Original Article

Bulletin of Pure and Applied Sciences.
Vol. 36E (Math & Stat.), No.2, 2017.P.306-310
Print version ISSN 0970 6577
Online version ISSN 2320 3226
DOI: 10.5958/2320-3226.2017.00032.7

EVALUATION OF THE AVAILABILITY AND M.T.T.F. OF TWO UNIT STANDBY REDUNDANT COMPLEX SYSTEMS

Vivek Tyagi*, Viresh Sharma, Ajay Tyagi*****

Author Affiliation:
*Deptt. of Statistics, N.A.S. (P.G.) College, Meerut, Uttar Pradesh 250003
E-mail: avikrishna94@yahoo.co.in
******Deptt. of Mathematics, N.A.S. (P.G.) College, Meerut, Uttar Pradesh 250003
E-mail: vireshsharma1@yahoo.com
*******Research Scholar, Mewar University, Gangrar, Chittorgarh, Rajasthan 312901
E-mail: ajaytyagi2009@gmail.com

Corresponding Author:
Viresh Sharma, Deptt. of Mathematics, N.A.S. (P.G.) College, Meerut, Uttar Pradesh 250003

E-mail: vireshsharma1@yahoo.com

Received on 29.07.2017, **Accepted on** 21.11.2017

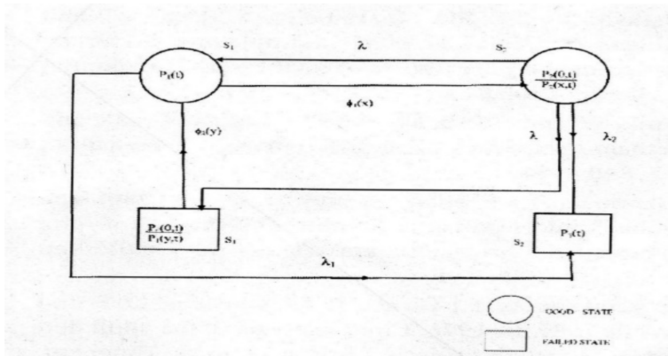
Abstract

The authors have considered a complex system composed of two units in parallel redundancy which has been studied to evaluate M.T.T.F. and availability. Supplementary variable technique has been utilised to derive various state probabilities and the operational availability. The repair facility is available in either one unit or the whole system fails. After failure of one unit, the system is undertaken for repair, of course, the other unit is in operative mode. If during the repair of the first unit, the second unit also fails then the system is assumed to go to down state.

Keywords: M.T.T.F., Supplementary variable, Repairable system, Redundancy, Availability.

1. INTRODUCTION

The authors have developed a model consisting of two units in parallel redundancy which has been studied to evaluate availability and M.T.T.F. under logical failures. It has been assumed that the system remains in four states during the operation stage. Initially both the units are good, while in state 2, only one unit is good, whereas, on the other hand states 3 and 4 are failed states. Logical failure can occur in states 1 and 2, which cause the system to go into the state 3 which is a failed state. Using the supplementary variable and Laplace transform technique, the time dependent probabilities with the help of Abel's theorem and MTTF of the complex system being in various states have been computed. The operator unit may fail partially or totally. In case of total failure of the first unit, stand by unit causes the operational system to work with full efficiency. It is assumed that whenever there is a hardware failure system goes to partial failure mode first and then to total failure while, due to common cause failure, the system fails totally. In case of partial failure of both units and common cause failure. The failure time distributions are exponential whereas repairs are general. Any unit may fail either partially or completely. The failure and repair follow exponential and general distributions, respectively. The repair facility is available when either one unit or whole system fails. After failure of one unit, when the system is undertaken for repairing, the other unit is in operational mode. If during the repair of the first unit the second unit also fails then the system is assumed to go to down state.



Transition of diagram

2. ASSUMPTIONS

(i) At time $t = 0$, the system is in good state.

(ii) The system has two states: good and failed.

(iii) Repair facility is available only when the system is either in position S_2 or in position S_4.

(iv) The failure and repair times for the system follow exponential and general distributions respectively.

(v) System is in failed state of all the units are failed.

3. NOTATIONS AND STATES

S_1 : good state and main unit is working

S_2 : good state and standby unit is working and under repair

S_3 : Failed State

S_4 : Failed State under repair

$D / Dt / Dx / Dy / Dz / Dw$:

$$\frac{d}{dt} / \frac{\partial}{\partial t} / \frac{\partial}{\partial x} / \frac{\partial}{\partial y} / \frac{\partial}{\partial z} / \frac{\partial}{\partial w}$$

λ : exponential constant failure rate for one unit from state 1 to 2 and state 2 to 4

1 : Constant failure from states 1 to 3

2 : Constant failure from states 2 to 4

$_1(x)$: transition rate of repair for one unit from states 2 to 1

$_2(x)$: transition rate of repair for one unit from states 4 to 1

$P(s)$: Laplace transform of the function F(t)

$P_1(t)$: P (at time t the system in state S_1)

$P_3(t)$: P (at time t the system in state S_3)

$P_2(x,t)$: P (the system is in state S_2 at time t due to minor failure and elapsed repair time lies in the interval (x, x^+)

$P_4(y,t)$: P (the system is in state S_4 at time t and elapsed repair time lies in the interval $(y, y^+ y)$.s

4. FORMULATION OF MATHEMATICAL MODELS

The set of difference differential equations associated with the mathematical model are as follows:

$$\left(\tfrac{d}{dt}+2\lambda+\lambda_1\right)P_1(t)=\int_0^\infty P_2(x,t)\phi_1(x)\,dx+\int_0^\infty P_4(y,t)\phi_2(y)\,dy \tag{1}$$

$$\left[\frac{d}{dx}+\frac{d}{dt}+\lambda_2+\phi_1(x)+\lambda\right]P_2(x,t)=0 \tag{2}$$

$$\frac{d}{dt}P_3(t)=\lambda_1P_1(t)+\lambda_2P_2(t) \tag{3}$$

$$\left[\frac{d}{dt}+\frac{d}{dy}+\phi_2(x)\right]P_4(y,t)=0 \tag{4}$$

The boundary conditions are as follows :

$$P_2(0,t)=\lambda P_1(t) \tag{5}$$

$$P_4(0,t)=\lambda P_2(t) \tag{6}$$

Initial conditions are as follows:

We assume that system is initially in a normal state S_1 i.e., $P_1(0)=1$, otherwise zero. (7)

Solution of the Model

Taking the Laplace-transform of the equations (1) to (6) then we get

$$[s+\lambda+\lambda_1]\bar{P}_1(s)=\int_0^\infty \bar{P}_1(x,s)\phi_1(x)\,dx+\int_0^\infty \bar{P}_2(x,s)\phi_2(t)\,dx \tag{8}$$

$$\left[s+\frac{\partial}{\partial x}+\lambda_2+\phi_1(x)+\lambda\right]\bar{P}_2(x,s)=0 \tag{9}$$

$$s\bar{P}_3(s)=\lambda_1\bar{P}_1(s)+\lambda_2\bar{P}_2(s) \tag{10}$$

$$\left[s+\frac{\partial}{\partial y}+\phi_2(y)+\lambda\right]\bar{P}_4(y,s)=0 \tag{11}$$

$$\bar{P}_2(0,s)-\lambda\bar{P}_1(s)+\lambda_2\bar{P}_1(s) \tag{12}$$

$$\bar{P}_4(0,s)=\lambda\bar{P}_2(s) \tag{13}$$

After solving the above equations (5) to (13), we get

$$\bar{P}_2(s)=\lambda Z_1(s+\lambda+\lambda_2)\bar{P}_1(s) \tag{14}$$

$$\bar{P}_4(s)=\lambda^2 Z_1(s+\lambda+\lambda_2)Z_2(s)\bar{P}_1(s) \tag{15}$$

where

$$Z_i(s)=\frac{1-S_1(s)}{s} \tag{16}$$

$$\bar{P}_1(s)=\frac{1}{B(s)} \tag{17}$$

and

$$B(s)=s+\lambda+\lambda_1-\lambda\bar{S}_1(s+\lambda+\lambda_2)-\lambda^2 Z_1(s+\lambda+\lambda_2)\bar{S}_2(s) \tag{18}$$

Now,

$$\bar{P}_2(s)=\lambda Z_1(s+\lambda+\lambda_2)\frac{1}{B(s)} \tag{19}$$

$$\bar{P}_3(s)=\left[\lambda_2+\lambda Z_1(s+\lambda+\lambda_2)+\lambda_1\right]\frac{1}{sB(s)} \tag{20}$$

Using above equations, we have

$$\ddddot{P}_4(s) = \lambda^2 Z_1(s+\lambda+\lambda_2) Z_2(s) \frac{1}{B(s)} \tag{21}$$

$$P_1(s) + P_2(s) + P_3(s) + P_4(s) = \frac{1}{s} \tag{22}$$

Up and Down State Probabilities

The system is in up state and down state at time t, then

$$\bar{P}_{up}(s) = \bar{P}_1(s) + \bar{P}_2(s)$$

$$= \left[1 + \lambda Z_1(s+\lambda+\lambda_2)\right] \frac{1}{B(s)}$$

$$\bar{P}_{down}(s) = 1 - \bar{P}_{up}(s)$$

Steady State Behaviour of the System

Using Abel's lemma, we get

$$\bar{P}_{up} = \left[1 + \lambda Z_1(\lambda+\lambda_2)\right] \frac{1}{B(s)}$$

where ϕ_1 and ϕ_2 are constant repair, we get

$$\bar{P}_1(s) = \frac{1}{B_1(s)}$$

where

$$B_1(s) = s + \lambda + \lambda_1 - \frac{\lambda\phi_1}{s+\lambda+\lambda_2+\phi_1} - \frac{\lambda^2\phi_2}{(s+\phi_2)(s+\lambda+\lambda_2+\phi_1)}$$

$$\bar{P}_2(s) = \frac{1}{B_1(s)} \cdot \frac{\lambda}{(s+\lambda+\lambda_3+\phi_1)}$$

$$\bar{P}_3(s) = \frac{1}{sB_1(s)} \cdot \frac{\left(\lambda_2 + \lambda + (s+\lambda+\lambda_2+\phi_1)(\lambda_1+\lambda_2)\right)}{(s+\lambda+\lambda_2+\phi_1)}$$

$$\bar{P}_4(s) = \frac{1}{sB_1(s)} \cdot \frac{\lambda^2}{(s+\lambda+\lambda_2+\phi_1)(s+\phi_2)}$$

$$\bar{P}_{up}(s) = \bar{P}_1(s) + \bar{P}_2(s)$$

$$= \frac{s+\lambda+\lambda_2+\phi_2}{(s+\lambda+\lambda_2+\phi_1)B_1(s)}$$

$$\bar{P}_1(s) = \frac{(s+1)(s+\lambda+\lambda_2+1)}{s^3 + s^2\alpha_1 + s\alpha_3 + \alpha_2} . (\phi_1 = \phi_2 = 1)$$

where

$$\alpha_1 = \lambda + \lambda_1 + \lambda_2 + 2$$

$$\alpha_2 = \lambda\lambda + \lambda\lambda_2 + \lambda_1\lambda_2 + \lambda_1$$

$$\alpha_3 = \lambda^2 - 1 + \alpha_3 + \alpha_2$$

5. NUMERICAL COMPUTATION

Consider an electronic two unit standby redundant system with various parameters as

We take $\lambda_1 = 0.04, \lambda_2 - 0.05$ and $\phi_1 = 1$ and $\phi_2 = 1$

$$P_{up}(t) = 0.989^{-0.0105t} + 0.256e^{-0.01951t} + 0.237e^{-0.091} \quad (23)$$

Mean time to system failure is given by

$$\text{M.T.T.F.} = \frac{\lambda + \lambda_2}{(\lambda + \lambda_2)(\lambda + \lambda_1)} \quad (24)$$

Now take to constant repair and failure rates then put $\phi_1 = 1$ and $\phi_2 = 1$ $\lambda_1 = 0.02$, $\lambda_2 = 0.03$, $\lambda = 0.02$, 0.04, 0.06, 0.08, 0.10, 0.12, 0.14, 0.16 in equation (24) we get table

Table 1:

S.NO.	Time	P up(t)	P down(t)
1	0	1	0
2	1	0.9892	0.0108
3	2	0.9884	0.0116
4	3	0.9791	0.0209
5	4	0.9659	0.0341
6	5	0.9431	0.0569
7	6	0.9582	0.0418
8	7	0.8956	0.1044
9	8	0.8756	0.1244

Table 2:

S.NO.	λ	M.T.T.F.
1	0.02	40.52
2	0.04	34.29
3	0.06	30.23
4	0.08	15.45
5	0.1	11.32
6	0.12	7.49
7	0.14	4.67
8	0.16	3.98
9	0.18	3.13

6. INTERPRETATION OF THE RESULT

Table-1computes the availability of the system at an instant t, the availability of the system at any time decreases slowly and tends to zero
Table-2 compute M.T.T.F. for different values of constant failure rate and M.T.T.F. decreases.

REFERENCES

1. Mohmoud, M,, Mokheles, M.A., Saleh E.H., Availability analysis of a repair system with common cause failures and one standby unit', Microelectron, Reliability Vol.27, pp 741-754 (1987).
2. Singh, J., Goyal,P. Availability analysis of a standby complex system having imperfect switch over device', Microelectron, Reliability Vol. 35, pp. 285-286 (1996).
3. Pandey, D., Jacob Mendus:'Cost analysis, Availability and MTTF of a 3-state stand by complex system under common cause and human failures', Microelectron Reliability Vol.35, pp. 91-95 (1995).
4. Gupta P.P., Kumar, S.K.:'Stochastic behaviour of three state complex repairable system with three types of failure' Microelectron, Reliability Vol.25, pp. 853-885 (1985).
5. Mine,H., Oski, Akura,T.:'Some consideration for multiple unit redundant system with generalized repair time distribution', IEEE Trans. On Reliability R-28, pp. 325-326 (1999).