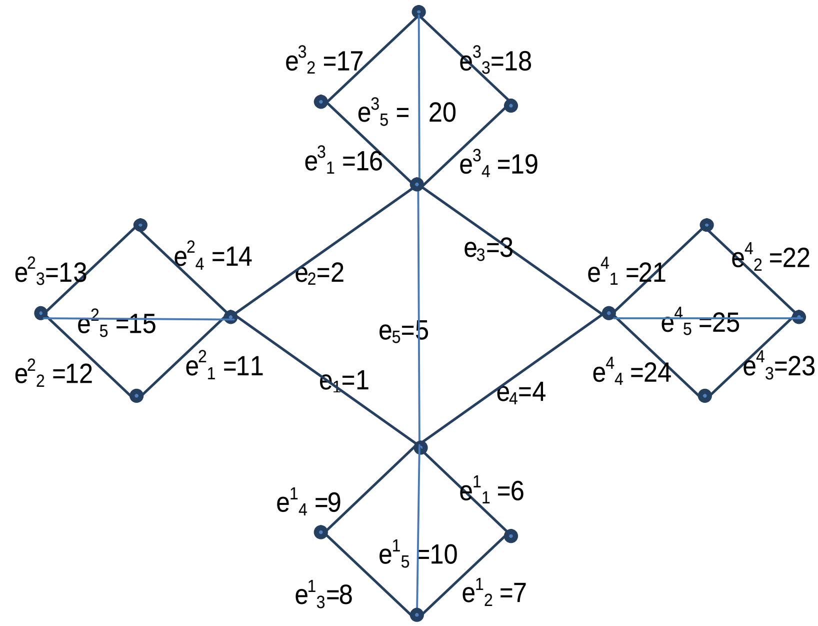

Fig. 6: Star of flag S_4: edge labels are shown

3.7 Theorem: Chain of S_4 is vertex prime.

Proof: On each Sn there iare two vertices each of degree 2.To obtain a chain on Sn of length 2 i.e.chain(Sn,2) , two copies of snare fused at degree 2 vertex on it.The process is repeated for (m-10 times to obtain G=chain(Sn,m).Note that V(G) = m|V(G)|-m+1 and E(G) =m. |E(G)|

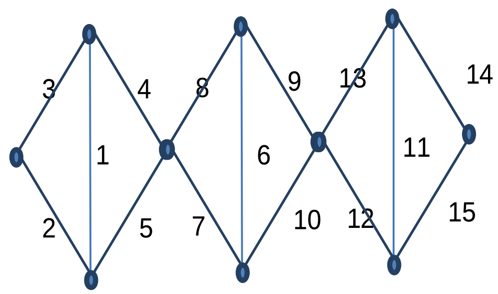

Fig 7: chain (S4,3) with vpl

G=Chain(Sn,4) is obtained as follows Take two sets of points u1,u2,..um and $v_1,v_2,\dots v_m$.Take Additional set of m+1 points given by $p_1,p_2,..p_m,p_{m+1}$and edges are given by E(G)= $\{(u_iv_i)/i= 1,2,..m\}$ $U\{u_ip_i)/ i = 1,2,m\}U\{(u_ip_{i+1})/i = 1,2,..m.\}U\{(v_ip_i)/i= 1,2,..m\}U\{(v_ip_{i+1})/i= 1,2,..m\}$

Define $f:E(G)\rightarrow\{1,2,\dots|E(G)|\}$ as follows:

$f(u_iv_i)=i+5(i-1)$

$f(v_ip_i)=2+5(i-1)$;

$f(u_ip_i)=3+5(i-1)$;

$f(u_ip_{i+1})=4+5(i-1)$;

$f(v_ip_{i+1})=5+5(i-1)$The resultant labeling is vertex prime.

4. FUTURE SCOPE

We have considered only a few types of graphs that can be obtained from Shel graph. Typical structure of shel provides a lot of scope for this. The star of Sn can be further extended to obtain 2-star,3-star..q-star. The star we have discussed above may be taken as 1-star shel. Having obtained 1-star we attach a shen at each vertex of the outer shel. That will give us 2-star.etc.

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