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Optimized Time and Price Sensitive Fuzzy Inventory Model for Seasonal Deteriorating Products

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ABSTRACT

Under partial backlogging, this study develops an improved fuzzy inventory model for deteriorating seasonal products with time and price-sensitive demand. By adding triangular fuzzy numbers for deterioration rates, holding costs, and production costs, the new model enhances earlier approaches. To ensure practicality, the model is solved using the Graded Mean Integration Representation (GMIR) technique. The effect of changing system parameters on inventory performance is assessed through sensitivity analysis. The findings demonstrate that fuzzy techniques are effective in increasing profitability and making decisions in industries that produce deteriorating products.

Keywords: Fuzzy inventory model, deterioration rate, profit function

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1. INTRODUCTION

Perishable commodities like food, drugs, and crops have their inventory managed through complex modeling to avoid loss through degradation. The selling price and time-sensitive demand of these items call for the inclusion of fuzzy logic in dealing with uncertainty in the most critical parameters, which include degradation rate and carrying cost. The present work enhances previously proposed models through a more stringent fuzzy technique to enable greater accuracy in making decisions.

Existing inventory model research has been on deterministic and probabilistic models. Whitin (1957) developed deteriorating product models, then exponential decay models by Ghare and Schrader (1963). Modern developments have involved stock-dependent demand (Teng et al., 2005) and price-sensitive demand (Soni, 2013). Fuzzy models, developed by Zadeh (1965), have enhanced precision in dealing with uncertainty. Our research advances these works by combining fuzzy set theory with a seasonal deteriorating item inventory model having time- and price-varying demand.

2 MODEL FORMULATION

2.1 Assumptions

1. A single seasonal product is considered.
2. The demand depends on both selling price and time .
3. Partly backlogged shortages.
4. Production cost, holding cost, and deterioration rate are all triangular fuzzy numbers.
5. The planning horizon is limitless.

2.2 Notations

- $I(t)$: level of Inventory at time t
- $D(p,t)$: Demand function dependent on price (p) and time (t)
- θ: Fuzzy deterioration rate
- h: Fuzzy holding cost
- R: Production rate
- S: Setup cost
- T: Time cycle
- p: Selling price
- h: Holding cost
- θ: Deterioration rate
- S: setup cost.

2.3 DIFFERENTIAL EQUATIONS FOR INVENTORY DYNAMICS

During production ($0 \le t \le \beta$):

$$\frac{dI(t)}{dt} = R - \theta I(t) - D(p,t) \tag{1}$$

During depletion ($\beta \le t \le T$):

$$\frac{dI(t)}{dt} = -\theta I(t) - D(p,t) \tag{2}$$

For the production phase ($0 \le t \le \beta$):

$$I(t) = e^{-\theta t}\left(\int (R - D(p,t))e^{\theta t}dt + C\right) \tag{3}$$

where C is the integration constant.

For the depletion phase ($\beta \le t \le T$):

$$I(t) = e^{-\theta t}\left(-\int D(p,t)e^{\theta t}dt + C'\right) \tag{4}$$

where C' is another integration constant.

3. PROFIT FUNCTION

The total profit function consists of the following components:

3.1 Revenue (R_v)

$$R_v = p \times D(p,T) \tag{5}$$

3.2 Holding Cost (C_h)

$$C_h = h\int_0^T I(t)dt \tag{6}$$

3.3 Deterioration Cost (C_d)

$$C_d = \theta\int_0^T I(t)dt \tag{7}$$

3.4 Setup Cost (C_s)

$$C_s = \frac{S}{T} \tag{8}$$

3.5 Production Cost (C_p)

$$C_p = C_{unit} \times R \tag{9}$$

where C_{unit} is the unit production cost, and R is the replenishment rate.

3.6 TOTAL PROFIT CALCULATION

The formula for the overall average profit per unit of time is:

$$M(T,p) = \frac{R_p-(C_h+C_d+C_s+C_p)}{T} \tag{10}$$

Expanding the terms:

$$M(T,p) = \frac{p\times D(p,T)-\left[h\int_0^T I(t)dt+\theta\int_0^T I(t)dt+\frac{S}{T}+C_{unit}R\right]}{T} \tag{11}$$

Using the inverntory differential equations and integrating $I(t)$, we approximate:

$$\int_0^T I(t)dt \approx \frac{R-D(p,T)}{\theta} \tag{12}$$

Substituting this into the profit function:

$$M(T,p) = \frac{p\times D(p,T)-(h+\theta)\frac{R-D(p,T)}{\theta}-\frac{S}{T}-C_{unit}R}{T} \tag{13}$$

4. SOLUTION APPROACH

Applying the GMIR method, the fuzzy parameters are defuzzified to obtain a crisp equivalent model. The optimal values of T and p are determined by solving the system of equations:

$$\frac{\partial M(T,p)}{\partial T} = 0, \quad \frac{\partial M(T,p)}{\partial p} = 0 \tag{14}$$

where $M(T,p)$ represents the total average profit function.

5. NUMERICAL EXAMPLE

Given:

- $\tilde{\theta} = (0.0003, 0.0004, 0.0005)$, $\tilde{h} = (0.002, 0.004, 0.006)$
- $p = 250$, $\beta = 0.6$, $S = 300$, $\delta = 0.5$

Solving using GMIR, we obtain:

- Optimal $T = 1.58$ days
- Optimal $p = 241.3$

- Maximum profit = 627.3

6. SENSITIVITY ANALYSIS

The impact of varying deterioration rates and holding costs on profit was analyzed. Results indicate:

1. As the deterioration rate increases, the inventory availability decreases, leading to higher selling prices and increased profit.
2. The overall profit is not much impacted by holding expenses.

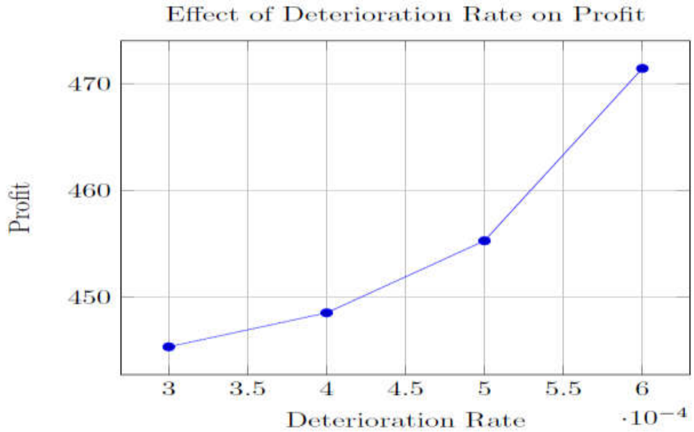

Figure 1: Effect of Deterioration Rate on Profit

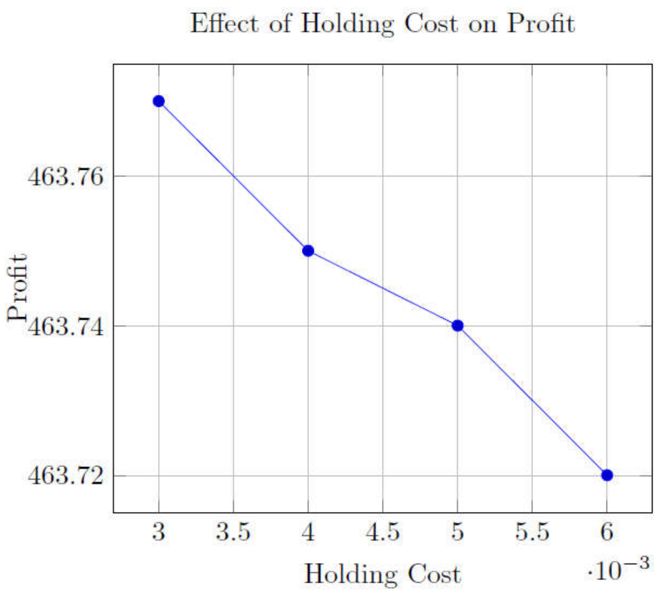

Figure 2: Effect of Holding Cost on Profit

7. CONCLUSION

With regard to seasonal deteriorating items with time- and price-sensitive demand, this study develops a fuzzy optimal inventory model. The GMIR technique effectively addresses parameter uncertainty, enhancing inventory decision-making. For next research, environmental factors like supply chain disruption and carbon footprint could be included in the fuzzy inventory model.

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